JOINT CARRIER FREQUENCY OFFSET AND CHANNEL ESTIMATION IN OFDM BASED NON-REGENERATIVE WIRELESS RELAY NETWORKS

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ABSTRACT

Cooperative relaying is an effective approach to combat wireless fading. However, the reliability enhancements depend strongly on the accuracy of the carrier frequency offset (CFO) compensation and channel estimation algorithms. In this paper, we show that CFO compensation at relay is necessary in non-regenerative OFDM based wireless relay networks with relays employing space-time coding. A simple training scheme available for estimating space-time channel is exploited to estimate the CFO at the relay. Maximum likelihood (ML) and least squares (LS) based joint CFO and channel estimators are constructed for estimating the CFO and the product channel (effective channel from source to destination) at destination. For non-regenerative OFDM based wireless relays, we prove that the LS based estimator is equivalent to the ML based estimator. Theoretical mean square error for the product channel estimator is also derived.

Index Terms— Carrier frequency offset, channel estimation, wireless relays, amplify and forward, Alamouti coding

1. INTRODUCTION

Recently, there is an immense interest in wireless relay networks employing distributed space-time coding [1]-[3]. The well known benefits of orthogonal frequency division multiplexing (OFDM) in combating inter symbol interference (ISI) and frequency selective fading channels have led wireless relay networks to use OFDM for data transmission [3]. The use of regenerative relays could drastically increase the system's cost as the relay is required to perform advanced signal processing operations for decoding the signal [1]. On the other hand, non-regenerative relays are not required to perform signal decoding and hence are cheaper and easier to implement.

In non-regenerative space-time coded relay networks, the relay is required to perform time-reversion and complex conjugation operations on the received signal [3]. Performing these operations directly on the received analog RF signal is infeasible and hence the relay is required to down convert, sample and store the received signal. Therefore, the presence of carrier frequency offset (CFO) at relay is unavoidable.

In this paper, we show that the relay needs to perform CFO compensation so that the destination can reap the benefits from space-time coding. The received signal at destination is again subjected to CFO and frequency selective fading channel between the relay and destination. Furthermore, the noise from relay is also transmitted to the destination, causing the total noise at destination to be colored. The destination needs to estimate the CFO and the product channel, i.e., the effective channel from source to destination. In this scenario, the well known result is to use maximum likelihood (ML) based estimator for joint CFO and channel impulse response (CIR) estimation [4]. However, in this paper, we show that the least squares (LS) based estimator is indeed the ML estimator for joint CFO and CIR estimation in OFDM based non-regenerative space-time coded wireless relay networks.

Notation: All vectors are column vectors. Vectors (matrices) are denoted by small (upper) case bold letters. Matrix $\mathbf{I}_N$ represents $(N \times N)$ Identity matrix and $\mathbf{0}$ can represent either a vector or matrix with all elements equal to zero. The Kronecker product, linear convolution, statistical expectation, absolute value, conjugate, l_2-norm, transpose, and transpose conjugate operations are denoted by $\otimes$, $\star$, $E(.)$, $|.|$, $(.)^*$, $\|.\|$, $(.)^T$ and $(.)^H$, respectively. Function $\angle(.)$ denotes the phase of $(.)$. Function $\mathrm{diag}(\mathbf{a})$ denotes a diagonal matrix with elements of $\mathbf{a}$ as its diagonal elements. If $\mathbf{A}$ is a matrix, then $\mathbf{A}(:, a : b)$ denotes a matrix containing ath column vector to bth column vector of $\mathbf{A}$.

2. SYSTEM MODEL

We consider a three node quasi-time synchronous wireless relay network consisting of a source, relay and destination, each operating in half-duplex mode. The source and destination are each equipped with single antenna, whereas the relay is equipped with two antennas. The data transmission from the source to destination (transmission cycle) takes place over two phases. In the first phase, signal is transmitted only from the source to relay. The relay does not decode the received signal, instead it performs simple time domain operations to generate a space-time signal, which is then transmitted to destination. In the second phase, the source remains silent and the relay transmits the space-time signal to the destination. OFDM is used for data transmission and each antenna transmits two OFDM symbols during each phase. Let L_1 and L_2 denote the maximum number of channel taps for the source to relay and relay to destination channels, respectively and scalar $P \geq max\{L_1 - 1, L_2 - 1\}$ denotes the number of cyclic prefix (CP) samples. In order to combat ISI, the source (relay) augments CP to the signal before transmitting it to relay (destination). The CIR vectors for source to ith relay antenna and ith relay antenna to destination channels are denoted by $\mathbf{h}_{S,i}$ and $\mathbf{h}_{i,D}$, respectively for $i = 1, 2$.

2.1. Phase 1

In the first phase, relay receives the signal from source and discards the samples corresponding to CP. Discarding the CP samples at relay is necessary for constructing the space-time signal suitable for frequency selective fading channel conditions. Vector $\mathbf{r}_{R,i,l}$ contains N ISI free samples received by the ith relay antenna during lth OFDM symbol duration and it can be expressed as

$$\mathbf{r}_{R,i,l} = \mathbf{H}_{S,i}\mathbf{W}_N^H\mathbf{x}_l + \mathbf{v}_{R,i,l} \tag{1}$$

Table 1. Operations performed at Relay

Antenna	OFDM Symbol $\mathbf{s}_{i,l}$	
	$l=2n$	$l=2n+1$
$i=1$	$\mathbf{r}_{R,1,2n}/\sqrt{2}$	$\mathbf{r}_{R,1,2n+1}/\sqrt{2}$
$i=2$	$\zeta\left(\mathbf{r}^*_{R,2,2n+1}\right)/\sqrt{2}$	$-\zeta\left(\mathbf{r}^*_{R,2,2n}\right)/\sqrt{2}$

for $i = 1,2$ and $l = 2n, 2n+1$ where n denotes the transmission cycle index. In (1), $\mathbf{W}_N$ is the $(N \times N)$ discrete Fourier transform (DFT) matrix, $\mathbf{H}_{S,i}$ is a $(N \times N)$ circulant matrix with $[\mathbf{h}^T_{S,i}, \mathbf{0}]^T$ as its first column vector, vector $\{\mathbf{x}_l\}_{l=2n,2n+1}$ contains N data symbols transmitted in lth OFDM symbol and $\mathbf{v}_{R,i,l}$ is the $(N \times 1)$ additive white Gaussian noise (AWGN) whose covariance matrix is $\sigma^2_R\mathbf{I}_N$. The relay performs only complex conjugation and sample reordering operations. For any vector $\mathbf{a} = [a_0, a_1, \cdots, a_{N-1}]^T$, we define $\zeta(\mathbf{a}) = [a_{N-1}, a_{N-2}, \cdots, a_0]^T$. If matrix $\mathbf{A} = [\mathbf{a}_0, \cdots, \mathbf{a}_{N-1}]$ then $\zeta(\mathbf{A}) = [\zeta(\mathbf{a}_0), \cdots, \zeta(\mathbf{a}_{N-1})]$. If elements of $\mathbf{a}$ are stored in a register then $\zeta(\mathbf{a})$ is obtained by just reading the register in reverse order. Table 1 lists the space-time signal vector $\mathbf{s}_{i,l}$ to be transmitted by the ith relay during the lth OFDM symbol duration. The scaling factor $1/\sqrt{2}$ is required to equally allocate the signal power across the two relay antennas. The relay augments CP of length P samples to the space-time signal and then transmits it to the destination during the second phase.

Some of the key differences between the system model in our paper and that in [3] are as follows: 1) source in [3] used DFT for modulating data symbol $\mathbf{x}_{2n+1}$ whereas in our work only inverse DFT (IDFT) is used for modulation and hence ensures compatibility with existing systems, 2) the $\zeta(.)$ function used in [3] involved an additional circular shift which adds additional burden to the relay, 3) only frequency flat fading channel was considered in [3] whereas we consider frequency selective fading channel, 4) unlike our system the relay in [3] did not discard the CP as the channel was just a complex scalar and 5) the nodes were assumed not to be time synchronized in [3], whereas we assume that the nodes are quasi-time synchronized.

2.2. Phase 2

The received signal vector obtained after the removal of CP at destination for the lth OFDM symbol duration is given by

$$\mathbf{r}_{D,l} = \mathbf{H}_{1,D}\mathbf{s}_{1,l} + \mathbf{H}_{2,D}\mathbf{s}_{2,l} + \mathbf{v}_{D,l} \tag{2}$$

for $l = 2n, 2n+1$. In (2), $\mathbf{H}_{i,D}$ is a circulant matrix with $[\mathbf{h}^T_{i,D}, \mathbf{0}]^T$ as its first column vector for $i = 1,2$ and $\mathbf{v}_{D,l}$ is the $(N \times 1)$ AWGN vector whose covariance matrix is given by $\sigma^2_D\mathbf{I}_N$. Let $\mathbf{y}_{D,n}$ be a $(2N \times 1)$ vector given by

$$\begin{aligned}\mathbf{y}_{D,n} =& [(\mathbf{W}_N\mathbf{r}_{D,2n})^T\,(\mathbf{W}_N\mathbf{r}_{D,2n+1})^H]^T\\ =& \mathbf{H}_p\left[\mathbf{x}^T_{2n}, \mathbf{x}^H_{2n+1}\right]^T + \mathbf{f}_n\end{aligned} \tag{3}$$

where vector $\mathbf{f}_n$ contains the effective noise from relay which is circularly convolved with relay to destination channels and the AWGN at the destination. The $(2N \times 2N)$ block diagonal channel matrix $\mathbf{H}_P$ is given by

$$\mathbf{H}_p = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{\Lambda}_{1,D}\mathbf{\Lambda}_{S,1} & \mathbf{A} \\ -\mathbf{A}^* & \mathbf{\Lambda}^*_{1,D}\mathbf{\Lambda}^*_{S,1}\end{bmatrix} \tag{4}$$

where $\mathbf{\Lambda}_{S,i} = \mathbf{W}_N\mathbf{H}_{S,i}\mathbf{W}^H_N$ and $\mathbf{\Lambda}_{i,D} = \mathbf{W}_N\mathbf{H}_{i,D}\mathbf{W}^H_N$ for $i = 1,2$ and $\mathbf{A} = \mathbf{Z}_N\left(\frac{2\pi}{N}\right)\mathbf{\Lambda}_{2,D}\mathbf{\Lambda}^*_{S,2}$ with

$$\mathbf{Z}_N(\phi) = \mathrm{diag}\left(\left[1, e^{j\phi}, e^{j2\phi}, \cdots, e^{j(N-1)\phi}\right]^T\right). \tag{5}$$

Note that $\mathbf{H}_p$ has the Alamouti coded form, i.e.,

$$\mathbf{H}^H_p\mathbf{H}_p = \left(|\mathbf{\Lambda}_{1,D}|^2|\mathbf{\Lambda}_{S,1}|^2 + |\mathbf{\Lambda}_{2,D}|^2|\mathbf{\Lambda}_{S,2}|^2\right)\mathbf{I}_{2N}/2. \tag{6}$$

Therefore, the decision statistics for $\mathbf{x}_{2n}$ and $\mathbf{x}_{2n+1}$ can be obtained through simple Alamouti decoding. Furthermore, the matrix $\mathbf{H}_p$ can be easily estimated by using the following training symbols [5]:

$$\mathbf{x}_{2n} = \mathbf{p} \text{ and } \mathbf{x}_{2n+1} = -\mathbf{p} \tag{7}$$

where $\mathbf{p}$ is a $(N \times 1)$ vector such that $\mathbf{QQ}^* = \mathbf{I}_N$ with

$$\mathbf{Q} = \mathrm{diag}(\mathbf{p}). \tag{8}$$

3. PRESENCE OF CFO

Let f_S, f_R and f_D be the frequencies of the oscillators at the source, relay and destination, respectively. The normalized CFOs experienced at the relay and destination are given by $\epsilon_r = (f_S - f_R)T$ and $\epsilon_d = (f_R - f_D)T$, respectively, where T is the OFDM symbol duration. The angular CFO (ACFO) experienced at the relay and destination are given by $\phi_r = 2\pi\epsilon_r/N$ and $\phi_d = 2\pi\epsilon_d/N$, respectively. In the presence of ACFO, the received signal vector $\mathbf{r}_{R,i,l}$ in the first phase is given by

$$\mathbf{r}_{R,i,l} = \alpha_l\mathbf{Z}_N(\phi_r)\mathbf{H}_{S,i}\mathbf{W}^H_N\mathbf{x}_l + \mathbf{v}_{R,i,l} \tag{9}$$

for $i = 1,2$ and $l = 2n, 2n+1$ where $\alpha_l = e^{j(l(N+P)+P)\phi_r}$. If the relay generates the space-time signal $\mathbf{s}_{i,l}$ for $i = 1,2$ and $l = 2n, 2n+1$ without compensating for the ACFO ϕ_r, then the received signal vectors at the destination are given as follows: (the noise terms are omitted for the sake of clarity)

$$\begin{aligned}\mathbf{r}_{D,2n} =& \frac{\mathbf{Z}_N(\phi_d)}{\sqrt{2}}\Big[b_{2n}\mathbf{H}_{1,D}\mathbf{Z}_N(\phi_r)\mathbf{H}_{S,1}\mathbf{W}^H_N\mathbf{x}_{2n}\\ &+ c_n\mathbf{H}_{2,D}\mathbf{Z}_N(\phi_r)\zeta\left(\mathbf{H}^*_{S,2}\right)\left(\mathbf{W}^H_N\mathbf{x}_{2n+1}\right)^*\Big]\end{aligned} \tag{10}$$

$$\begin{aligned}\mathbf{r}_{D,2n+1} =& \frac{\mathbf{Z}_N(\phi_d)}{\sqrt{2}}\Big[b_{2n+1}\mathbf{H}_{1,D}\mathbf{Z}_N(\phi_r)\mathbf{H}_{S,1}\mathbf{W}^H_N\mathbf{x}_{2n+1}\\ &- d_n\mathbf{H}_{2,D}\mathbf{Z}_N(\phi_r)\zeta\left(\mathbf{H}^*_{S,2}\right)\left(\mathbf{W}^H_N\mathbf{x}_{2n}\right)^*\Big]\end{aligned} \tag{11}$$

where $c_n = e^{j[(2n(N+P)+P)(\phi_d-\phi_r)-(2N+P-1)\phi_r]}$, $d_n = e^{j[\{(2n+1)(N+P)+P\}(\phi_d-\phi_r)+(P+1)\phi_r]}$ and $b_l = e^{j(l(N+P)+P)(\phi_r+\phi_d)}$ for $l = 2n, 2n+1$. The presence of ACFO destroys the Alamouti code structure in the resultant channel matrix $\mathbf{H}_p$. Furthermore, (10) and (11) imply that the resultant channel matrix varies with index n and hence must be estimated for every transmission cycle. This could render the implementation of the wireless relay system infeasible. Matrix $\mathbf{Z}_N(\phi_r)$ cannot be factored out and lumped together with $\mathbf{Z}_N(\phi_d)$ in (10) and (11) and hence simultaneously compensating for ϕ_r and ϕ_d at the destination is not possible. Therefore, the relay is required to compensate for ϕ_r before constructing the space-time signals.

3.1. ACFO Estimation at Relay

Using the pilot symbols given in (7), the ML estimate for ϕ_r is given by [6]

$$\hat{\phi}_r = \angle\left(-\sum_{i=1}^{2}\mathbf{r}^H_{R,i,2n}\mathbf{r}_{R,i,2n+1}\right)/(N+P). \tag{12}$$

The above estimator is computationally simple and can be easily implemented in relay at minimal cost. The estimate $\hat{\phi}_r$ is used by the relay to adjust its carrier frequency such that the resultant CFO at relay is negligible.

4. RELAY SYNCHRONIZED WITH SOURCE

The received signal vectors at the destination, in the absence of ϕ_r but in the presence of ϕ_d, are given by

$$\begin{aligned}\mathbf{r}_{D,2n} =&\gamma_{2n}\mathbf{Z}_N(\phi_d)\Big[\mathbf{H}_{1,D}\mathbf{H}_{S,1}\mathbf{W}_N^H\mathbf{x}_{2n}\\&+\mathbf{H}_{2,D}\;\zeta\left(\mathbf{H}_{S,2}^*\right)\left(\mathbf{W}_N^H\mathbf{x}_{2n+1}\right)^*\Big]/\sqrt{2}+\mathbf{k}_{D,2n}\\\mathbf{r}_{D,2n+1} =&\gamma_{2n+1}\mathbf{Z}_N(\phi_d)\Big[-\mathbf{H}_{2,D}\;\zeta\left(\mathbf{H}_{S,2}^*\right)\left(\mathbf{W}_N^H\mathbf{x}_{2n}\right)^*\\&+\mathbf{H}_{1,D}\mathbf{H}_{S,1}\mathbf{W}_N^H\mathbf{x}_{2n+1}\Big]/\sqrt{2}+\mathbf{k}_{D,2n+1}\end{aligned} \tag{13}$$

where $\gamma_l = e^{j(l(N+P)+P)\phi_d}$ for $l = 2n, 2n+1$ and the $(N \times 1)$ noise vectors are given by

$$\begin{aligned}\mathbf{k}_{D,2n} =&\gamma_{2n}\mathbf{Z}_N(\phi_d)\left[\mathbf{H}_{1,D}\mathbf{v}_{R,1,2n}\right.\\&\left.+\mathbf{H}_{2,D}\;\zeta\left(\mathbf{v}_{R,2,2n+1}^*\right)\right]/\sqrt{2}+\mathbf{v}_{D,2n}\\\mathbf{k}_{D,2n+1} =&\gamma_{2n+1}\mathbf{Z}_N(\phi_d)\left[\mathbf{H}_{1,D}\mathbf{v}_{R,1,2n+1}\right.\\&\left.-\mathbf{H}_{2,D}\;\zeta\left(\mathbf{v}_{R,2,2n}^*\right)\right]/\sqrt{2}+\mathbf{v}_{D,2n+1}.\end{aligned} \tag{14}$$

The covariance matrix for the noise vectors $\{\mathbf{k}_{D,l}\}_{l=2n,2n+1}$ is given by

$$\mathbf{R} = \mathbf{Z}_N(\phi_d)\mathbf{R}_0\mathbf{Z}_N^H(\phi_d) \tag{15}$$

where $\mathbf{R}_0$ is a $(N \times N)$ circulant matrix expressed as follows:

$$\mathbf{R}_0 = \sigma_D^2\mathbf{I}_N + \sigma_R^2\left(\mathbf{H}_{1,D}\mathbf{H}_{1,D}^H + \mathbf{H}_{2,D}\mathbf{H}_{2,D}^H\right)/2. \tag{16}$$

This circulant structure is available in non-regenerative wireless relay networks as the noise from the relay is circularly convolved with the relay to destination channels.

5. JOINT ACFO AND CIR ESTIMATION

The received signal vectors at the destination corresponding to the pilot symbols given in (7) are stacked to form a $(2N \times 1)$ vector $\mathbf{q} = [\mathbf{r}_{D,2n}^T,\ \mathbf{r}_{D,2n+1}^T]^T$ which can be expressed as

$$\mathbf{q} = \mathbf{G}_{\phi_d}\mathbf{D}_p\mathbf{U}_p\mathbf{h}_p + \tilde{\mathbf{k}} \tag{17}$$

where

$$\mathbf{G}_{\phi_d} = \begin{bmatrix}\gamma_{2n}\mathbf{Z}_N(\phi_d) & \mathbf{0}\\ \mathbf{0} & \gamma_{2n+1}\mathbf{Z}_N(\phi_d)\end{bmatrix} \tag{18}$$

and noise vector $\tilde{\mathbf{k}} = [\mathbf{k}_{D,2n}^T, \mathbf{k}_{D,2n+1}^T]^T$ whose covariance matrix is given by $\tilde{\mathbf{R}} = \mathbf{I}_2 \otimes \mathbf{R}$. Let L be defined as $L = L_1 + L_2 - 1$. The $(2L \times 1)$ product channel vector $\mathbf{h}_p$ is given by

$$\mathbf{h}_p = \left[(\mathbf{h}_{1,D} \star \mathbf{h}_{S,1})^T, (\mathbf{h}_{2,D} \star \zeta(\mathbf{h}_{S,2}^*))^T\right]^T. \tag{19}$$

The $(2N \times 2N)$ unitary matrix $\mathbf{D}_p$ is defined as follows:

$$\mathbf{D}_p = \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{W}_N^H & \mathbf{0}\\ \mathbf{0} & \mathbf{W}_N^H\end{bmatrix}\begin{bmatrix}\mathbf{Q} & -\mathbf{Q}^*\\ -\mathbf{Q} & -\mathbf{Q}^*\end{bmatrix}. \tag{20}$$

The $(2N \times 2L)$ matrix $\mathbf{U}_p$ is given by

$$\mathbf{U}_p = \sqrt{N}\begin{bmatrix}\mathbf{U}_a & \mathbf{0}\\ \mathbf{0} & \mathbf{U}_b\end{bmatrix} \tag{21}$$

where

$$\begin{aligned}\mathbf{U}_a =&\mathbf{W}_N(:,1:L)\\\mathbf{U}_b =&[\mathbf{W}_N(:,N-L_1+1:N), \mathbf{W}_N(:,1:L_2-1)].\end{aligned} \tag{22}$$

Let matrix $\mathbf{X}$ be defined as $\mathbf{X} = \mathbf{D}_p\mathbf{U}_p$. Note that the matrix $\mathbf{X}$ has full column rank of $2L$ and $\mathbf{X}^H\mathbf{X} = N\mathbf{I}_{2L}$. The ML and LS based estimates for $\mathbf{h}_p$ are [4]

$$\begin{aligned}\mathbf{h}_p^{\mathrm{ML}} &= \left(\mathbf{S}_\phi^H\mathbf{S}_\phi\right)^{-1}\mathbf{S}_\phi^H\tilde{\mathbf{R}}^{\frac{-1}{2}}\mathbf{q}\\\mathbf{h}_p^{\mathrm{LS}} &= \left(\mathbf{X}^H\mathbf{G}_\phi^H\mathbf{q}\right)/N\end{aligned} \tag{23}$$

where $\mathbf{S}_\phi = \tilde{\mathbf{R}}^{-1/2}\mathbf{G}_\phi\mathbf{X}$ and the diagonal matrix $\mathbf{G}_\phi$ is obtained by replacing ϕ_d by variable ϕ in $\mathbf{G}_{\phi_d}$. Substituting the estimates for $\mathbf{h}_p$ into the respective ML and LS cost functions, the ML and LS based estimates for ϕ_d are obtained as follows:

$$\begin{aligned}\phi_d^{\mathrm{ML}} =&arg\min_{\phi\in[-\pi,\pi)}\left\|(\mathbf{I}_{2N} - \mathbf{S}_\phi(\mathbf{S}_\phi^H\mathbf{S}_\phi)^{-1}\mathbf{S}_\phi^H)\tilde{\mathbf{R}}^{\frac{-1}{2}}\mathbf{q}\right\|^2\\\phi_d^{\mathrm{LS}} =&arg\min_{\phi\in[-\pi,\pi)}\left\|\left(\mathbf{I}_{2N} - (\mathbf{X}\mathbf{X}^H)/N\right)\mathbf{G}_\phi^H\mathbf{q}\right\|^2.\end{aligned} \tag{24}$$

A grid search is employed to obtain the estimate for ϕ_d using (24). The ML based ACFO estimator requires inversion of $(2L \times 2L)$ matrix $\mathbf{S}_\phi^H\mathbf{S}_\phi$ for every search point, whereas no matrix inversion is involved in LS based ACFO estimator. Note that

$$\begin{aligned}&\mathrm{ML}: \tilde{\mathbf{R}}^{-1/2}\mathbf{q} = \tilde{\mathbf{R}}^{-1/2}\mathbf{G}_{\phi_d}\mathbf{X}\mathbf{h}_p + \tilde{\mathbf{R}}^{-1/2}\tilde{\mathbf{k}}\\&\mathrm{LS}: \quad \mathbf{G}_{\phi_d}^H\mathbf{q} = \mathbf{X}\mathbf{h}_p + \mathbf{G}_{\phi_d}^H\tilde{\mathbf{k}}.\end{aligned} \tag{25}$$

The signal subspace for the ML and LS based ACFO estimators are given by the column space of matrices $\mathbf{X}$ and $\tilde{\mathbf{R}}^{-1/2}\mathbf{G}_{\phi_d}\mathbf{X}$, respectively. Both the estimators exploit the orthogonality between the signal and noise subspaces. The noise whitening matrix $\tilde{\mathbf{R}}^{-1/2}$ is full rank and does not change the signal subspace dimension nor the signal to noise ratio (SNR) [4]. As a result, the above ACFO estimators will have the same performance but the LS based ACFO estimator has reduced computational complexity. Once ϕ_d is estimated, the corresponding product channel vector $\mathbf{h}_p$ can be estimated by substituting $\phi = \phi_d^t$ in (23) for $t = \{\mathrm{ML}, \mathrm{LS}\}$.

5.1. Equivalence between LS and ML based CIR Estimators

In [7], the ML estimate of covariance matrix was shown to be equivalent to the estimate obtained using periodogram provided that the covariance matrix is circulant. In our system model, the covariance matrix of the total noise at destination is circulant in the absence of ϕ_d. In the following, we show that the ML based CIR estimator is indeed the LS based CIR estimator.

Assuming that ϕ_d has been perfectly estimated, one can perform the following unitary transformation to $\mathbf{q}$ in (19) as follows:

$$\tilde{\mathbf{q}} = \mathbf{B}\mathbf{D}_P^H\mathbf{G}_{\phi_d}^H\mathbf{q} = \sqrt{N}\mathbf{a}_p + \mathbf{B}\mathbf{D}_P^H\mathbf{G}_{\phi_d}^H\tilde{\mathbf{k}} \tag{26}$$

where

$$\mathbf{B} = \begin{bmatrix}\mathbf{W}_N^H & \mathbf{0}\\ \mathbf{0} & [\mathbf{U}_b, \mathbf{W}_N(:,L_2:N-L_1)]^H\end{bmatrix} \tag{27}$$

and $\mathbf{B}^H\mathbf{B} = \mathbf{I}_{2N}$. The $(2N \times 1)$ vector $\mathbf{a}_p$ is obtained to be

$$\mathbf{a}_p = \left[(\mathbf{h}_{1,D} \star \mathbf{h}_{S,1})^T, \mathbf{0}, (\mathbf{h}_{2,D} \star \zeta(\mathbf{h}_{S,2}^*))^T, \mathbf{0}\right]^T. \tag{28}$$

Let $\tilde{\tilde{\mathbf{q}}}$ be a $(2L \times 1)$ vector defined as

$$\tilde{\tilde{\mathbf{q}}} = \mathbf{V}\tilde{\mathbf{q}} = \sqrt{N}\mathbf{h}_p + \mathbf{V}\mathbf{B}\mathbf{D}_p^H\mathbf{G}_{\phi_d}^H\tilde{\mathbf{k}}, \tag{29}$$

where $\mathbf{V}$ is a $(2L \times 2N)$ matrix given by

$$\mathbf{V} = \begin{bmatrix} \mathbf{I}_L & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_L & \mathbf{0} \end{bmatrix}. \tag{30}$$

Note that the ML estimate for $\mathbf{h}_p$ obtained from (17) and (26) are the same. Furthermore, the ML estimate for $\mathbf{h}_p$ obtained from (30) and (26) are the same since the elements of $\tilde{\mathbf{q}}$ where elements of $\mathbf{a}_p$ are zeros carry no information about $\mathbf{h}_p$. The ML estimate for $\mathbf{h}_p$ obtained from (29) is given by

$$\mathbf{h}_p^{\mathrm{ML}} = \mathbf{V}\mathbf{B}\mathbf{D}_p^H \mathbf{G}_{\phi_d}^H \mathbf{q}/\sqrt{N}. \tag{31}$$

As $\mathbf{V}\mathbf{B} = \mathbf{U}_p^H/\sqrt{N}$, it can be concluded from observing (23) and (31) that

$$\mathbf{h}_p^{\mathrm{ML}} = \mathbf{h}_p^{\mathrm{LS}}. \tag{32}$$

The mean square error (MSE) for the estimated $\mathbf{h}_p$ obtained using (31) is

$$E[\|\mathbf{h}_p - \mathbf{h}_p^{\mathrm{ML}}\|^2 = tr\left(\mathbf{V}\mathbf{B}\mathbf{D}_p^H\left(\mathbf{I}_2 \otimes \mathbf{R}_0\right)\mathbf{D}_p\mathbf{B}^H\mathbf{V}^T\right)/N \tag{33}$$

Exploiting the circulant structure of $\mathbf{R}_0$, we have

$$\begin{aligned} E[\|\mathbf{h}_p - \mathbf{h}_p^{\mathrm{ML}}\|^2 &= tr\left(\mathbf{V}\left(\mathbf{I}_2 \otimes \mathbf{R}_0\right)\mathbf{V}^T\right)/N = 2L\ \mathbf{R}_0(1,1)/N \\ &= 2L\left(\sigma_D^2 + \sigma_R^2(\mathbf{h}_{1,D}^H\mathbf{h}_{1,D} + \mathbf{h}_{2,D}^H\mathbf{h}_{2,D})/2\right)/N. \end{aligned} \tag{34}$$

6. SIMULATION RESULTS

In this section, we illustrate the MSE performance of ML and LS based joint ACFO and CIR estimators and compare it with the Cramer-Rao lower bound (CRLB). The number of subcarriers $N = 64$, length of CP $P = 16$ and the number of taps in CIR is $L_1 = L_2 = 10$. The normalized CFOs ϵ_r and ϵ_d are assumed to be a uniformly distributed over the interval $[-0.35, 0.35]$. Estimator in (13) is used to compensate for ϕ_r at relay. The ML and LS based estimators given in (23) and (24) are used to obtain the simulation results. The estimates are obtained using only the received signal over 2 OFDM symbol durations. The grid size for grid search used to estimate the ACFO is 0.0001. The SNR is defined as the ratio of desired signal power to the total noise power at destination. The MSE for ϵ_d and $\mathbf{h}_p$ are computed as $E\left[((\phi_d - \phi_d^t)N/(2\pi))^2\right]$ and $E\left[(\|\mathbf{h}_p - \mathbf{h}_p^t\|^2/\mathbf{h}_p^H\mathbf{h}_p)\right]$, respectively, where $t = \{\mathrm{ML}, \mathrm{LS}\}$. The CRLB for MSE corresponding to ACFO and CIR estimates are obtained following the steps in [4]. Observing Figs. 1 and 2, it is found that MSE performance of ML and LS based estimators overlap and achieve the CRLB. This confirms our theoretical findings. Furthermore, it can be seen that the theoretical MSE obtained in (34) overlaps with the CRLB and the estimators' MSE performance.

7. CONCLUSION

In this paper, we have shown that the relay must perform CFO compensation in non-regenerative wireless relay networks with relays employing space-time coding. The covariance matrix of the total noise at destination is shown to be circulant in the absence of CFO at destination. Furthermore, the ML and LS based estimators for ACFO and CIR estimation are shown to be identical. Thus, the simple LS based estimator can be used to obtain the ML estimates for ACFO and CIR. The theoretical MSE obtained for the product channel estimate was verified through computer simulations. Lastly, this work provides some insight into the design of joint CFO and channel estimation algorithms when the relays are spatially distributed and have their carrier frequencies synchronized with the source.

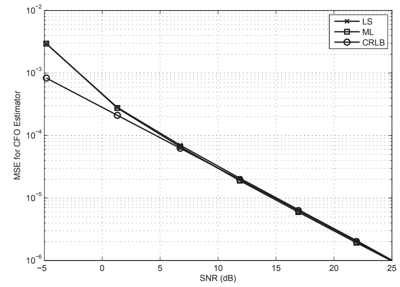

Fig. 1. MSE Performance for CFO Estimator.

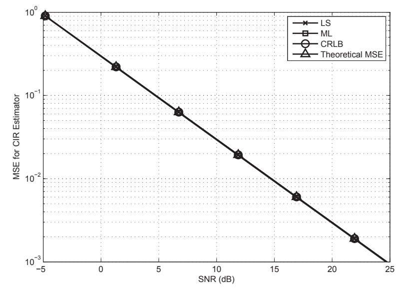

Fig. 2. MSE Performance for CIR Estimator.

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