ON THE PTS METHOD AND BER-MINIMIZING POWER ALLOCATION OF THE MULTI-CHANNEL OFDM SYSTEM

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ABSTRACT

In multiple-access multicarrier systems, it is possible to serve multiple users at high data rates from a single base station. In this paper, we propose the multi-channel partial transmit sequences (MCPTS) method for multiple-access OFDM peak-to-average power ratio (PAR) reduction. By applying the PTS phase rotations to each access channel separately and assuming that the rotations can be detected by each user as a channel fading, MCPTS avoids two major disadvantages of standard PTS: i) the phase rotations can be detected without side information, and ii) the set of possible phase rotation values does not need to be finite. In addition, an iterative method of joint MCPTS and power allocation is proposed to minimize the average BER. The results show a significant BER improvement.

Index Terms— Orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAR), power allocation

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been widely adopted by various modern communication standards because of its high spectral efficiency and low complexity over frequency-selective fading channels [1, 2]. However, a potentially large dynamic range, which is usually measured by peak-to-average power ratio (PAR), exists in the time-domain OFDM waveform and rendering the power amplifier (PA) inefficient. OFDM-based frequency-division multiple access systems were also defined in communication standards to support multiuser communications. The frequency spectrum is channelized into adjacent fragments on each of which a (or multiple) separate OFDM user(s) can communicate. For instance, 5-20MHz and 1.25-20MHz channelization schemes were specified in IEEE 802.11a and 802.16 standards, respectively [1, 2]. In the downlink scenario, the base station transmits multi-channel signals through a single high-power PA concurrently. The waveforms overlap in the time domain thus the PAR problem is even more challenging.

Many PAR reduction methods have been proposed for OFDM systems, e.g. clipping methods [3], partial transmit sequence (PTS) [4] and waveform optimization methods [5, 6]. However, the PAR reduction problem for multi-channel OFDM systems has not been well addressed. In this paper, a distortionless PTS-based PAR reduction algorithm, referred to as the multi-channel PTS (MCPTS) method, is proposed for the synchronous multi-channel OFDM system. It achieves a significant PAR reduction performance by optimizing over the phases of channels and avoids several disadvantages existing in the traditional PTS method. Additionally, the joint phase and power optimization problem is formulated in this paper for the peak-power limited PA, as well as a suboptimal iterative MCPTS and power allocation approach.

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2. SYSTEM MODEL

The structure diagram of the base station transmitter in the synchronous multi-channel OFDM system is shown in Fig. 1. Synchronous transmission is assumed in this paper, where the start and end times of OFDM symbols among all channels are synchronized. More general asynchronous system will be addressed in the future work. Independent OFDM signals are transmitted on M different frequency bands, whose center frequencies are f_m and satisfy a non-overlap condition, $f_{m+1} - f_m \geq \frac{1}{2}(B_{m+1} + B_m)$ $(m = 1, \cdots, M-1)$ where B_m is the frequency bandwidth of the mth channel.

For each of the OFDM signals, data are transmitted on N_m orthogonal subcarriers which make up the OFDM symbol, denoted as $\mathbf{X}_m = [X_{m,-N_m/2}, \cdots, X_{m,N_m/2-1}]^T$. For notational simplicity, $B_m = B$ and $N_m = N$ are assumed in this paper. An L-times oversampling inverse FFT (LN-point IFFT) operation is performed to generate the mth channel's baseband time-domain samples

$$x_m[n] = \frac{1}{\sqrt{LN}} \sum_{k=0}^{LN-1} X_{m,k}^{\mathrm{ZP}} e^{\frac{j2\pi kn}{LN}},\ n = 0, \cdots, LN-1, \quad (1)$$

where $\mathbf{X}_m^{\mathrm{ZP}} = [X_{m,0}, \cdots, X_{m,\frac{N}{2}-1}, 0, \cdots, 0, X_{m,-\frac{N}{2}}, \cdots, X_{m,-1}]^T$ is generated by zero-padding $\mathbf{X}_m$ with $(L-1)N$ zeros. Then, the baseband signals will be up-converted to passband and combined as the input to the PA, i.e.

$$x_p[n] = \sum_{m=1}^{M} x_m[n] e^{\frac{j2\pi nTf_m}{LN}}, \quad (2)$$

where T is the symbol duration. The center frequency of the above passband signal can be found as $f_{\mathrm{pass}} = \frac{1}{2}(f_1 + f_M) +$

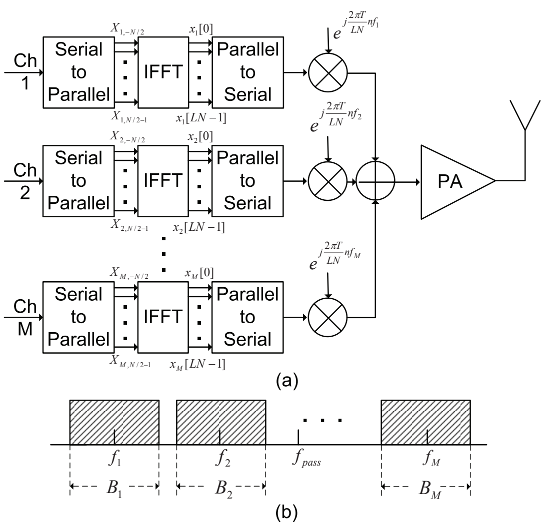

Fig. 1. The (a) structure and (b) frequency spectrum diagram of the multi-channel OFDM base station.

$\frac{1}{4}(B_M - B_1)$ and the equivalent baseband signal is $\mathbf{x} = [x[0], \cdots, x[LN-1]]^T$ with $x[n] = x_p[n]e^{-\frac{j2\pi nTf_{\text{pass}}}{LN}}$, which can be calculated by IFFT. Since the baseband PAR can be used to infer the passband dynamic range [7], in this paper we consider the symbol-wise PAR defined as

$$\text{PAR}(\mathbf{x}) = \frac{\|\mathbf{x}\|_\infty^2}{\frac{1}{LN}\|\mathbf{x}\|_2^2}. \tag{3}$$

PAs are generally peak-power limited. For a PA with predistortion, the soft limiter characteristic can be adopted [8]. Assuming an output peak-power limit of P_{peak}, nonlinear distortion occurs when $|x[n]|^2 > P_{\text{peak}}$ (i.e. clipping). Symbol-wise linear scaling (SWLS) can be used so that no clipping occurs, in which the OFDM symbol is modified such that $\mathbf{x}_{\text{SWLS}} = P_{\text{peak}}^{1/2}\mathbf{x}/\|\mathbf{x}\|_\infty$. Thus, reduction in the PAR leads to increase in the average transmit power [3].

3. MULTI-CHANNEL PTS (MCPTS)

Partial transmit sequence (PTS) approach has been proposed to reduce the PAR of OFDM signals [4]. The basic idea is to produce U time-domain representations for the same OFDM symbol and transmit the representation with the smallest PAR value. The set of OFDM subcarrier indices is partitioned into S disjoint sets $\mathbf{S}_s$ ($s = 1, \cdots, S$), having $\bigcup_{s=1,\cdots,S} \mathbf{S}_s = \{-\frac{N}{2}, \cdots, \frac{N}{2} - 1\}$ and $\mathbf{S}_p \bigcap \mathbf{S}_q = \emptyset$ ($p \neq q,\ 1 \leq p, q \leq S$). U independent phase sequences with independent phase shifts on each set $\mathbf{S}_s$, i.e. $\Theta^{(u)} = [e^{j\theta_1^{(u)}}, \cdots, e^{j\theta_S^{(u)}}]^T$ ($u \in \{1, \cdots, U\}$), are available to both transmitters and receivers. The superscript is used for the index of the multiple representations and the subscript is used for the index of partitions. In the time domain, U different representations are obtained via

$$\mathbf{x}^{(u)} = \text{IFFT}[\mathbf{X}^{(u)}] = \text{IFFT}[\mathbf{X} \circ \Phi^{(u)}],\ u = 1, \cdots, U, \tag{4}$$

where $\circ$ denotes element-wise multiplication, $\Phi^{(u)} = [\phi_{-\frac{N}{2}+1}^{(u)}, \cdots, \phi_{\frac{N}{2}}^{(u)}]^T$ and $\phi_k^{(u)} = e^{j\theta_s^{(u)}}$ if $k \in \mathbf{S}_s$. Since phase rotations do not change the average power, we have $\|\mathbf{x}\|_2^2 = \|\mathbf{x}^{(u)}\|_2^2$ ($\forall u \in \{1, \cdots, U\}$). The $\tilde{u}$th sequence will be transmitted whose PAR is the smallest, i.e. $\tilde{u} = \arg \min_{u \in \{1, \cdots, U\}} \text{PAR}(\mathbf{x}^{(u)})$. Inevitably, the index $\tilde{u}$ has to be sent as side information such that the phase shifts can be inversed at the receiver side.

In this section, the PTS method is extended to the multi-channel OFDM system, referred as the multi-channel PTS (MCPTS) method. In the synchronous transmitter, the whole frequency-domain symbols $\mathbf{X}_m$ ($m = 1, \cdots, M$) can be regarded as a single data block. The individual symbols to the data block is the same as the subcarrier subsets to the OFDM symbol in the PTS method. Therefore, for the multi-channel system, there is a natural and simple partition scheme determined by the channelization. The number of partitions is $S = M$ and $\mathbf{S}_s$ is the indices of $\mathbf{X}_m$ ($s = m \in \{1, \cdots, M\}$). The M-by-1 phase sequence $\Theta = [e^{j\theta_1}, \cdots, e^{j\theta_M}]^T$ will then be determined to minimize the PAR value, formulating the MCPTS approach in the following way

$$\underset{\theta_m \in [0, 2\pi)}{\text{minimize}} \quad \text{PAR}(\tilde{\mathbf{x}}) \tag{5}$$

$$\text{subject to} \quad \tilde{\mathbf{x}}_m = \text{IFFT}[e^{j\theta_m}\mathbf{X}_m] = e^{j\theta_m}\mathbf{x}_m \tag{6}$$

$$\tilde{x}[n] = \sum_{m=1}^{M} \tilde{x}_m[n]e^{\frac{j2\pi nT}{LN}(f_m - f_{\text{pass}})}. \tag{7}$$

In the end, the signal after adaptive linear scaling will be transmitted, i.e. $\mathbf{x}_{\text{SWLS}} = \tilde{\mathbf{x}}P_{\text{peak}}^{1/2}/\|\tilde{\mathbf{x}}\|_\infty$. Each of the partitions in the MCPTS method is just a single channel. With the assumption of ideal channel estimation, this property provides MCPTS a couple of merits that make it favored for this system:

1. Neither side information nor receiver-side modification is needed. The phase rotation is equivalently part of the channel response and can be recovered by the channel estimation capability of each OFDM channel;
2. Since no side information should be transmitted, phase rotation sequences can take on any value, leading to better PAR reduction performance. Optimization techniques, such as the particle swarm optimization (PSO) [9], can be used to solve (5)-(7).

4. JOINT MCPTS AND BER-MINIMIZING POWER ALLOCATION

For peak-power-limited PA, reducing PAR can increase the effective average output power. However, for fading channels, power allocation should also be designed so that the potential average power increase can be effectively utilized. Accordingly, we propose that each subcarrier in each channel be scaled so that the average power of the kth subcarrier in the mth channel is $P_{m,k}$, i.e. $\tilde{X}_{m,k} = X_{m,k}(P_{m,k}/E[|X_{m,k}|^2])^{\frac{1}{2}}$. We assume the transmitter has the channel state information

(CSI), including the frequency response $h_{m,k}$ and the variance of the Gaussian channel noise $\sigma^2_{m,k}$. The signal-to-noise ratio (SNR) of a given subcarrier is

$$\text{SNR}_{m,k} = P_{m,k}\frac{|h_{m,k}|^2}{\sigma^2_{m,k}} = \frac{P_{m,k}}{\hat{\sigma}^2_{m,k}}, \quad (8)$$

where $\hat{\sigma}^2_{m,k} = \sigma^2_{m,k}/|h_{m,k}|^2$ is the equivalent channel noise power. The values for $P_{m,k}$ will be determined to minimize the average bit error rate (BER).

Given the peak power constraint, the joint PAR reduction and average BER-minimizing power allocation problem can be formulated as:

$$\underset{P_{m,k},\,\theta_m}{\text{minimize}} \quad \frac{1}{MN}\sum_{m=1}^{M}\sum_{k=-N/2}^{N/2-1} f_{\text{ber}}\left(\sqrt{\text{SNR}_{m,k}}\right) \quad (9)$$

$$\text{subject to} \quad \sum_{m=1}^{M}\sum_{k=-N/2}^{N/2-1} P_{m,k} = \|\tilde{\mathbf{x}}\|_2^2 \le \frac{LNP_{\text{peak}}}{\text{PAR}(\tilde{\mathbf{x}})} \quad (10)$$

$$\tilde{x}[n] = \sum_{m=1}^{M}\tilde{x}_m[n]e^{\frac{j2\pi nT}{LN}(f_m - f_{\text{pass}})} \quad (11)$$

$$\tilde{\mathbf{x}}_m = \text{IFFT}\{e^{j\theta_m}\bar{\mathbf{X}}_m\},\ \forall m \in \{1,\cdots,M\},$$

where $f_{\text{ber}}(\sqrt{\text{SNR}})$ is the bit error rate for the constellation of interest. Eq. (10) is a very complicated function of both $P_{m,k}$ and θ_m. Because the convexity of this problem is not clear, it is hard to solve.

Next, we propose an iterative algorithm to find a suboptimal solution to this problem. First, for an initial objective average power $P^0_{\text{av}} \triangleq \frac{LNP_{\text{peak}}}{\text{PAR}^0}$, the BER-minimizing power allocation solves the Lagrangian problem

$$\frac{\partial}{\partial P_{m,k}}\left[-\sum_{m=1}^{M}\sum_{k=-N/2}^{N/2-1} f_{\text{ber}}\left(\sqrt{\frac{P_m}{\hat{\sigma}^2_{m,k}}}\right) - \lambda P_{m,k}\right] = 0 \quad (12)$$

$$\sum_{m=1}^{M}\sum_{k=-N/2}^{N/2-1} P_{m,k} = P^0_{\text{av}}. \quad (13)$$

Assuming QPSK modulation with Gray mapping, where $f_{\text{ber}}(x) = \text{erfc}(x/\sqrt{2})/2$ [10, P. 271], Eq. (12) becomes

$$\frac{1}{\sqrt{\hat{\sigma}^2_{m,k}P_{m,k}}}e^{-\frac{P_{m,k}}{2\hat{\sigma}^2_{m,k}}} = 2\sqrt{2\pi}\lambda. \quad (14)$$

Although $P_{m,k}$ is one-to-one with λ, the functional relationship cannot be described in closed form. But by combining Eq. (13) and Eq. (14), $P_{m,k}$ can be determined numerically.

Secondly, because PAR is related to the allocated power, the PAR minimization in MCPTS needs to take the scaling $|\bar{X}_{m,k}|^2 = P_{m,k}$ into consideration. By replacing $\mathbf{x}_m$ and $\mathbf{X}_m$ in Eq. (6) with $\bar{\mathbf{x}}_m$ and $\bar{\mathbf{X}}_m$, the minimum PAR can be found as shown in the MCPTS method in (5)-(7), i.e. $\text{PAR}_{\min} = \arg\min_{\Theta} \text{PAR}(\tilde{\mathbf{x}})$.

Denote $0 < \beta < 1$ as the convergence parameter. If $\text{PAR}_{\min} > \text{PAR}^0$ or $\text{PAR}_{\min} < \beta\text{PAR}^0$, either the objective average power cannot be reached or the MCPTS yields a much greater average power. In both cases, a new iteration begins with $\text{PAR}^0 = \text{PAR}_{\min}$ and $P^0_{\text{av}} = \frac{LNP_{\text{peak}}}{\text{PAR}_{\min}}$. Otherwise, the algorithm converges. A sub-optimal solution in terms of both the power allocation and the phase sequence is achieved. The iteration stops if this convergence condition is met or the maximum iteration number is reached.

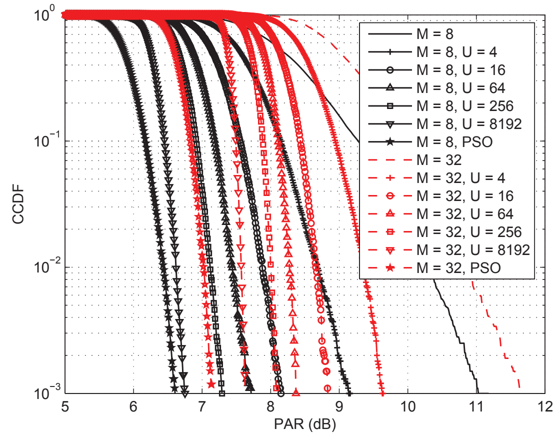

Fig. 2. CCDF curves of the PAR of the original and MCPTS multi-channel OFDM signals for different numbers of channels ($M = 8, 32$) and numbers of phase sequences ($U = 4, 16, 64, 256, 8192$) as well as the PSO method.

5. RESULTS

We present some examples to illustrate the performance improvements. In the simulations, adjacent channels were assumed without using guard bands, i.e. $f_{m+1} - f_m = B$. The number of subcarriers was $N = 64$ for every channel.

5.1 MCPTS

The complementary cumulative distribution function (CCDF) curves of the PAR are plotted in Fig. 2. The constellation was QPSK although our experiences indicated that the PAR reduction performance is insensitive to the constellation choice. When the PSO method is used, the globally minimum PAR can be achieved. However, more than 1,000 iterations were generally necessary for the 10-particle PSO to achieve a good convergence. Instead, similar to PTS, a limited number of phase sequences can be used for searching a small PAR. It provides a tradeoff between PAR reduction performance and complexity.

5.2 Joint MCPTS and power allocation

In this section, the number of channels $M = 8$ and the oversampling rate $L = 8$ were used. For simplicity, we assumed $P_{m,k} = P_m$ and $h_{m,k} = h_m$, but $\{h_m\}$ were independent and identically-distributed complex Gaussian random variables. The channel noise was white Gaussian with $\sigma^2_{m,k} = \sigma^2$. The peak signal-to-noise power ratio (PSNR) can be defined as

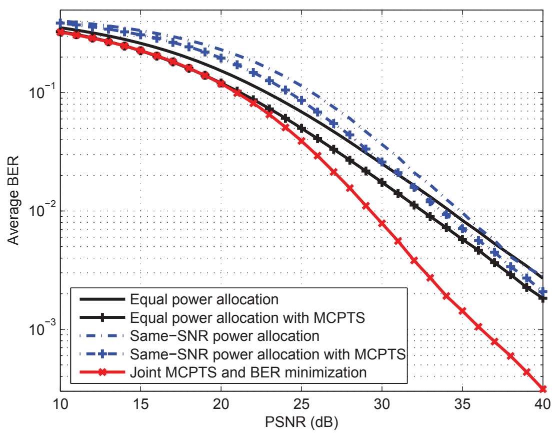

Fig. 3. Average BER versus PSNR curves for $M = 8$ multi-channel OFDM systems.

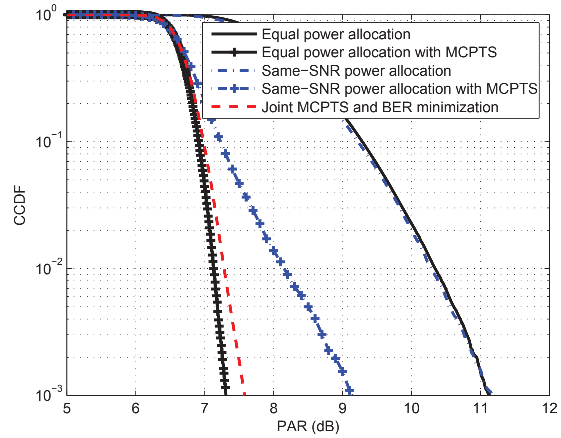

Fig. 4. CCDF curves of the PAR for $M = 8$ multi-channel OFDM systems.

$$\text{PSNR} \triangleq \frac{P_{\text{peak}} \sum_{m=1}^{M} E[|h_m|^2]}{M\sigma^2}. \quad (15)$$

Initialization $\text{PAR}^0 = 10\text{dB}$ and convergence parameter $\beta = 0.95$ were used.

Fig. 3 shows the average BER versus PSNR curves. Two comparative methods are used, where the average power is either equally allocated among channels, i.e. $P_m = P$, or allocated so that every channel has the same SNR and average BER, i.e. $\frac{P_m}{P_n} = \frac{\hat{\sigma}_m^2}{\hat{\sigma}_n^2}$ [11, Chp. 9]. In both methods, we force the condition $P_{\text{av}} = \sum_{m=1}^{M} P_m$. MCPTS can be easily combined with these methods and improve the BER performance. Also, the curve of the proposed joint MCPTS and BER-minimizing power allocation method is plotted in Fig. 3. It is clear that the proposed joint optimization method leads to much lower BER at high PSNR values.

The resulting PAR of the iterative joint method might be, however, greater than using MCPTS for a constant power allocation scheme, e.g. the equal power allocation with MCPTS as shown in Fig. 4. Intuitively, the joint method optimizes both the magnitude and the phase of each OFDM channel. Although the potential average power increase is less than using only MCPTS, the power is more efficiently utilized among channels.

6. CONCLUSION

In this paper, the multi-channel partial transmit sequence PAR reduction method is proposed for multi-channel OFDM systems. We illustrated how MCPTS can be used in multi-channel systems to reduce the PAR without side information. In addition, using the assumption of perfect CSI at transmitters, we demonstrated how the BER can be further decreased with a joint MCPTS and BER-minimizing power allocation approach. For future research, we would like to address the PAR reduction issues in general multiple access systems.

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