OUTAGE MINIMIZED RELAY SELECTION WITH PARTIAL CHANNEL INFORMATION

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ABSTRACT

Selection relaying is a promising technique for practical implementation of cooperative systems with multiple relays. However, to select the best relay, globe channel knowledge is required at the selecting entity. In this paper, we consider the relay selection problem in dual-hop amplify-and-forward (AF) communication systems with partial channel state information (CSI). We present relay selection strategies aiming at minimizing the outage probability with different kinds of channel knowledge available, including perfect, statistical and quantized CSI. Simulation results show that near optimal performance is achievable with a few bits feedback to the selecting entity. Thus the signaling overhead of relay selection can be greatly reduced with quantized CSI feedback.

Index Terms— Wireless relay networks, amplify-and-forward, relay selection, partial channel information.

1. INTRODUCTION

Wireless relay networks have attracted a lot of research attention in recent years [1][2][3]. The benefits of relayed transmission include increasing the system capacity, reducing the transmit power, and extending the cover range. Generally, there are two kinds of relaying schemes, amplify-and-forward (AF) relaying and decode-and-forward (DF) relaying. The AF relaying protocol is of more interest due to its low complexity.

In an AF system with multiple relays, several relaying protocols have been proposed in the literature, including repetition-based relaying [4], distributed space-time coding (DSTC) [5], and selection relaying [6]. For the repetition-based protocol, the relays forward the received signal in orthogonal channels (e.g., time slots), which leads to low bandwidth efficiency. In [5], the authors proposed DSTC relaying where linear dispersion space-time code was applied to wireless relay networks to enhance the system performance. However, DSTC requires perfect time synchronization among the relays, which is a crucial issue for practical systems. A selection relaying scheme was introduced in [6][7][8], where only one of the relays is selected for signal forwarding according to the instantaneous CSI of all the channels. It is shown in [6] and [7] that the selection relaying scheme achieves lower outage probability and error probability than the repetition-based orthogonal relaying. In [8], the authors proposed a distributed relay selection scheme for dual-hop AF systems. However, such distributed scheme requires synchronization among all the relays, and is difficult to realize in practical systems. On the other hand, centralized scheme in [6] requires globe knowledge of all the channels to select the best relay, which may result in considerable protocol overhead. In [9], the authors considered the relay selection problem in AF systems when only the instantaneous source-relay channel information is available at the selecting entity. However, simulation results show that the achievable diversity order is limited to be one regardless of the number of relays. Relay selection with partial CSI has also been investigated in [10] for dual-hop DF systems.

In this paper, we study the relay selection problem in dual-hop AF systems with partial channel state information. We evaluate the cumulative distribution function (CDF) of the end-to-end signal-to-noise ratio (SNR) of the AF system with different kinds of CSI available, including perfect, statistical and quantized CSI. Based on these CDFs, optimal relay selection schemes are presented to minimize the outage probability. In case of when quantized CSI is available, we propose a target rate based quantizer to partition the SNR range. It is shown that with a few bits feedback, the performance of the AF system with quantized CSI is almost the same as that with perfect CSI.

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2. SYSTEM MODEL

We consider a dual-hop system where the source communicates with the destination with the help of *K* relays. For simplicity, we assume there is no direct link between the source and the destination. All the nodes are half-duplex and can not transmit and receive simultaneously. The relayed transmission is performed in two time slots as in [6]. In the first slot, the source broadcasts the data to the *K* relays. In the second time slot, one of these *K* relays is selected for forwarding the received signal to the destination. We assume the relay selection is done at the source, which is possible in the downlink transmission, where the source is the base station (BS) and the destination is the mobile station (MS). In order to keep the mobile station as simple as possible, the BS determines which relay will be active in the second time slot.

If the k^{th} relay is selected for transmission, the end-to-end SNR of the dual-hop AF system is given by [6]

$$\gamma_k = \frac{\gamma_{s,k}\gamma_{k,d}}{\gamma_{s,k} + \gamma_{k,d} + 1} \tag{1}$$

where $\gamma_{s,k}$ is the SNR of the channel between the source and the k^{th} relay, and $\gamma_{k,d}$ is the SNR of the channel between the k^{th} relay and the destination. All the channels are assumed to be block flat-fading and Rayleigh distributed. The probability density function (PDF) and the CDF of $\gamma_{i,j}, i, j = s, d, 1, ..., K$, are given by

$$f_{\gamma_{i,j}}(\gamma) = (1/\bar{\gamma}_{i,j})\exp(-\gamma/\bar{\gamma}_{i,j}) \tag{2}$$

and

$$F_{\gamma_{i,j}}(\gamma) = 1 - \exp(-\gamma/\bar{\gamma}_{i,j}), \tag{3}$$

respectively, where $\bar{\gamma}_{i,j}$ denotes the average SNR of the channel between node *i* and node *j*.

3. RELAY SELECTION WITH PARTIAL CSI

In this section, we study the relay selection strategy to minimize the outage probability. When the k^{th} relay is selected, the mutual information is given by

$$C_k = \frac{1}{2}\log_2(1+\gamma_k) \tag{4}$$

The probability that the mutual information falls below the target rate *R* when the k^{th} relay is selected is given by

$$P_{out}(C_k < R) = P(\gamma_k < 2^{2R} - 1) \tag{5}$$

To minimize the outage probability, we have to chose the relay for which the outage probability conditioned on the available CSI at the selecting entity (the source) is minimized

$$\begin{aligned} k_{opt} &= \arg\min_k P_{out}(C_k < R|\text{CSI}) \\ &= \arg\min_k F_{\gamma_k}(\gamma_{th}|\text{CSI}) \end{aligned} \tag{6}$$

where $\gamma_{th} = 2^{2R}-1$, and $F_{\gamma_k}(\gamma|\text{CSI})$ is the CDF of γ_k conditioned on the available CSI at the source. If perfect CSI of all the channels is available, i.e., $\gamma_{s,k}$ and $\gamma_{k,d}$ are perfect known by the source, it is apparent that select the relay with minimal outage probability is equivalent to select the relay with the maximal SNR

$$k_{opt} = \arg\max_k \gamma_k \tag{7}$$

3.1. Selection with Perfect and Statistical CSI

In practical systems, it is unrealistic for the source to have perfect knowledge of all the channels, especially those between the relays and the destination. In this subsection, we consider the case that the channels between the source and the relays are perfect known , while only statistical knowledge of the relay-destination channels is available at the source.

In this case, $\gamma_{s,k}$, and $\bar{\gamma}_{k,d}, k = 1, 2, ..., K$, are known by the source. It is easy to show that the CDF of γ_k conditioned on $\gamma_{s,k}$ and $\bar{\gamma}_{k,d}$ is given by

$$F_{\gamma_k}(\gamma|\gamma_{s,k}, \bar{\gamma}_{k,d}) = \begin{cases} 1 - \exp\left(-\frac{\gamma(\gamma_{s,k}+1)}{\bar{\gamma}_{k,d}(\gamma_{s,k}-\gamma)}\right) & \gamma < \gamma_{s,k} \\ 1 \quad \gamma \geq \gamma_{s,k} \end{cases} \tag{8}$$

The optimal relay that with the minimal expected outage probability can be determined by substituting (8) into (6)

$$k_{opt} = \arg\min_k F_{\gamma_k}(\gamma_{th}|\gamma_{s,k}, \bar{\gamma}_{k,d}) \tag{9}$$

3.2. Selection with Perfect and Quantized CSI

In the previous section, we assumed that only the statistical CSI of the relay-destination channels is available. In such case, only very low-rate feedback is required, which can greatly reduce the signaling overhead. However, as we will show in Section V, severe performance loss is observed in this case. As a tradeoff between signaling overhead and system performance, we here consider another case that the source-relay channels are perfectly known whereas quantized and statistical CSI are available for the relay-destination channels.

Without loss of generality, we assume that a *N*-bit quantizer with 2^N quantization levels is employed for each relay-destination channel. The whole SNR range is subdivided into 2^N sections, and the index of the section that contains the instantaneous SNR value is feedback to the source.

Let the section containing $\gamma_{k,d}$ be $\Lambda_k = [\xi_{k,q}, \xi_{k,q+1})$ for the channel between the k^{th} relay and the destination, where $\xi_{k,q}$ and $\xi_{k,q+1}$ is the lower and upper bound of the q^{th} section, $q = 1, 2, ..., 2^N$. Then the conditional PDF and CDF of $\gamma_{k,d}$ given Λ_k are given by

$$f_{\gamma_{k,d}}(\gamma|\Lambda_k) = \begin{cases} 0 & \gamma < \xi_{k,q} \\ \frac{f_{\gamma_{k,d}}(\gamma)}{F_{\gamma_{k,d}}(\xi_{k,q+1}) - F_{\gamma_{k,d}}(\xi_{k,q})} & \xi_{k,q} \leq \gamma < \xi_{k,q+1} \\ 0 & \gamma \geq \xi_{k,q+1} \end{cases} \tag{10}$$

and

$$F_{\gamma_{k,d}}(\gamma|\Lambda_k) = \begin{cases} 0 & \gamma < \xi_{k,q} \\ \frac{F_{\gamma_{k,d}}(\gamma) - F_{\gamma_{k,d}}(\xi_{k,q})}{F_{\gamma_{k,d}}(\xi_{k,q+1}) - F_{\gamma_{k,d}}(\xi_{k,q})} & \xi_{k,q} \leq \gamma < \xi_{k,q+1} \\ 1 & \gamma \geq \xi_{k,q+1} \end{cases} \tag{11}$$

respectively, where $f_{\gamma_{k,d}}(\gamma)$ and $F_{\gamma_{k,d}}(\gamma)$ are given in (2) and (3).

Since $\gamma_{s,k}$ is known by the source, the CDF of γ_k conditioned on $\gamma_{s,k}$ and Λ_k is

$$F_{\gamma_k}(\gamma|\gamma_{s,k}, \Lambda_k) = \begin{cases} F_{\gamma_{k,d}}\left(\frac{\gamma(\gamma_{s,k}+1)}{\gamma_{s,k}-\gamma}|\Lambda_k\right) & \gamma < \gamma_{s,k} \\ 1 & \gamma \geq \gamma_{s,k} \end{cases} \tag{12}$$

After some manipulations, (12) can be expressed as

$$F_{\gamma_{k,d}}(\gamma|\gamma_{s,k}, \Lambda_k) = \begin{cases} 0 & \gamma < g_1(\gamma_{s,k}, \xi_{k,q}) \\ J_k(\gamma) & g_1(\gamma_{s,k}, \xi_{k,q}) \leq \gamma < g_1(\gamma_{s,k}, \xi_{k,q+1}) \\ 1 & \gamma \geq g_1(\gamma_{s,k}, \xi_{k,q+1}) \end{cases} \tag{13}$$

where $g_1(x, y) \triangleq xy/(x+y+1)$,
$J_k(\gamma) = A_{k,q}\left(e^{-\xi_{k,q}/\bar{\gamma}_{k,d}} - e^{-\gamma(\gamma_{s,k}+1)/(\bar{\gamma}_{k,d}(\gamma_{s,k}-\gamma))}\right)$, and $A_{k,q} = (\exp(-\xi_{k,q}/\bar{\gamma}_{k,d}) - \exp(-\xi_{k,q+1}/\bar{\gamma}_{k,d}))^{-1}$.

As before, the relay selected for transmission can be easily determined by substituting (13) into (6).

3.3. Selection with Quantized CSI of all channels

To further reduce the signaling overhead, we here consider a more general case that only quantized and statistical CSI of all the channels are available at the source. If the feedback information Ψ_k of the source-k^{th} relay channel indicates that $\gamma_{s,k} \in [\phi_{k,\ell}, \phi_{k,\ell+1})$, then the conditional PDF and CDF of $\gamma_{s,k}$ are given by

$$f_{\gamma_{s,k}}(\gamma|\Psi_k) = \begin{cases} 0 & \gamma < \phi_{k,\ell} \\ \frac{f_{\gamma_{s,k}}(\gamma)}{F_{\gamma_{s,k}}(\phi_{k,\ell+1}) - F_{\gamma_{s,k}}(\phi_{k,\ell})} & \phi_{k,\ell} \leq \gamma < \phi_{k,\ell+1} \\ 0 & \gamma \geq \phi_{k,\ell+1} \end{cases} \tag{14}$$

and

$$F_{\gamma_{s,k}}(\gamma|\Psi_k) = \begin{cases} 0 & \gamma < \phi_{k,\ell} \\ \frac{F_{\gamma_{s,k}}(\gamma) - F_{\gamma_{s,k}}(\phi_{k,\ell})}{F_{\gamma_{s,k}}(\phi_{k,\ell+1}) - F_{\gamma_{s,k}}(\phi_{k,\ell})} & \phi_{k,\ell} \leq \gamma < \phi_{k,\ell+1} \\ 1 & \gamma \geq \phi_{k,\ell+1} \end{cases} \tag{15}$$

respectively.

Based on (11) and (15), the CDF of γ_k conditioned on Ψ_k and Λ_k is found to be (see the appendix)

$$F_{\gamma_k}(\gamma|\Psi_k,\Lambda_k) = \begin{cases} 0 \quad \gamma < g_1(\phi_{k,\ell},\xi_{k,q}) \\ 1 - I_k(\gamma) \\ \qquad g_1(\phi_{k,\ell},\xi_{k,q}) \le \gamma < g_1(\phi_{k,\ell+1},\xi_{k,q+1}) \\ 1 \quad \gamma \ge g_1(\phi_{k,\ell+1},\xi_{k,q+1}) \end{cases} \tag{16}$$

where

$$I_k(\gamma) = B_{k,\ell}\left(e^{-\frac{\beta_k(\gamma)}{\bar{\gamma}_{s,k}}} - e^{-\frac{\phi_{k,\ell+1}}{\bar{\gamma}_{s,k}}}\right)$$
$$+B_{k,\ell}\left(1 - A_{k,q}e^{-\frac{\xi_{k,q}}{\bar{\gamma}_{k,d}}}\right)\left(e^{-\frac{\alpha_k(\gamma)}{\bar{\gamma}_{s,k}}} - e^{-\frac{\beta_k(\gamma)}{\bar{\gamma}_{s,k}}}\right)$$
$$+A_{k,q}B_{k,\ell}\sqrt{\frac{\gamma(1+\gamma)}{\bar{\gamma}_{s,k}\bar{\gamma}_{k,d}}}e^{-\frac{\gamma}{\bar{\gamma}_{s,k}}-\frac{\gamma}{\bar{\gamma}_{s,d}}}\int_{\mu_k(\gamma)(\alpha_k(\gamma)-\gamma)}^{\mu_k(\gamma)(\beta_k(\gamma)-\gamma)}$$
$$\exp\left(-\sqrt{\frac{\gamma(\gamma+1)}{\bar{\gamma}_{s,k}\bar{\gamma}_{k,d}}}\left(\frac{1}{x}+x\right)\right)\mathrm{d}x \tag{17}$$

with $\alpha_k(\gamma) = \max(\phi_{k,\ell}, g_2(\gamma,\xi_{k,q+1}))$, $\beta_k(\gamma) = \min(\phi_{k,\ell+1}, g_2(\gamma,\xi_{k,q}))$, $\mu_k(\gamma) = \sqrt{\frac{\bar{\gamma}_{k,d}}{\bar{\gamma}_{s,k}\gamma(\gamma+1)}}$, and $B_{k,\ell} = (\exp(-\phi_{k,\ell}/\bar{\gamma}_{s,k}) - \exp(-\phi_{k,\ell+1}/\bar{\gamma}_{s,k}))^{-1}$.

4. TARGET RATE BASED QUANTIZATION

In section III, we presented relay selection strategies when quantized and statistical CSI are available. In this section, we propose a target rate based quantizer to partition the SNR range. Notice that the end-to-end SNR γ_k can be upper bounded as [11]

$$\gamma_k = \frac{\gamma_{s,k}\gamma_{k,d}}{\gamma_{s,k}+\gamma_{k,d}+1} \le \min(\gamma_{s,k},\gamma_{k,d}). \tag{18}$$

The equality holds if and only if $\gamma_{s,k} = 0$ or $\gamma_{k,d} = 0$. Thus if $\gamma_{s,k}$ or $\gamma_{k,d}$ is small than the threshold SNR γ_{th}, it follows immediately that the end-to-end SNR $\gamma_k < \gamma_{th}$, i.e., an outage occurs. So it would not make any sense to have more than one quantization section for SNRs below γ_{th}. On the other hand, if both $\gamma_{s,k}$ and $\gamma_{k,d}$ are larger than or equal to γ_{th}, it is still possible that γ_k is smaller than γ_{th}. So it is necessary to partition the SNR above γ_{th}. In the following, we take $\gamma_{k,d}$ as an example to illustrate the proposed quantization scheme. For the N-bit quantizer with 2^N quantization levels, the quantization sections are $[\xi_{k,0},\xi_{k,1})$, $[\xi_{k,1},\xi_{k,2})$, ... , $[\xi_{k,2^N-1},\xi_{k,2^N})$, where $\xi_{k,0} = 0$, $\xi_{k,2^N} = +\infty$. As discussed before, the upper bound for the first section is set to be $\xi_{k,1} = \gamma_{th}$. For the other sections, we use a simple equiprobable SNR partition scheme, in which the probability that the instantaneous SNR value falls in each section is the same for all sections. The bound for the q^{th} section, $q = 2, 3, ..., 2^N - 1$, can be determined by

$$F_{\gamma_{k,d}}(\xi_{k,q}) = F_{\gamma_{k,d}}(\xi_{k,1}) + \frac{q-1}{2^N-1}(1 - F_{\gamma_{k,d}}(\xi_{k,1})). \tag{19}$$

Substituting (3) into (19) , we have

$$\xi_{k,q} = \gamma_{th} + \bar{\gamma}_{k,d}\log\left(\frac{2^N-1}{2^N-q}\right). \tag{20}$$

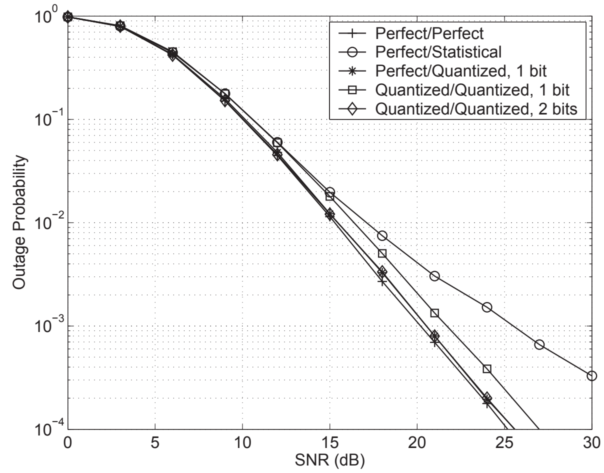

Fig. 1. Outage probability of a 2-relay AF system with different kinds of CSI available, *R* = 1bps/Hz.

5. SIMULATION RESULTS

During simulation, we consider a symmetric network that the channels between the source and the relays are independent identically distributed (i.i.d.), $\bar{\gamma}_{s,k} = \bar{\gamma}_{s,r}, \forall k$, and those between the relays and the destination are also i.i.d, $\bar{\gamma}_{k,d} = \bar{\gamma}_{r,d}, \forall k$. Furthermore, we assume the relays are close to the destination and the mean gains of the relay-destination channels are 10dB higher than the mean gains of the source-relay channels.

Fig. 1 compares the outage performance of a 2-relay AF system with different kinds of CSI available. The target information rate is $R = 1$ bit/s/Hz. It can be seen that for the perfect/statistical case, severe performance loss is observed in the high SNR region. For example, at an outage probability of 3×10^{-4}, there is more than 7dB performance loss as compared with the perfect/perfect CSI case. For the perfect/quantized case, only a single feedback bit is sufficient to achieve almost the same performance as in case that perfect CSI of all the channels is available. For the quantized/quantized case with single bit feedback for each channel, there is only 1.5dB performance loss at an outage probability of 10^{-3}. The gap reduces to be within 0.5dB when there are two bits feedback.

Fig. 2 shows the outage probability for AF systems with different target rates when SNR=24dB. As can be seen, much higher rates can be achieved with better CSI for lower outage probability. For instance, at an outage probability of 10^{-3}, the achievable information rate is 1bps/Hz, 1.6bps/Hz, 2.35bps/Hz and 2.7bps/Hz for the perfect/statistical, perfect/quantized (1bit), quantized/quantized (1bit) and perfect/perfect case in a 4-relay AF system, respectively. While for higher target rates, the gaps between these four cases are small. All these results show that the quantized/quantized case provides a good trade off between signaling overhead and system performance, and is very attractive for practical implementation.

6. CONCLUSION

We presented relay selection strategies for dual-hop AF systems with various different kinds of CSI available at the selecting entity. A target rate based quantizer is also proposed to partition the SNR range when quantized CSI is available. Computer simulations are carried

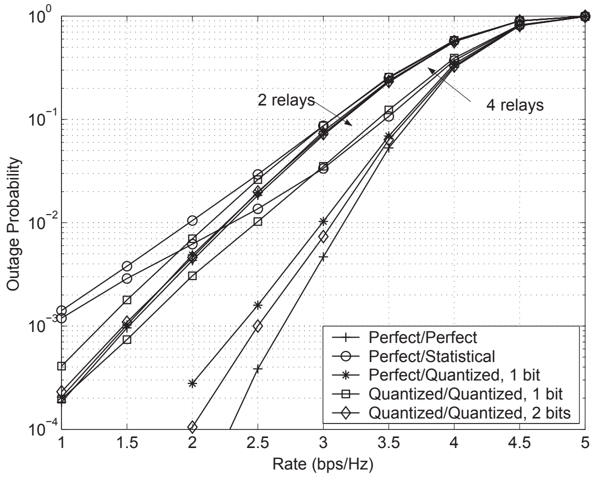

Fig. 2. Outage probability as a function of target information rate for SNR = 24dB.

out to validate the propose strategies. It is shown that only a few quantization bits are required for the quantized/quantized case to achieve near optimal outage performance as in case when perfect CSI is available. These simulation results suggest that considerable signaling overhead can be reduced for dual-hop AF systems with quantized CSI based selection relaying.

7. REFERENCES

[1] A. Nosratinia, T. E. Hunter, and A. Hedayat, "Cooperative communication in wireless networks," *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 74–80, Oct. 2004.

[2] R. Pabst, B. H Walke, D. C. Schultz, and et. al., "Relay-based deployment concepts for wireless and mobile broadband radio," *IEEE Commun. Mag.*, vol. 42, no. 9, pp. 80–89, Sept. 2004.

[3] M.O. Hasna and M.-S. Alouini, "Performance analysis of two-hop relayed transmissions over rayleigh fading channels," in *IEEE 56th Vehicular Technology Conference*. IEEE, Sept. 2002, vol. 4, pp. 1992–1996.

[4] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks : Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[5] Y. Jing and B. Hassibi, "Distributed space-time coding in wireless relay networks," *IEEE Trans. Wireless Commun.*, vol. 5, no. 12, pp. 3524–3536, Dec. 2006.

[6] Y. Zhao, R. Adve, and T. J. Lim, "Improving amplify-and-forward relay networks: optimal power allocation versus selection," *IEEE Trans. Wireless Commun.*, vol. 6, no. 8, pp. 3114–3123, Aug. 2007.

[7] Y. Zhao, R. Adve, and T. J. Lim, "Symbol error rate of selection amplify-and-forward relay systems," *IEEE Commun. Letters*, vol. 10, no. 11, pp. 757–759, Nov. 2006.

[8] A. Bletsas, A. Khisti, D. P. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE J. Select. Areas Commun.*, vol. 24, pp. 659–672, Mar. 2006.

[9] I. Krikidis, J. Thompson, S. McLaughlin, and N. Goertz, "Amplify-and-forward with partial relay selection," *IEEE Commun. Letters*, vol. 12, no. 4, pp. 235–237, April 2008.

[10] A. Muller and J. Speidel, "Relay selection in dual-hop transmission systems: Selection strategies and performance results," in *ICC '08*, Beijing, China, May 2008, pp. 4998–5003.

[11] P. A. Anghel and M. Kaveh, "Exact symbol error probability of a cooperative network in a rayleigh-fading environment," *IEEE Trans. Wireless Commun.*, vol. 3, no. 5, pp. 1416–1421, Sept. 2004.

Appendix

Since the feedback information indicates that $\gamma_{s,k} \in [\phi_{k,\ell}, \phi_{k,\ell+1})$, $\gamma_{k,d} \in [\xi_{k,q}, \xi_{k,q+1})$, then the end-to-end SNR γ_k is bounded by

$$g_1(\phi_{k,\ell}, \xi_{k,q}) \le \gamma_k = g_1(\gamma_{s,k}, \gamma_{k,d}) < g_1(\phi_{k,\ell+1}, \xi_{k,q+1}). \quad (21)$$

Thus we have

$$F_{\gamma_k}(\gamma|\Psi_k, \Lambda_k) = \begin{cases} 0 & \gamma < g_1(\phi_{k,l}, \xi_{k,q}) \\ 1 & \gamma \ge g_1(\phi_{k,l+1}, \xi_{k,q+1}) \end{cases} . \quad (22)$$

When $g_1(\phi_{k,\ell}, \xi_{k,q}) \le \gamma_k < g_1(\phi_{k,\ell+1}, \xi_{k,q+1})$, the CDF of γ_k is given by

$$\begin{aligned} F_{\gamma_k}(\gamma|\Psi_k, \Lambda_k) &= P\left(\frac{\gamma_{s,k}\gamma_{k,d}}{\gamma_{s,k}+\gamma_{k,d}+1} < \gamma|\Psi_k, \Lambda_k\right) \\ &= \int_0^{\gamma} P\left(\gamma_{k,d} > \frac{\gamma(x+1)}{x-\gamma}|\Lambda_k\right) f_{\gamma_{s,k}}(x|\Psi_k)\,\mathrm{d}x \\ &+ \int_{\gamma}^{+\infty} P\left(\gamma_{k,d} < \frac{\gamma(x+1)}{x-\gamma}|\Lambda_k\right) f_{\gamma_{s,k}}(x|\Psi_k)\,\mathrm{d}x \\ &= \int_0^{\gamma} f_{\gamma_{s,k}}(x|\Psi_k)\,\mathrm{d}x \\ &+ \int_{\gamma}^{+\infty} \left[1 - P\left(\gamma_{k,d} \ge \frac{\gamma(x+1)}{x-\gamma}|\Lambda_k\right)\right] f_{\gamma_{s,k}}(x|\Psi_k)\,\mathrm{d}x \\ &= 1 - I_k(\gamma) \end{aligned} \quad (23)$$

where

$$I_k(\gamma) = \int_{\gamma}^{+\infty} \left[1 - F_{\gamma_{k,d}}(g_2(\gamma, x)|\Lambda_k)\right] f_{\gamma_{s,k}}(x|\Psi_k)\,\mathrm{d}x \quad (24)$$

with $g_2(x, y) \triangleq x(y+1)/(y-x)$.

When $x < g_2(\gamma, \xi_{k,q+1})$, $g_2(\gamma, x) > g_2(\gamma, g_2(\gamma, \xi_{k,q+1})) = \xi_{k,q+1}$, thus for $x < g_2(\gamma, \xi_{k,q+1})$, $F_{\gamma_{k,d}}(g_2(\gamma, x)|\Lambda_k) = 1$. Also, when $x < \phi_{k,\ell}$ or $x \ge \phi_{k,\ell+1}$, $f_{\gamma_{s,k}}(x|\Psi_k) = 0$, then $I_k(\gamma)$ can be further simplified as

$$\begin{aligned} I_k(\gamma) &= \int_{\alpha_k(\gamma)}^{\beta_k(\gamma)} \left[1 - F_{\gamma_{k,d}}(g_2(\gamma, x)|\Lambda_k)\right] f_{\gamma_{s,k}}(x|\Psi_k)\,\mathrm{d}x \\ &+ \int_{\beta_k(\gamma)}^{\phi_{k,\ell+1}} f_{\gamma_{s,k}}(x|\Psi_k)\,\mathrm{d}x \end{aligned} \quad (25)$$

where $\alpha_k(\gamma) = \max(\phi_{k,l}, g_2(\gamma, \xi_{k,q+1}))$,
$\beta_k(\gamma) = \min(\phi_{k,l+1}, g_2(\gamma, \xi_{k,q}))$.

Finally, Substituting (11) and (14) into (25) yields the desired results in (16) and (17).