NARROWBAND INTERFERENCE MITIGATION IN BICM OFDM SYSTEMS

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ABSTRACT

Orthogonal frequency division multiplexing (OFDM) is noted for its resistance to narrowband interference when equipped with forward error correction. This technique along with erasure insertion is adequate when the the signal-to-interference ratio (SIR) is moderate, around 0 dB, however, when the interference is severe, i.e. SIR = -20 dB, a non-orthogonal interference corrupts a large number of the data subcarriers due to spectral leakage. This situation requires a large number of erasures that compromise the code's error correction capability. The prediction-error filter (PEF) is proposed as an erasure insertion mechanism that localizes the erasures to the tones surrounding the interference, while not affecting the remaining tones. The simulation results indicate excellent performance, for the bit-interleaved coded modulated (BICM) OFDM system using the PEF and BPSK modulation when the SIR = -20 dB in a frequency-selective fading channel.

***Index Terms*—** Orthogonal frequency division multiplexing (OFDM), interference suppression, convolutional codes, prediction-error filtering, multipath channels

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a popular transmission scheme that has been used in many emerging wireless communications standards (i.e. WLAN, WiMax, UWB). OFDM involves converting the input into multiple low-rate streams that are transmitted on separate narrowband subcarriers that are allowed to overlap orthogonally in frequency, providing an increase in spectral efficiency. This also converts a frequency-selective fading channel over the OFDM bandwidth into subcarriers that each experience flat fading, allowing for simple one-tap equalization per subcarrier. Forward error correction coding and interleaving are necessary for these OFDM systems in a realistic environment.

For all of the benefits of OFDM, there are also drawbacks, such as the high peak-to-average power ratio (PAPR) and carrier frequency offset (CFO). This paper examines the performance of a bit-interleaved coded modulated (BICM) OFDM systems [1] in the presence of narrowband interference (NBI). Many of the new standards utilize the unlicensed bands, thus there is the possibility that the system will have to share the same frequency band with other communications systems. For instance, WiMAX is a narrowband interferer for UWB systems. This is especially true when discussing OFDM-based cognitive radios [2] that are required to modify its spectrum in order to avoid primary users. In other cases, NBI may arise from radio non-linearities.

This work was supported by the UCSD Center for Wireless Communications (UCDG Grant # Com 06-10216).

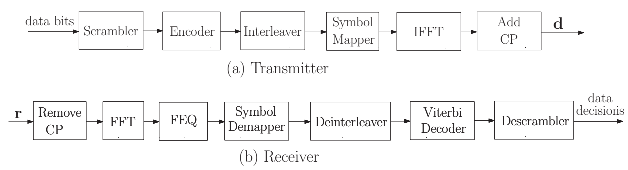

Fig. 1. OFDM System Model.

There has been a great deal of research concerning mitigation of NBI in an uncoded OFDM system (see references in [3]). More recently there has been some work relating to NBI suppression in coded OFDM systems (see [3, 4]). Snow et al. [5] primarily examine the problem of efficient error rate analysis for coded multicarrier systems, but also briefly mention that a coded OFDM system with genie-aided erasure insertion is not adequate for low signal-to-interference ratios (SIRs). More recently, these authors [6] examined the use of a two-stage approach to mitigate WiMAX interference (which has been shown to be approximately Gaussian) in a coded MB-OFDM UWB system. A spectral estimate of the interference is generated using the maximum entropy method (which is based on linear prediction) during a silent period. This estimate is then used to update the bit metric weighting on each subcarrier prior to decoding.

In this work, the case of a BICM OFDM system is considered in the presence of strong NBI (i.e., SIR $\ll$ 0 dB) operating in a frequency-selective channel. In [7] it was shown that coding with erasure insertion is not adequate for a severe non-orthogonal interferer because many erasures are required when the interference power leaks (i.e. spectral leakage) into a large number of subcarriers. In this paper, the prediction-error filter (PEF) is proposed as an erasure insertion mechanism that localizes erasures around the interference by placing a notch in the frequency spectrum of the received signal. The PEF uses the narrowband nature of the interference (as compared to the OFDM spectrum) to remove the interference. and can be easily implemented adaptively using the low complexity least-mean square (LMS) algorithm. Simulation results indicate that this system provides excellent performance as compared to a BICM OFDM system in the presence of no NBI.

2. SYSTEM MODEL

The OFDM system model of interest is depicted in Fig. 1. There are N subcarriers that carry data. The message is composed of i.i.d. data bits with average power equal to E_b. These bits are first passed through a scrambler, a convolutional encoder, and a puncturer. Note that the average power of the coded data on each tone is given by $E_c = RE_b$, where R is the rate of the code. This output is fed into a serial-to-parallel (S/P) block, interleaved using a block interleaver, and finally mapped onto each subcarrier based on the QAM constellation of choice with average power equal to E_s. The frequency-domain data symbols located on the subcarriers are given by,

$$\mathbf{D} = [D_0, D_1, \ldots, D_{N-1}]^T, \tag{1}$$

where $(\cdot)^T$ denotes the transpose operator. The data symbols are subsequently modulated into time-domain samples using the inverse discrete Fourier transform (IDFT),

$$\mathbf{d} = \mathbf{F}^H \mathbf{D}, \tag{2}$$

where $(\cdot)^H$ represents conjugate (Hermitian) transpose and $\mathbf{F}$ is the N-point DFT matrix. Note that $\mathbf{F}$ is a unitary operator, i.e. $\mathbf{F}\mathbf{F}^H = \mathbf{F}^H\mathbf{F} = \mathbf{I_N}$, where $\mathbf{I_N}$ is the $N \times N$ identity matrix.

A cyclic prefix, composed of the last G samples of $\mathbf{d}$ is prepended to give, $\mathbf{d}_{\text{CP}}$, in order to mitigate the effects of intersymbol interference (ISI) and intercarrier interference (ICI) caused by the channel.

Assuming perfect timing and frequency synchronization, the received signal in the time-domain is

$$\mathbf{r}_{\text{CP}} = \mathbf{h} * \mathbf{d}_{\text{CP}} + \mathbf{x}_{\text{CP}} + \mathbf{n}_{\text{CP}}, \tag{3}$$

where $\mathbf{x}_{\text{CP}}$ and $\mathbf{n}_{\text{CP}}$ are the length $N + G$ vectors of interference and noise samples, respectively, $\mathbf{h}$ is the L vector containing the taps of a multipath channel, and $*$ represents linear convolution. The components, h_l, are modeled as i.i.d. zero-mean, circularly Gaussian random variables, with variance equal to $\frac{1}{L}$. The time-domain noise samples are distributed as zero-mean Gaussian random variables with variance equal to σ_n^2. The interference term in the time-domain is given by

$$x_n = \sqrt{E_x} e^{j(2\pi f_i n T_s + \theta)} = \sqrt{E_x} e^{j\left(\frac{2\pi}{N}(m+\alpha)n + \theta\right)}, \tag{4}$$

where E_x is the interferer power, T_s is the original symbol period, θ is a random phase that is distributed uniformly, $\mathcal{U}[-\pi, \pi]$. The frequency of the interference is defined using the original sampling frequency, f_s, as $f_i = (m + \alpha)\frac{f_s}{N}$. In this formulation, the value m, is the tone closest to the interference, and $\alpha \sim \mathcal{U}\left[-\frac{1}{2}, \frac{1}{2}\right]$.

After cyclic prefix removal, the output of the DFT in the frequency-domain is given by,

$$\begin{aligned} \mathbf{R}' &= \mathbf{FHd} + \mathbf{Fx} + \mathbf{Fn} \\ &= \tilde{\mathbf{H}}\mathbf{D} + \mathbf{X} + \mathbf{N} \end{aligned} \tag{5}$$

where $\mathbf{H}$ is the $N \times N$ circulant channel matrix composed of the channel taps and $\tilde{\mathbf{H}}$ is the $N \times N$ diagonal matrix of the eigenvalues of the $\mathbf{H}$ [8].

In general, one-tap equalization in OFDM is accomplished using a single complex value that removes the effects of the phase and amplitude. However, here the details of the equalization process are expanded to provide further insight. Let $\mathbf{H} = \tilde{\mathbf{H}}_\phi \tilde{\mathbf{H}}_\rho$, where $\tilde{\mathbf{H}}_\phi$ is the diagonal matrix of the phase information of the channel and $\tilde{\mathbf{H}}_\rho$ is the diagonal matrix of the magnitude of the eigenvalues, each component distributed as a Rayleigh random variable. The coherently demodulated received vector is then given as

$$\mathbf{R} = \tilde{\mathbf{H}}_\phi^H \mathbf{R}' = \tilde{\mathbf{H}}_\rho \mathbf{D} + \tilde{\mathbf{H}}_\phi^H (\mathbf{X} + \mathbf{N}). \tag{6}$$

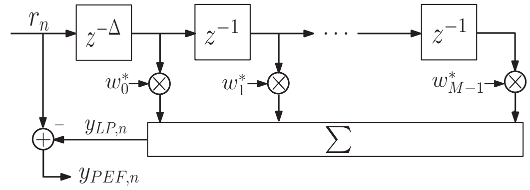

Fig. 2. M-tap prediction-error filter.

The output of the DFT on subcarrier k is given by,

$$R_k = \tilde{H}_{\rho,k} D_k + \tilde{H}_{\phi,k}^* (X_k + N_k), \tag{7}$$

where N_k noise component on subcarrier k with the same statistics as in the time-domain and X_k is the interference value that is located on subcarrier k. When the interference is non-orthogonal, the interference is spread across all the tones, which is described as,

$$X_k = \sqrt{\frac{E_x}{N}} \frac{\left(1 - e^{j2\pi\alpha}\right) e^{j\theta}}{1 - e^{j\frac{2\pi}{N}(m-k-\alpha)}}. \tag{8}$$

This scenario is similar to when ICI is present. When the interference is orthogonal to the subcarriers ($\alpha = 0$), the interference after the DFT reduces to

$$X_k = \begin{cases} \sqrt{NE_x} e^{j\theta}, & k = m \\ 0, & k \neq m \end{cases}. \tag{9}$$

The interference only affects tone m for this case.

These frequency-domain symbols are passed through a one-tap per tone frequency-domain equalizer, demapped into bits with possible erasure insertion, deinterleavered, and decoded using the Viterbi algorithm [9] with soft inputs. Finally, the signal is descrambled using the same scrambling sequence that was utilized at the receiver.

3. ERASURE INSERTION USING THE PREDICTION-ERROR FILTER

3.1. Prediction-Error Filter

In this work, the prediction-error filter (PEF) is considered as the means for removing the interference in the time-domain. The PEF is a well studied structure that is a variant of the linear predictor (LP). The LP is a structure that uses the correlation between past samples to form an estimate of the current sample [9]. The PEF has the property that it removes the correlation between samples, thereby whitening the spectrum.

This technique has also been used to remove narrowband interference in many applications [9, 10]. The filter is able to predict the interferer due to its narrowband properties. A block diagram of the PEF is shown in Fig. 2. It is a transversal filter with M taps. The decorrelation delay (Δ) ensures that the current sample of the OFDM time-domain signal is decorrelated from the samples in the filter when calculating the error term. Note that the received symbols are encoded, thus there will be some correlation that is induced by the encoder. However, the presence of the interleaver, allows the assumption that the symbols are essentially i.i.d, thus $\Delta = 1$ is a sufficient choice, giving the one-step predictor.

The output of the PEF, $y_{\text{PEF},n}$, is defined as the subtraction of the estimate of the interference from the current input sample,

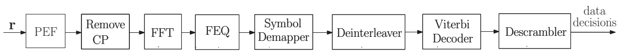

Fig. 3. OFDM receiver with prediction-error filter

$$y_{\text{PEF},n} = r_n - \sum_{l=0}^{M-1} w_l^* r_{n-\Delta-l}. \tag{10}$$

where w_l are the weights of the predictor. The optimal weights of the PEF under the minimum mean-squared error (MMSE) criterion can be found using the orthogonality principle [10]. M equations are obtained and the weights are found using the method of undetermined coefficients [11],

$$w_l = Je^{-j\frac{2\pi}{N}(m+\alpha)(l+\Delta)}, \quad J = \frac{E_x}{E_s + \sigma_n^2 + ME_x}. \tag{11}$$

For the scenario of interest in this paper, the interference power is much larger than both the signal power and the noise power. Therefore, the SIR and the noise-to-interference (NIR) can be assumed to be very small (i.e. SIR $\ll$ 0 dB, NIR $\ll$ 0 dB [10]) allowing the coefficient to be approximated as $J \approx \frac{1}{M}$.

This filter is easily implemented adaptively using the least-mean square (LMS) algorithm. The convergence properties are well described [12].

3.2. Receiver with Prediction-Error Filtering

The PEF is implemented before the removal of the cyclic prefix as seen in Fig. 3. Let the one-step predictor weights be defined as $\mathbf{c} = [1, -w_0^*, \ldots, -w_{M-1}^*]$ and the convolution of the filter and channel be defined as $\mathbf{a} = \mathbf{c} * \mathbf{h}$. It is assumed that the overall length of $\mathbf{a}$ is less than the length of the cyclic prefix (i.e. $L + M - 1 \leq G$) to ensure that there is no ISI or ICI. Therefore, the effective channel matrix, $\mathbf{A}$, is circulant. Then, the filtered signal in the frequency-domain can be written as

$$\begin{aligned} \mathbf{R}' &= \mathbf{FAF}^H\mathbf{D} + \mathbf{FC}_{\text{noise}}(\mathbf{x}_{\text{cp}} + \mathbf{n}_{\text{cp}}) \\ &= \tilde{\mathbf{A}}\mathbf{D} + \mathbf{FC}_{\text{noise}}(\mathbf{x}_{\text{cp}} + \mathbf{n}_{\text{cp}}) \end{aligned} \tag{12}$$

where $\tilde{\mathbf{A}}$ is the diagonal matrix of the eigenvalues of $\mathbf{A}$, $\mathbf{x}_{\text{cp}}$ and $\mathbf{n}_{\text{cp}}$ are vectors of interference and noise samples, respectively that are not cyclically extended (length $N+G$), and $\mathbf{C}_{\text{noise}}$ is the $N \times (N+G)$ filtering matrix for the noise and interference that is defined as

$$\begin{aligned} \mathbf{C}_{\text{noise}} = \big[&\mathbf{0}_{N,G-(L+M-1)}, \\ &\text{Toeplitz}([c_M^*, \mathbf{0}_{1,N-1}]^T, [\mathbf{c}^*, \mathbf{0}_{1,N-1}])\big], \end{aligned} \tag{13}$$

where $\mathbf{0}_{i,j}$ is the $i \times j$ zero matrix. The Toeplitz operator, Toeplitz $(column, row)$, generates a Toeplitz matrix from a column vector and a row vector. Note that $\tilde{\mathbf{A}}$ in (12) can also be defined as

$$\tilde{\mathbf{A}} = \tilde{\mathbf{H}}\tilde{\mathbf{C}}, \tag{14}$$

where $\tilde{\mathbf{C}} = \sqrt{N}\mathbf{F}\begin{bmatrix}\mathbf{c}^* & \mathbf{0}_{1,N-(M+1)}\end{bmatrix}^T$. This vector is the sampled frequency response of the notch filter. Let $\tilde{\mathbf{A}} = \tilde{\mathbf{A}}_\phi \tilde{\mathbf{A}}_\rho$, where $\tilde{\mathbf{A}}_\rho = |\tilde{\mathbf{A}}|$ and $\tilde{\mathbf{A}}_\phi = e^{\angle\tilde{\mathbf{A}}}$. Thus, the coherently demodulated received vector is given as

$$\mathbf{R} = \tilde{\mathbf{A}}_\phi^H \mathbf{R}' = \tilde{\mathbf{A}}_\rho \mathbf{D} + \tilde{\mathbf{A}}_\phi^H \mathbf{FC}_{\text{noise}}(\mathbf{x}_{\text{cp}} + \mathbf{n}_{\text{cp}}). \tag{15}$$

Let the uncanceled interference and noise be grouped into one general noise term,

$$\tilde{\mathbf{N}} = \tilde{\mathbf{A}}_\phi^H \mathbf{FC}_{\text{noise}}(\mathbf{x}_{\text{cp}} + \mathbf{n}_{\text{cp}}). \tag{16}$$

It is clear from (16) that the noise samples, $\tilde{N}_k$, are correlated due to the PEF matrix, $\mathbf{C}_{\text{noise}}$. It is also noted that the noise samples $\tilde{N}_k$ are assumed to be Gaussian random variables. As stated in [10], this system is difficult to analyze when the noise samples are not strictly independent, however, due to the fact that the noise power in the main tap ($c_0 = 1$) is much larger than in the remaining taps (approximately $\frac{1}{M}$ for the scenario of interest in this paper), it is reasonable to assume that the noise samples are independent. Therefore, let $\sigma_{\tilde{N},k}^2$ be the variance of the noise on tone k, given by

$$\begin{aligned} \sigma_{\tilde{N}}^2(k) &= E\left[\tilde{N}_k \tilde{N}_k^H\right] \\ &= \left[\tilde{\mathbf{A}}_\phi^H \mathbf{FC}_{\text{noise}}\left(\mathbf{R}_x + \sigma_n^2 \mathbf{I}_N\right) \mathbf{C}_{\text{noise}}^H \mathbf{F}^H \tilde{\mathbf{A}}_\phi\right]_{kk} \end{aligned} \tag{17}$$

where $\mathbf{R}_x$ is the $N \times (N+G)$ autocorrelation matrix of the interference, whose components are given by $r_x(l) = E_x e^{j\frac{2\pi}{N}(m+\alpha)l}$. Note that the variances given in (17) are scaled according to the notch filter that is used to suppress the interference.

Finally, the amplitude is removed and the estimates of the transmitted data symbols are given by

$$\hat{\mathbf{D}} = \mathbf{R}/\tilde{\mathbf{A}}_\rho. \tag{18}$$

4. RESULTS

4.1. Simulation Parameters

The OFDM system is equipped with $N = 64$ data-carrying subcarriers and a cyclic prefix of $G = 16$ samples. The data on each subcarrier is BPSK modulated. The encoder uses the industry-standard generator polynomials, $\mathrm{g}_0 = 133_8$ and $\mathrm{g}_1 = 171_8$, giving the rate as $R = \frac{1}{2}$. Note that higher rates can be obtained by puncturing the output of the encoder. One codeword encompasses 50 OFDM symbols. A block interleaver is utilized and the subcarrier modulation mapping is performed according to Gray encoding.

The channel is modeled as independent complex Gaussian random variables as discussed in Sec. II. It is assumed that the channel is constant over each OFDM symbol, and subsequently changes upon transmission of the next symbol. It is also assumed for this work that the channel is known at the receiver. The narrowband interference is randomly distributed within the spectrum for each OFDM symbold transmission. The PEF weights given in (11) are assumed to be known at the receiver, though they can quickly be obtained through adaptive means.

The deinterleaver is simply the dual of the interleaver process. The Viterbi decoder is used to decode the codeword utilizing soft decisions out of the symbol demapper. Specifically, log-likelihood ratios (LLRs) are calculated using the effective channel matrix ($\tilde{\mathbf{A}}_\rho$) and the overall noise variance given in (17).

It is also noted that the PEF does not insert true erasures (i.e. an LLR representing no information), however, true erasures can be additionally added using the Bayesian insertion rule proposed in [13] and used previously in [7]. An erasure is inserted after filtering, if the following expression is satisfied:

$$\frac{\max_j \pi_j f(R_k|D_j)}{\sum_{i=0}^{P-1} \pi_i f(R_k|D_i)} \leq (1-\tau), \tag{19}$$

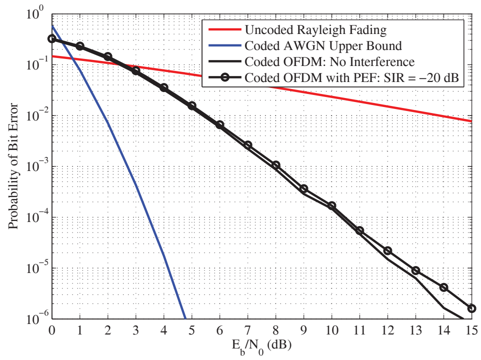

Fig. 4. Bit error rate of BICM OFDM with PEF versus E_b/N_0, $L = 5$, $M = 12$, SIR = -20 dB.

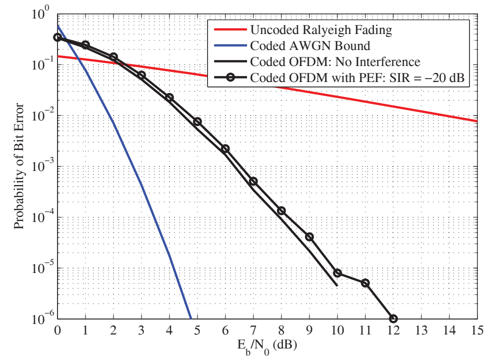

Fig. 5. Bit error rate of BICM OFDM with PEF versus E_b/N_0, $L = 10$, $M = 7$, SIR = -20 dB.

where π_i is the a priori probability of transmitting D_i, $f(R_k|D_i)$ is the conditional probability density function (pdf) of R_k given that D_i was transmitted, P is the size of the constellation, and $\tau \in [0, 1]$ is a threshold. Note that $\tau = 1$ is the case of no erasure insertion.

4.2. Simulation Results

Fig. 4 demonstrates the simulation curve of the BICM OFDM system and the PEF with one narrowband interferer, SIR = -20 dB. The multipath channel is composed of $L = 5$ taps, and the remaining guard interval is used for the PEF. Thus, $M = 12$ taps are used to cancel out the interference. For reference, the simulation of the coded OFDM system with no interference is also plotted. It is seen that in the case of a severe narrowband interferer, the PEF is effective in canceling out the interference, and effectively places erasures in the codeword around the interferer location. The redundancy provided by the code allows for these erasures to be corrected. Also, plotted are the theoretical results for the bound on coded AWGN and for uncoded Rayleigh fading. It is apparent that this system's improvement over uncoded Rayleigh fading is due to the frequency diversity that is obtained through the application of forward error correction coding across the subcarriers. Results for the scenario of $L = 10$, $M = 7$, SIR = -20 dB are shown in Fig. 5. This plot demonstrates that even with a shorter PEF, the results are still excellent. One can notice that there is a larger deviation between the case of interference and no interference for $M = 7$ as compared to $M = 12$.

5. CONCLUSION

In this paper, a bit-interleaved coded modulated (BICM) OFDM system in the presence of strong NBI was investigated. The prediction-error filter (PEF) is proposed to mitigate the interference in the time-domain, effectively inserting a few erasures around the interference, while not degrading the tones away from the notch. Simulation results demonstrate that a BICM OFDM system with PEF provides excellent performance and approximates the performance of a similar system in the presence of no narrowband interference.

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