JOINT POWER ALLOCATION BASED ON LINK RELIABILITY FOR MIMO SYSTEMS ASSISTED BY RELAY

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Abstract: A new optimization criterion is proposed to minimize error probability for the proposed joint optimal power allocation (PA) of the MIMO systems enhanced by relay in this paper. It is proved that the cost function obtained is only convex with respect to (w.r.t.) the power parameters of the source or those of the relay separately, but not convex w.r.t. the whole parameters. In order to use convex optimization methods with high efficiency to solve this complicated problem, a tight upper bound of the sum MSE (mean squared error) is derived, and employed to modify the cost function in order to obtain a convex problem. It is verified through simulation results that the proposed PA scheme outperforms the existing one.
Keywords: MIMO, relay, power allocation, convex optimization.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) technology has been developed to improve the spectral efficiency and link reliability of wireless communication systems by exploiting the spatial multiplexing gain, spatial diversity and array gain [1]. Unfortunately, it becomes difficult to apply MIMO with the antennas' number increasing due to its requirement on array size. Wireless relay is proved to be a promising technique to increase the coverage of cells and to combat shadowing effects [2]. Traditionally, three typical relay strategies, namely, the amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF) strategies, have been proposed for relay wireless systems [3]-[5]. Among the three, AF is the most commonly used strategy due to its low complexity and low power consumption. Meanwhile, MIMO enhanced by relay is becoming a hot research topic considering that the performance of MIMO systems could be further enhanced by employing relay links in [6]. It should be mentioned that these results concerning the channel capacity have been obtained from the information theory perspective.

However, power allocation (PA) is of crucial importance in AF because of the noise amplification induced by the relay. Many power allocation schemes have been proposed for multi-hop single-input single-output (SISO) relay networks [7][8]. It has been known that the performance of multi-hop systems can be improved significantly by using PA. However, only a few papers dealing with PA in MIMO systems enhanced by relay can be found in existing literatures [9][10]. In [9], a non-regenerative MIMO relay was designed by maximizing the instantaneous channel capacity, leading to a number of parallel SISO subchannels and a waterfilling PA scheme for the subchannels. In this method, as the PA was considered for relay solely, rather than the source and relay jointly, the channel capacity of the entire MIMO relay networks has not been maximized.

In this paper, we consider the joint PA problem of a MIMO system enhanced by relay with one source, one relay and one destination. Our objective is to develop a joint PA scheme for the source and relay such that the sum MSE of the overall system is minimized.

The following notations are adopted. $(\cdot)^H$ denotes Hermitian transpose; $z \sim \mathcal{CN}(\boldsymbol{\mu}, \Sigma)$ indicates that z is a circularly symmetric complex Gaussian random vector with mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$; $\boldsymbol{I}_n$ denotes the $n \times n$ identity matrix; $\mathbb{C}^{l \times m}$ the set of complex matrices with l rows and m columns. $rank(\boldsymbol{A})$ the rank of the matrix $\boldsymbol{A}$; $\mathbf{0} \le \boldsymbol{a} \le \mathbf{1}$ means $0 \le a_i \le 1,\ i = 1, 2, \cdots, K$; $[x]^+$ means $\max\{x, 0\}$.

2. SYSTEM MODEL

In this paper, we consider a MIMO system enhanced by relay with one source, one relay and one destination, each equipped with multiple antennas. It is assumed that the AF strategy is used in the relay [4]. In the first time slot, the source transmits data stream to the relay, while in the second slot, the relay processes and re-transmits the signal

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to the destination. It is also assumed that there is no direct transmission from the source to the destination all the time.

Given an input signal $\boldsymbol{s}$, the output signal of the relay system can be expressed as

$$\begin{aligned}\boldsymbol{y} &= \boldsymbol{W}_2\boldsymbol{H}_2\boldsymbol{y}_R + \boldsymbol{n}_2 \\ &= \underbrace{\boldsymbol{W}_2\boldsymbol{H}_2\boldsymbol{W}_R\boldsymbol{H}_1\boldsymbol{W}_1}_{\boldsymbol{H}_e}\boldsymbol{s} + \underbrace{\boldsymbol{W}_2\boldsymbol{H}_2\boldsymbol{W}_R\boldsymbol{n}_1 + \boldsymbol{W}_2\boldsymbol{n}_2}_{noise}\end{aligned} \tag{1}$$

where $\boldsymbol{H}_e$ is the equivalent channel , the second term is the additive noise of the overall channel. $\boldsymbol{W}_1$, $\boldsymbol{W}_R$ and $\boldsymbol{W}_2$ represent, respectively, the weighting matrices for the source, relay and the destination, and $\boldsymbol{H}_1$ denotes the MIMO channel for the first hop and $\boldsymbol{H}_2$ for the second hop. Let M, L, N be, respectively, the number of antennas of the source, that of the relay, and that of the destination. The noises of the two hop channels are assumed to be AWGN as described by $\boldsymbol{n}_1 \sim \mathcal{CN}\left(0,\sigma_1^2\boldsymbol{I}_L\right)$ and $\boldsymbol{n}_2 \sim \mathcal{CN}\left(0,\sigma_2^2\boldsymbol{I}_N\right)$.

A uniform PA has been assumed for the source in [9]. We construct the weighting matrices $\boldsymbol{W}_1$, $\boldsymbol{W}_R$ and $\boldsymbol{W}_2$ by using the SVDs of $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$.

Performing the SVD of $\boldsymbol{H}_i$ $(i=1,2)$ gives

$$\boldsymbol{H}_i = \boldsymbol{U}_i\Lambda_i\boldsymbol{V}_i^H, i=1,2 \tag{2}$$

where $\boldsymbol{U}_i$ and $\boldsymbol{V}_i$ $(i=1,2)$ are unitary matrices and Λ_i is a diagonal matrix composed of the singular values of $\boldsymbol{H}_i$ $(i=1,2)$. Here, we assume that $\Lambda_1 = \mathrm{diag}\left\{\sqrt{\alpha_1}, \sqrt{\alpha_2}, \cdots, \sqrt{\alpha_{rank(\boldsymbol{H}_1)}}\right\}$ with $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_{rank(\boldsymbol{H}_1)} > 0$ and $\Lambda_2 = \mathrm{diag}\left\{\sqrt{\beta_1}, \sqrt{\beta_2}, \cdots, \sqrt{\beta_{rank(\boldsymbol{H}_2)}}\right\}$ with $\beta_1 \geq \beta_2 \geq \cdots \geq \beta_{rank(\boldsymbol{H}_2)} > 0$. We now propose to construct $\boldsymbol{W}_1$ and $\boldsymbol{W}_R$ as follows

$$\boldsymbol{W}_1 = \sqrt{(1-\tau)P_o}\boldsymbol{V}_1\boldsymbol{A} \tag{3}$$

$$\boldsymbol{W}_R = \sqrt{\tau P_o}\boldsymbol{V}_2\boldsymbol{B}\boldsymbol{U}_1^H \tag{4}$$

where P_o is the total power of the system, τ is the ratio of the power consumed by the relay to the total power P_o, and $\boldsymbol{A}$ and $\boldsymbol{B}$ are two diagonal matrices to be designed. Without loss of generality, we assume that $\boldsymbol{A} \triangleq \mathrm{diag}\left\{\left(\sqrt{a_1}, \sqrt{a_2}, \cdots, \sqrt{a_M}\right)^{\mathrm{T}}\right\}$ and $\boldsymbol{B} \triangleq \mathrm{diag}\left\{\left(\sqrt{b_1}, \sqrt{b_2}, \cdots, \sqrt{b_L}\right)^T\right\}$. The diagonal elements of $\boldsymbol{A}$ and $\boldsymbol{B}$ represent, respectively, the power weighting coefficients for the antennas of the source and those of the relay.

In our scheme, $\boldsymbol{W}_2$ is simply chosen as

$$\boldsymbol{W}_2 = \boldsymbol{U}_2^H \tag{5}$$

When the parameter τ and the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are determined, the output signal $\boldsymbol{y}$ can be expressed as

$$\boldsymbol{y} = \sqrt{P_o^2\tau(1-\tau)} \cdot \Lambda_2\boldsymbol{B}\Lambda_1\boldsymbol{A}\boldsymbol{s} + \left[\sqrt{P_o\tau}\Lambda_2\boldsymbol{B}\boldsymbol{U}_1^H\boldsymbol{n}_1 + \boldsymbol{U}_2^H\boldsymbol{n}_2\right] \tag{6}$$

which can be viewed as the output of $K = \min\left\{rank\left(\boldsymbol{H}_1\right), rank\left(\boldsymbol{H}_2\right)\right\}$ parallel SISO subchannels.

In this paper, we consider a joint optimal PA for the source and the relay by utilizing the sum MSE as the criterion. As will be shown, the difficulty of such a joint optimization problem lies in its nonconvexity. To overcome this difficulty, a tight upper bound of the sum MSE will be derived and utilized to formulate a convex problem.

3. FORMULATION OF JOINT POWER ALLOCATION PROBLEM

In this section, we first derive the SNR expression of each subchannel in the equivalent SISO system. Then, by using the derived SNR along with the criterion of sum MSE minimization, a joint PA optimization problem is formulated. It is then shown that the cost function based on the sum MSE is not convex w.r.t. the complete parameter set, yet it is convex w.r.t. part of the parameters.

A. SNR derivation for each subchannel

First of all, the source power should be limited and normalized, which imposes the following constraint on the weighting coefficients a_i

$$\sum_{i=1}^{M} a_i \leq 1,\ a_i \geq 0,\ \left(i=1,2,\cdots,M\right) \tag{7}$$

Considering that the transmit power by the ith antenna in the source is given by $P_o(1-\tau)a_i$, the power received by ith antenna in the relay can be expressed as $\left[P_o(1-\tau)a_i\right]\alpha_i + \sigma_1^2$. The total power of the relay is bounded by $P_o\tau$, thus, we can write the retransmit power of each antenna in the relay as

$$p_i = P_o\tau b_i \tag{8}$$

Thus, we have the following constraint on b_i as

$$\sum_{i=1}^{L} b_i \leq 1,\ b_i \geq 0,\ \left(i=1,2,\cdots,L\right) \tag{9}$$

Therefore, the power gain from each antenna in the relay is given by

$$r_i = \frac{p_i}{P_o(1-\tau)a_i\alpha_i + \sigma_1^2},\ \left(i=1,2,\cdots,L\right) \tag{10}$$

The signal power at the destination is given by

$$\left[P_o(1-\tau)a_i\right]\alpha_i r_i \beta_i \tag{11}$$

Substituting (10) in (11) gives

$$\frac{P_o(1-\tau)a_i\alpha_i\beta_i}{\sigma_1^2 + P_o(1-\tau)a_i\alpha_i}p_i \tag{12}$$

The total noise power at the destination can be given by

$$\sigma_1^2\left(r_i\beta_i\right) + \sigma_2^2 = \sigma_1^2\beta_i\frac{p_i}{\sigma_1^2 + P_o(1-\tau)a_i\alpha_i} + \sigma_2^2 \tag{13}$$

Using (12) and (13), the SNR at the destination can be expressed as

$$SNR_i = \frac{P_o^2 \tau (1-\tau)\alpha_i \beta_i a_i b_i}{\sigma_2^2 P_o (1-\tau)\alpha_i a_i + \sigma_1^2 (\sigma_2^2 + P_o \tau \beta_i b_i)}, \quad (\text{i}=1,2,\cdots, K) \tag{14}$$

Note that τ, a_i and b_i in (14) should satisfy the constraints in (7) and (9), and moreover, we have $a_{K+1} = \cdots = a_M = 0, b_{K+1} = \cdots = b_L = 0$ due to the fact that $K \le \min\{M, L\}$. As usual, one can assume that $\sigma_1^2 = \sigma_2^2 = 1$ without loss of generality. Then, the SNR expression with the desired power constraints can be given by

$$SNR_i = \frac{P_o^2 \tau (1-\tau)\alpha_i \beta_i a_i b_i}{1 + P_o (1-\tau)\alpha_i a_i + P_o \tau \beta_i b}, \quad (i=1,2,\cdots,K) \tag{15}$$
$$\text{s.t. } 0 \le \tau \le 1,\ \boldsymbol{a}^T \mathbf{1} \le 1,\ \boldsymbol{a} \ge \mathbf{0},\ \boldsymbol{b}^T \mathbf{1} \le 1, \text{ and } \boldsymbol{b} \ge \mathbf{0}$$

In (15), $\boldsymbol{a} = [a_1, a_2, \cdots, a_K]^T$, $\boldsymbol{b} = [b_1, b_2, \cdots, b_K]^T$, and $\mathbf{0}$ and $\mathbf{1}$ denote, respectively, the zero and the one vectors. The notations $\boldsymbol{a} \ge \mathbf{0}$ and $\boldsymbol{b} \ge \mathbf{0}$ mean that each element of $\boldsymbol{a}$ and $\boldsymbol{b}$ is greater than or equal to 0.

B. Power allocation using sum MSE (PA-E)

The transmission reliability is often pursued in communication systems. In [12], the MSE of the ith subchannel of a MIMO system is measured using the link SNR_i as

$$MSE_i = \frac{1}{1 + SNR_i}, \quad (i=1,2,\cdots,K) \tag{16}$$

based on which a so-called sum MSE can be defined based as

$$g = \sum_{i=1}^{K} MSE_i = \sum_{i=1}^{K} \frac{1}{1 + SNR_i} \tag{17}$$

This sum MSE has then been used for a joint design of the transmitter and receiver of MIMO systems in [12][13].

Using (15) along with (17), we can obtain the corresponding optimization problem as given below,

$$\min_{\tau,\boldsymbol{a},\boldsymbol{b}} \quad g(\boldsymbol{a},\boldsymbol{b},\tau) = \sum_{i=1}^{K} \frac{1 + P_o(1-\tau)\alpha_i a_i + P_o \tau \beta_i b_i}{1 + P_o(1-\tau)\alpha_i a_i + P_o \tau \beta_i b_i + P_o^2 \tau (1-\tau) \alpha_i \beta_i a_i b_i} \tag{18}$$
$$\text{s.t.} \quad 0 \le \tau \le 1,\ \boldsymbol{a}^T \mathbf{1} \le 1,\ \boldsymbol{a} \ge \mathbf{0},\ \boldsymbol{b}^T \mathbf{1} \le 1, \text{ and } \boldsymbol{b} \ge \mathbf{0}$$

By analyzing the cost function in (18), one can find that $g(\boldsymbol{a},\boldsymbol{b},\tau)$ is in general not convex w.r.t. the complete parameter set $(\boldsymbol{a};\boldsymbol{b};\tau)$, but it can be convex w.r.t. part of the parameters. The analysis result is briefed in the following theorem.

Theorem 1: The cost function $g(\boldsymbol{a},\boldsymbol{b},\tau)$ is *(i)* convex w.r.t. $\boldsymbol{a}$ for a fixed $(\boldsymbol{b};\tau)$; *(ii)* convex w.r.t. $\boldsymbol{b}$ for a fixed $(\boldsymbol{a};\tau)$; *(iii)* convex w.r.t. τ for a fixed $(\boldsymbol{a};\boldsymbol{b})$; and *(iv)* not convex w.r.t. $(\boldsymbol{a};\boldsymbol{b})$, or $(\boldsymbol{a};\boldsymbol{b};\tau)$. (Proof is absent for the paper space.)

Theorem 1 indicates that the problem in (18) cannot be solved by a convex optimization method.

4. JOINT POWER ALLOCATION

We now modify the cost function in (18) to obtain a convex optimization problem for the joint PA. As $1 + SNR_i$ has appeared in the sum MSE expression, we would like to investigate its lower and upper bounds. Let us consider $MSE_i = \frac{1}{1+SNR_i}$. Using (15) and (16), we have

$$\begin{aligned} MSE_i &= \frac{1}{1 + P_o(1-\tau)\alpha_i a_i} + \frac{1}{1 + P_o \tau \beta_i b_i} - \frac{1}{\left[1 + P_o(1-\tau)\alpha_i a_i\right](1 + P_o \tau \beta_i b_i)} \\ &\le \frac{1}{1 + P_o(1-\tau)\alpha_i a_i} + \frac{1}{1 + P_o \tau \beta_i b_i} \end{aligned} \tag{19}$$

On the other hand, noting that $\left[1 + P_o(1-\tau)\alpha_i a_i\right](1 + P_o \tau \beta_i b_i) \ge 1$ due to $\alpha_i > 0$, $\beta_i > 0$ and $0 \le \tau \le 1$, from the third equation of (19), one can have the following inequality

$$MSE_i \ge \frac{1}{1 + P_o(1-\tau)\alpha_i a_i} + \frac{1}{1 + P_o \tau \beta_i b_i} - 1 \tag{20}$$

The two inequalities in (19) and (20) can be combined as

$$\mu_i - 1 \le MSE_i \le \mu_i \tag{21}$$

where

$$\mu_i = \frac{1}{1 + P_o(1-\tau)\alpha_i a_i} + \frac{1}{1 + P_o \tau \beta_i b_i} \tag{22}$$

Clearly, (22) gives two bounds, $\mu_i - 1$ and μ_i, within which the MSE_i is limited. By using either of the two bounds to replace the expression of $1 + SNR_i$ in (18), one can obtain modified the objective function for the joint PA problem.

Let us use the upper-bound μ_i as given by (22) into (18), leading to a modified optimization problem as the joint power allocation based on the sum MSE minimization (JPA-E)

$$\min_{\tau,\boldsymbol{a},\boldsymbol{b}} \quad g_{ub}(\boldsymbol{a},\boldsymbol{b},\tau) = \sum_{i=1}^{K} \left(\frac{1}{1 + P_o(1-\tau)\alpha_i a_i} + \frac{1}{1 + P_o \tau \beta_i b_i} \right) \tag{23}$$
$$\text{s.t.} \quad 0 \le \tau \le 1,\ \boldsymbol{a}^T \mathbf{1} \le 1,\ \boldsymbol{a} \ge \mathbf{0},\ \boldsymbol{b}^T \mathbf{1} \le 1, \text{ and } \boldsymbol{b} \ge \mathbf{0}$$

Similarly, we have the following theorem to guarantee the convexity of $g_{ub}(\boldsymbol{a},\boldsymbol{b},\tau)$ w.r.t. $(\boldsymbol{a};\boldsymbol{b})$.

Theorem 2: The cost function $g_{ub}(\boldsymbol{a},\boldsymbol{b},\tau)$ is *(i)* convex w.r.t. $(\boldsymbol{a};\boldsymbol{b})$ for a fixed $\tau \in [0,1]$; *(ii)* convex w.r.t. τ in $[0,1]$ for a fixed $(\boldsymbol{a};\boldsymbol{b})$; and *(iii)* not convex w.r.t. $(\boldsymbol{a};\boldsymbol{b};\tau)$. (Proof is absent for the paper space.)

5. SIMULATION RESULTS

In this section, we present some of the simulation results in terms of the sum MSE obtained from the proposed methods with comparison to the method in [9]. The source, relay and destination have the same number of antennas, i.e. $M = L = N = 4$. The normalized SNR for the source and the relay is defined as $\rho_1 = \frac{P_o(1-\tau)}{\sigma_1^2 M}$ and $\rho_2 = \frac{P_o \tau}{\sigma_2^2 L}$, respectively, with $\sigma_1^2 = \sigma_2^2 = 1$.

Fig.1 shows the sum MSE plots achieved by our proposed JPA-E scheme as well as the method in [9]. The sum MSE is treated as a function of ρ_1 with the fixed $\rho_2 = 10\text{dB}$. It is not surprising that the proposed JPA-E outperforms the existing method as a result of its using the sum MSE as the PA cost function. It is observed from Fig.1 that the method [9] fails to give a satisfactory the sum MSE when ρ_1 is larger than $\rho_2 = 10\text{dB}$. Fig.2 presents the MSE versus SNR. From Fig.2, we can conclude that the proposed JPA-E outperforms the scheme in [9] regardless of the high or low SNR regime.

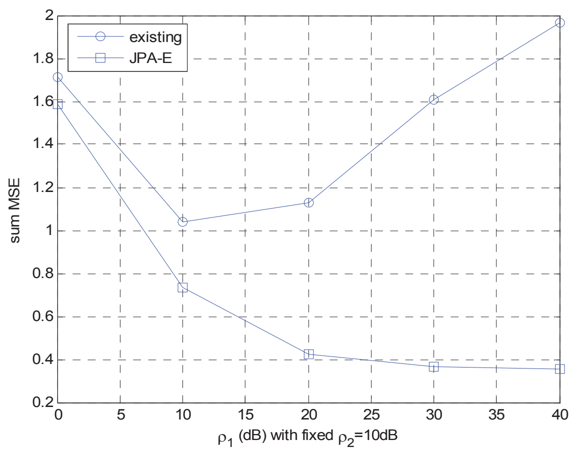

Fig. 1 Sum MSE versus ρ_1 with fixed $\rho_2 = 10\text{dB}$.

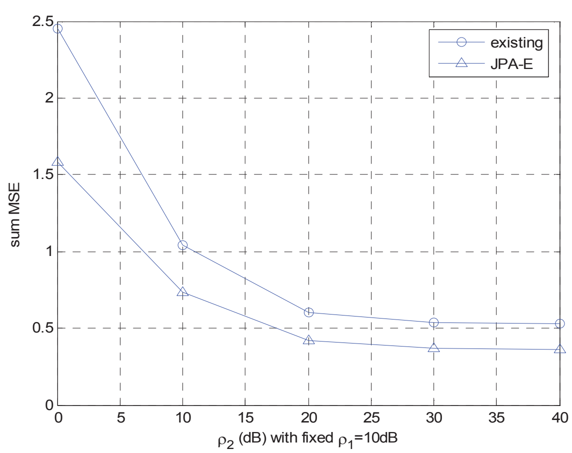

Fig. 2 Sum MSE versus ρ_2 with fixed $\rho_1 = 10\text{dB}$.

6. CONCLUSION

In this paper, we have investigated the joint PA issue in MIMO systems enhanced by relay. By using the sum MSE as the optimization criterion, a joint PA optimization problem has been formulated. The key contribution of the proposed method lies in the discovery of a tight bound for the sum MSE that simplifies the joint source and relay power allocation into a convex problem. A distinct feature of the new method is that the power allocation within the source and that within the relay are jointly optimal for any given power ratio of the two units.

7. REFERENCES

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