JOINT ESTIMATION OF I/Q IMBALANCE, CFO AND CHANNEL RESPONSE FOR OFDM SYSTEMS

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ABSTRACT

In this paper, we study the joint estimation of I/Q imbalance, CFO and channel response for OFDM systems. Using two repeated OFDM blocks for training, we propose a new method to solve the joint estimation problem. The implementation complexity is low. Simulation results show that the proposed method has a good performance and its BER is very close to the ideal case where all the parameters are known perfectly at the receiver.

***Index Terms*—** OFDM, I/Q, CFO, Channel estimation

1. INTRODUCTION

Recently, direct-conversion receiver receives a lot of attention because of its low cost implementation and low power consumption [1]. However some drawbacks have been found in direct-conversion receivers, such as in-phase and quadrature-phase (I/Q) imbalance, direct current (DC) offset and carrier frequency offset (CFO). These mismatches will degrade the system performance. The impact of I/Q imbalance on orthogonal frequency division multiplexing (OFDM) systems is investigated in [2]. Several frequency domain estimation methods for I/Q mismatch are proposed in [2]. Exploiting the fact that the size of the DFT matrix is usually larger than the channel order, the authors in [3] propose a time domain method for the joint estimation of I/Q mismatch and channel response. For CFO estimation, a method using two repeated OFDM training blocks is proposed in [4]. Assuming that the channel frequency response is smooth in OFDM systems, a frequency domain method is provided in [5] for the joint estimation of I/Q mismatch , CFO and channel response. In [6], a specific sequence is provided for the estimation of I/Q and CFO. In [7], an artificial frequency offset is needed at the transmitter for the joint estimation of I/Q and CFO. The proposed method in general needs 6 OFDM training blocks to achieve a good performance. On the other hand, the authors in [8][9] use 3 training blocks for training. The proposed methods in [8][9] suffer a performance loss when CFO is small. In [10], the maximum likelihood (ML) estimators for I/Q and CFO are derived by using an iteration method to solve the expectation and maximization (EM) problem. An adaptive approach for the compensation of CFO and I/Q mismatch is proposed in [11].

In this paper, we study the joint estimation of I/Q mismatch, CFO and channel response for OFDM systems. Using two repeated OFDM blocks for training, we propose a new method to solve the joint estimation problem. The implementation complexity of the proposed method is low. Simulation results show that the proposed method has a good performance and it is robust to different values of I/Q imbalance, CFO and channel length. The BER performance of the proposed method is very close to the ideal case where the CFO, I/Q and channel response are perfectly known at the receiver.

Notation: The symbols $\mathbf{A}^\dagger$, $\mathbf{A}^T$ and $\mathbf{A}^*$ denote respectively the transpose-conjugate, the transpose and the complex conjugate of $\mathbf{A}$.

2. SYSTEM DESCRIPTION

Consider an OFDM system with I/Q mismatch and CFO in Fig. 1. The vector containing modulation symbols is denoted by $\mathbf{s}$. Taking the M-point IDFT of $\mathbf{s}$, we have an $M \times 1$ vector $\mathbf{x}$. A cyclic prefix of length L is appended to $\mathbf{x}$ and the sequence is then sent through the channel. In this paper, the channel is modeled as an FIR filter $h(n)$ of order L with additive noise $q(n)$. When there are no CFO and I/Q mismatch, after removing the CP at the receiver, we obtain the baseband vector

$$\mathbf{r} = \mathbf{H}_{cir}\mathbf{x} + \mathbf{q}, \tag{1}$$

where $\mathbf{H}_{cir}$ is an $M \times M$ circulant matrix with the first column

$$\mathbf{h} = \begin{bmatrix} h(0) & \cdots & h(L) & 0 & \cdots & 0 \end{bmatrix}^T, \tag{2}$$

and $\mathbf{q}$ is an $M \times 1$ noise vector. Suppose that there are CFO and I/Q mismatch. Then the received vector is given by [5]

$$\mathbf{z} = \mu\mathbf{E}\mathbf{r} + \nu(\mathbf{E}\mathbf{r})^*, \tag{3}$$

where $\mathbf{E}$ is an $M \times M$ diagonal matrix

$$\mathbf{E} = \mathrm{diag}\begin{bmatrix} 1 & e^{j\frac{2\pi}{M}\theta} & \cdots & e^{j\frac{2\pi}{M}(M-1)\theta} \end{bmatrix}, \tag{4}$$

and θ is the normalized CFO. The I/Q parameters μ and ν are related to the phase mismatch ϕ and amplitude mismatch ϵ by

$$\mu = \frac{1+\epsilon e^{-j\phi}}{2},\ \ \nu = \frac{1-\epsilon e^{j\phi}}{2}, \tag{5}$$

It is found [2][5] that if the CFO and I/Q mismatch are not properly compensated at the receiver, the system performance will seriously degrade. Define the I/Q parameter

$$\alpha = \frac{\nu}{\mu^*}. \tag{6}$$

Then from (3), we can write the vector $\mu\mathbf{r}$ in terms of $\mathbf{z}$ as [5]

$$\mu\mathbf{r} = \mathbf{E}^*\frac{\mathbf{z}-\alpha\mathbf{z}^*}{1-|\alpha|^2}. \tag{7}$$

Thus if the CFO θ and the I/Q parameter α are known at the receiver, we can use the above equation to compensate these mismatches. Below we will show how to jointly estimate θ and α.

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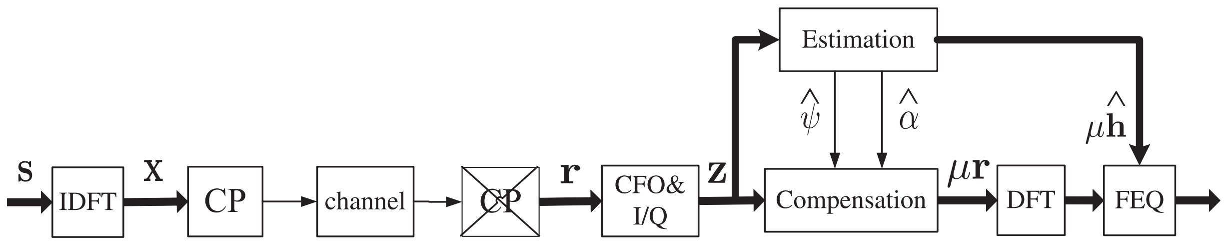

Figure 1: An OFDM system with I/Q mismatch and CFO

3. PROPOSED JOINT ESTIMATION METHOD

Suppose two repeated OFDM training blocks are transmitted from the transmitter. That means the training block is a length $2M+L$ vector of the form $\begin{bmatrix} \text{CP} & \mathbf{x}^T & \mathbf{x}^T \end{bmatrix}^T$. The vector $\mathbf{x}$, which is known to the receiver, can be arbitrary. Suppose that there are both CFO and I/Q mismatch and the channel remains constant during two OFDM training blocks. From (3), we see that the two $M \times 1$ vectors at the receiver have the form

$$\begin{aligned} \mathbf{z}_0 &= \mu\mathbf{y} + \nu(\mathbf{y})^* + \mathbf{q}_0 && (8) \\ \mathbf{z}_1 &= \mu e^{j2\pi\theta}\mathbf{y} + \nu(e^{j2\pi\theta}\mathbf{y})^* + \mathbf{q}_1, && (9) \end{aligned}$$

where the $M \times 1$ vector $\mathbf{y}$ is given by

$$\mathbf{y} = \mathbf{E}\mathbf{H}_{cir}\mathbf{x}. \quad (10)$$

Our goal is to jointly estimate the CFO, the I/Q parameter and the channel response from $\mathbf{z}_0$ and $\mathbf{z}_1$. Below we will first show how to solve the two subproblems: (A) given α, estimate θ and (B) given θ, estimate α and $h(n)$. Then the joint estimation of α, θ and $h(n)$ will be studied.

(A) Given α, estimate θ: Using (8) and (9), we can write $\mu\mathbf{y}$ and $e^{j2\pi\theta}\mu\mathbf{y}$ in terms of $\mathbf{z}_0$ and $\mathbf{z}_1$ respectively:

$$\begin{aligned} \mu\mathbf{y} &\approx \frac{\mathbf{z}_0 - \alpha\mathbf{z}_0^*}{1-|\alpha|^2} && (11) \\ e^{j2\pi\theta}\mu\mathbf{y} &\approx \frac{\mathbf{z}_1 - \alpha\mathbf{z}_1^*}{1-|\alpha|^2}. && (12) \end{aligned}$$

From the above equations, the estimate of CFO is given by

$$\widehat{\theta} = \frac{1}{2\pi}\text{angle}\left\{(\mathbf{z}_0 - \alpha\mathbf{z}_0^*)^\dagger(\mathbf{z}_1 - \alpha\mathbf{z}_1^*)\right\}, \quad (13)$$

where we have used the fact that $(\mu\mathbf{y})^\dagger(\mu\mathbf{y})$ and $(1-|\alpha|^2)^2$ are positive. Notice that the estimated CFO $\widehat{\theta}$ in the above formula is in the range

$$\frac{-1}{2} \leq \widehat{\theta} < \frac{1}{2}. \quad (14)$$

Only the fractional part of the CFO is estimated. The problem of estimating the integral part of the CFO will be discussed later.

(B) Given θ, estimate α and $h(n)$: Using the circular convolution property, we can rewrite (10) as

$$\mathbf{y} = \mathbf{E}\mathbf{X}_{cir}\mathbf{h}, \quad (15)$$

where $\mathbf{X}_{cir}$ is $M \times M$ circulant with the first column $\mathbf{x}$ and $\mathbf{h}$ is the $M \times 1$ vector defined in (2). Using (11), (12) and (15), we obtain two independent estimates of $\mu\mathbf{h}$ as

$$\mu\widehat{\mathbf{h}}_0 = \mathbf{X}_{cir}^{-1}\mathbf{E}^*\frac{\mathbf{z}_0 - \alpha\mathbf{z}_0^*}{1-|\alpha|^2} \quad (16)$$

$$\mu\widehat{\mathbf{h}}_1 = \mathbf{X}_{cir}^{-1}e^{-j2\pi\theta}\mathbf{E}^*\frac{\mathbf{z}_1 - \alpha\mathbf{z}_1^*}{1-|\alpha|^2}. \quad (17)$$

Observe that the last $(M-L-1)$ entries of the desired vector $\mu\mathbf{h}$ are zero. If α and θ are perfectly estimated, the last $(M-L-1)$ entries of $\mu\widehat{\mathbf{h}}_0$ and $\mu\widehat{\mathbf{h}}_1$ should be small. Thus, given θ, one can estimate α by minimizing the energy of these entries. Below we will show how to do this. Define $\mathbf{P}$ as the $(M-L-1)\times M$ matrix formed by the last $(M-L-1)$ rows of the identity matrix $\mathbf{I}_M$. The product $\mathbf{P}\mu\widehat{\mathbf{h}}_i$ will be an $(M-L-1)\times 1$ vector consisting of the last $(M-L-1)$ entries of $\mu\widehat{\mathbf{h}}_i$. For most applications, the I/Q mismatch is small so that $1-|\alpha|^2 \approx 1$. Using this approximation and (16) and (17), we have

$$\begin{aligned} \mathbf{P}\mu\widehat{\mathbf{h}}_0 &\approx \mathbf{A}\mathbf{z}_0 - \alpha\mathbf{A}\mathbf{z}_0^*, \\ \mathbf{P}\mu\widehat{\mathbf{h}}_1 &\approx e^{-j2\pi\theta}(\mathbf{A}\mathbf{z}_1 - \alpha\mathbf{A}\mathbf{z}_1^*), && (18) \end{aligned}$$

where the matrix $\mathbf{A}$

$$\mathbf{A} = \mathbf{P}\mathbf{X}_{cir}^{-1}\mathbf{E}^*. \quad (19)$$

From linear algebra theory, we know that the parameter α that minimizes $\|\mathbf{P}\mu\widehat{\mathbf{h}}_i\|^2$ is given by

$$\widehat{\alpha}_i = \frac{(\mathbf{A}\mathbf{z}_i^*)^\dagger(\mathbf{A}\mathbf{z}_i)}{\|\mathbf{A}\mathbf{z}_i^*\|^2}, \text{ for } i = 0, 1. \quad (20)$$

The minimized cost functions are given by

$$\|\mathbf{P}\mu\widehat{\mathbf{h}}_i\|^2_{min} = \|\mathbf{A}\mathbf{z}_i\|^2 - \frac{|(\mathbf{A}\mathbf{z}_i)^\dagger(\mathbf{A}\mathbf{z}_i^*)|^2}{\|\mathbf{A}\mathbf{z}_i^*\|^2}, \quad (21)$$

for $i = 0, 1$. Thus, given θ, we can obtain two independent estimates of α using (20). By taking the average, the estimate of the I/Q parameter is

$$\widehat{\alpha} = \frac{1}{2}\left(\widehat{\alpha}_0 + \widehat{\alpha}_1\right). \quad (22)$$

Substituting $\widehat{\alpha}$ into (16) and (17), we obtain two independent estimates of the channel response. By taking the average, we have

$$\mu\widehat{\mathbf{h}} = \frac{1}{2}\left(\mu\widehat{\mathbf{h}}_0 + \mu\widehat{\mathbf{h}}_1\right). \quad (23)$$

(C) Estimating the integral part of CFO: Let τ be the integral part of the CFO so that $\widehat{\theta}+\tau$ is the CFO. Substituting $\theta=\widehat{\theta}+\tau$ into $\|\mathbf{P}\mu\widehat{\mathbf{h}}_i\|^2_{min}$ in (21), we obtain different values for different integer τ. Using the observation that when τ is estimated perfectly, the cost function $\|\mathbf{P}\mu\widehat{\mathbf{h}}_i\|^2_{min}$ should be a minimum, the estimate of τ is therefore given by

$$\widehat{\tau}=\arg\min_{\tau\in\text{integer}}\left(\|\mathbf{P}\mu\widehat{\mathbf{h}}_0\|^2_{min}+\|\mathbf{P}\mu\widehat{\mathbf{h}}_1\|^2_{min}\right). \quad (24)$$

In practice, τ is usually a small integer. We only need to search for τ within a narrow range.

(D) Joint Estimation of CFO, I/Q and Channel Response: Using the results in (A), (B) and (C), we propose an algorithm to solve the joint estimation problem as follows:

1. In practice, the value of α is usually small. So we first assume that $\alpha\approx 0$. Substituting $\alpha=0$ into (13), we obtain an initial estimate of the fractional part of θ
$$\widehat{\theta}^0=\frac{1}{2\pi}\text{angle}\left\{\mathbf{z}_0^\dagger\mathbf{z}_1\right\}. \quad (25)$$
2. Estimate the integral part of CFO using (24). Replace $\widehat{\theta}^0$ with $\widehat{\theta}^0+\widehat{\tau}$.
3. Obtain an initial estimate of the I/Q parameter $\widehat{\alpha}^0$ by substituting $\widehat{\theta}^0$ into (20) and take the average as in (22).
4. Obtain the fractional part of CFO $\widehat{\theta}$ by substituting $\widehat{\alpha}^0$ into (13). Replace $\widehat{\theta}$ with $\widehat{\theta}+\widehat{\tau}$.
5. Obtain $\widehat{\alpha}$ by substituting $\widehat{\theta}$ into (20) and take the average as in (22).
6. Obtain $\mu\widehat{\mathbf{h}}$ by substituting $\widehat{\theta}$ and $\widehat{\alpha}$ into (16) and (17) and by taking the average as in (23).

The estimates of I/Q parameter, CFO and channel response are given by $\widehat{\alpha}$, $\widehat{\theta}$ and $\mu\widehat{h}(n)$ respectively.

(E) Complexity: From the expressions in (13), (16), (17), (20) and (21), we see that the main computation is the calculation of the vectors

$$\mathbf{X}_{cir}^{-1}\mathbf{E}^*\mathbf{z}_i \text{ and } \mathbf{X}_{cir}^{-1}\mathbf{E}^*\mathbf{z}_i^*. \quad (26)$$

All other calculations involved are scalar multiplications and vector additions. Because the matrix $\mathbf{X}_{cir}$ is circulant, $\mathbf{X}_{cir}^{-1}$ is also circulant. The computation of the two vectors in (26) can be implemented efficiently using circular convolution and its complexity is in the order of $M\log_2 M$. Thus the complexity of the proposed method is low.

4. SIMULATION RESULTS

In this section, we carry out Monte-Carlo experiments to verify the performance of the proposed method. The channel taps are i.i.d. complex Gaussian random variables with zero mean and normalized by $\sum_{l=0}^{L}\mathrm{E}\left\{|h(l)|^2\right\}=1$. Training symbols are QPSK randomly generated in each run. Two different cases of parameter settings are considered:

$$(A)\ \epsilon=1.1,\ \phi=10^\circ,\ \theta=-0.41,\ M=64,\ L=3,$$
$$(B)\ \epsilon=1.2,\ \phi=-15^\circ,\ \theta=1.73,\ M=64,\ L=15.$$

The MSE of the proposed estimators is defined as

$$\begin{aligned}\text{MSE(I/Q)} &= \mathrm{E}\left\{|\widehat{\alpha}-\alpha|^2\right\},\\ \text{MSE(Channel)} &= \frac{1}{L+1}\sum_{l=0}^{L}\mathrm{E}\left\{|\mu\widehat{h}(l)-\mu h(l)|^2\right\},\\ \text{MSE(CFO)} &= \mathrm{E}\left\{|\widehat{\theta}-\theta|^2\right\},\end{aligned}$$

respectively. Figure 2 shows the MSE performance of the proposed method. For comparison, we also draw the MSE performance of the IQ-CFO-FD method [5]. From these plots, we find that the proposed method outperforms the IQ-CFO-FD method in both cases, especially when the SNR is high. The difference between the two methods is more significant for Case B. This is because in Case B, the channel is longer ($L=15$) and the smoothness assumption on the channel frequency response in [5] is no longer valid. Therefore the performance of the IQ-CFO-FD method degrades significantly when the channel order increases from $L=3$ to $L=15$. On the other hand, the proposed method performs equally well for both cases. Also notice that in Case B, the CFO is $\theta=1.73$ and the proposed method is able to estimate both the integral and fractional parts of the CFO. Figure 3 shows the BER performance of the proposed method for Case B. The data symbols are QPSK. We find that without proper compensation, the system performance is very poor. The IQ-CFO-FD method does not perform well. The BER performance of the proposed method is very close to the ideal case where the I/Q parameter, CFO and channel response are perfectly known at the receiver.

5. CONCLUDING REMARKS

In this paper, we propose a new method for the joint estimation of I/Q mismatch, CFO and channel response for OFDM systems. Using two repeated OFDM blocks for training, we propose a new method to solve the joint estimation problem. The implementation complexity is low. Simulation results show that the proposed method has a good performance and it is robust to different values of I/Q mismatch, CFO and channel length. The BER performance is very close to the ideal case where all the parameters are perfectly known at the receiver.

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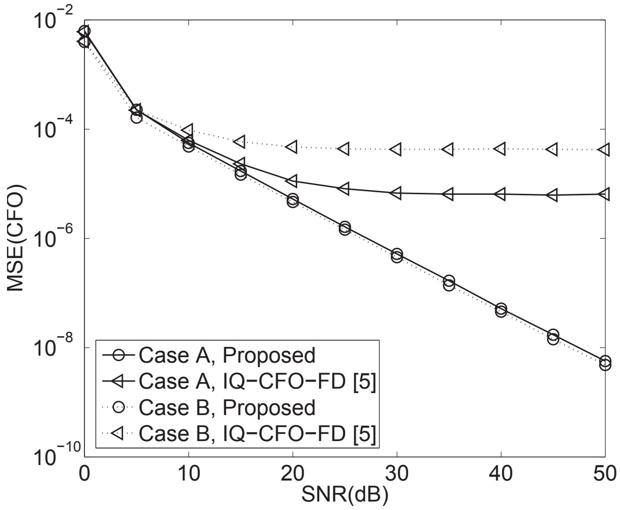
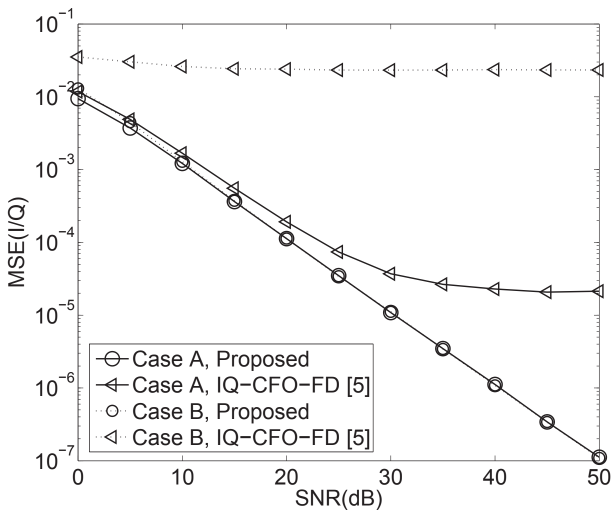
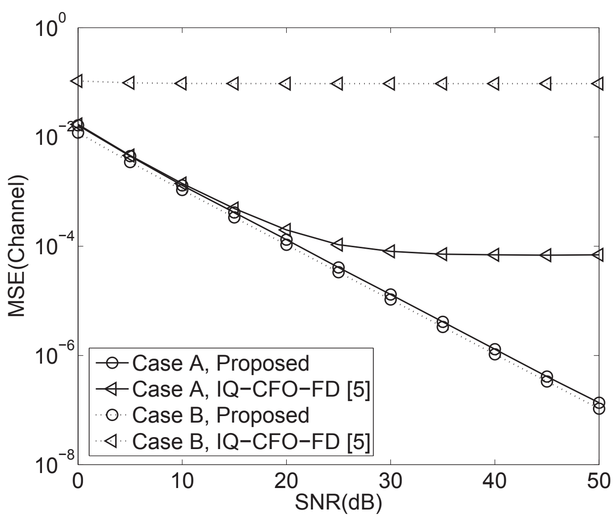


(a)



(b)



(c)

Figure 2: MSEs of the estimates of (a) θ, (b) α and (c) $\mu h(n)$.

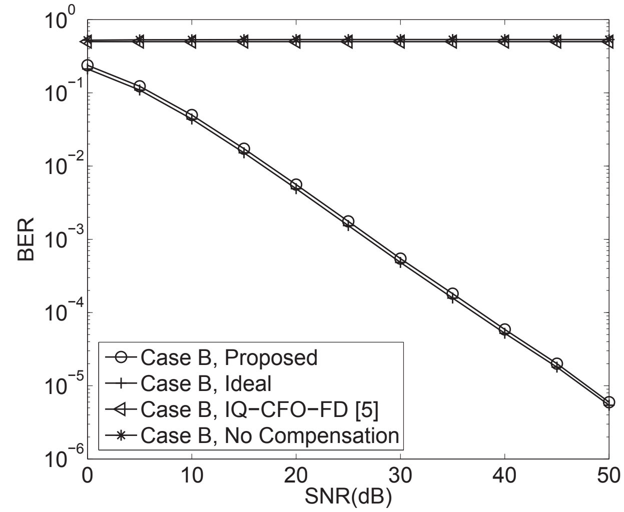


Figure 3: BER performance.

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