# SUBSPACE TRACKING OF FAST TIME-VARYING CHANNELS IN PRECODED MIMO-OFDM SYSTEMS

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# ABSTRACT

This paper presents a blind subspace-based tracking scheme for precoded MIMO-OFDM systems over rapidly time-varying wireless channels. Subspace-based tracking is normally considered for slow time-varying channels only. Thanks to the frequency correlation of the wireless channels, the proposed scheme is able to collect data not only from the time but also from the frequency domain to speed up the update of the required second-order statistics. After each update of the statistics, the subspace information is also updated using orthogonal iteration, and then a new channel estimate is computed. The proposed algorithm is verified in 3GPP-SCM suburban macro scenario, in which a mobile station is allowed to move in any direction with a constant speed of 100km/h. The simulation results show that the root mean square error of the channel estimates converges to the level of  $2 \times 10^{-2}$  within less than 5 OFDM symbols even for such a high Doppler rate.

*Index Terms*— Doppler effect, MIMO-OFDM systems, sub-space tracking, 3GPP-SCM, precoding matrix.

#### **1. INTRODUCTION**

Tracking time-varying (TV) channels with a large Doppler spread is a critical task, regardless of whether a nonblind or blind approach is used [1]. In this situation, a nonblind approach in general requires to employ training more frequently since the channel estimate becomes obsolete shortly after the training period ends. On the contrary, a blind approach totally eliminates the need of large amount of training data and therefore is favoured if complexity is not the main concern.

Among various blind approaches, a basis expansion model has been proposed to convert a TV single input single output (SISO) channel into a time-invariant (TI) single input multiple output (SIMO) channel, followed by a standard second-order statistics (SOS) based subspace method for blind channel estimation [2]. The idea of basis expansion was further extended for a TV-SIMO channel [3][4], and a generalized OFDM system over a TV-SISO channel [5]. Similarly, an interpolation model was also proposed to convert a TV-SISO channel into fixed parameters in a long-code code division multiple access system [6].

More recently, a zero padding SISO-OFDM system associated with either recursive least squares or least mean squares method for blind adaptive channel estimation was considered in [7]. It was reported that for an IFFT size of 64 and padding length of 16, the relative channel estimation errors converge to -27dB in 500 symbols when the maximum Doppler shift is limited to 100Hz and the SNR is 20dB. By properly choosing the so-called repetition index, a cyclic prefixing SISO-OFDM system was also proposed in [8]. It was reported that for a 64-point IFFT with a cyclic prefix length of 16, the bit error rate can reach the level of  $10^{-2}$  within 12 received blocks when the maximum Doppler shift is 50Hz and the SNR  $\geq$  25dB. To date, the studies on tracking of TV channels with a high Doppler rate, specifically in the context of a MIMO-OFDM system are scarce. In this paper, the subspace-based blind estimation exploiting frequency correlation over a TI channel [9][10] is extended to a TV scenario. The drawbacks of this approach, which requires a larger dimension of the ambiguity matrix, and a high-complexity singular value decomposition, are overcome by using a precoder at the transmitter side, and subspace-based tracking, respectively.

The notation used in this paper is as follows: A vector is denoted by a bold lower-case letter and a matrix is denoted by a bold uppercase letter. The hat notation  $\hat{\mathbf{X}}$  denotes an estimate of  $\mathbf{X}$ , and  $\Delta \mathbf{X} = \mathbf{X} - \hat{\mathbf{X}}$  denotes the difference between  $\mathbf{X}$  and  $\hat{\mathbf{X}}$ . E[x] denotes the expected value of the random variable x.  $\otimes$  denotes the Kronecker product, and  $\oslash$  denotes the elementwise division.  $\mathbf{X}^{\dagger}$  represents the pseudo-inverse of a matrix  $\mathbf{X}$ .

## 2. SYSTEM MODEL

We consider a MIMO-OFDM system with  $N_T$  transmit and  $N_R$  receive antennas, employing  $N_C$  subcarriers. Let the *m*th OFDM symbol transmitted from the *j*th transmit antenna be denoted as  $\mathbf{x}_j^m := \begin{bmatrix} x_j^m[0] & x_j^m[1] & \cdots & x_j^m[N_C - 1] \end{bmatrix}^T$ , where  $x_j^m[k]$  is the symbol transmitted at the *k*th subcarrier. Then the *m*th OFDM symbol transmitted over  $N_T$  transmit antennas can be written as  $\mathbf{x}^m := \begin{bmatrix} \mathbf{x}_1^{mT} & \mathbf{x}_2^{mT} & \cdots & \mathbf{x}_{N_T}^m \end{bmatrix}^T$ . At the receiver, let the *m*th received OFDM symbol from the *i*th receive antenna be denoted as  $\mathbf{y}_i^m := \begin{bmatrix} y_i^m[0] & y_i^m[1] & \cdots & y_i^m[N_C - 1] \end{bmatrix}^T$ , where  $y_i^m[k]$  is the symbol received at the *k*th subcarrier. Then the *m*th OFDM symbol received at the *k*th subcarrier. Then the *m*th ofDM symbol received over  $N_R$  receive antennas can be written as  $\mathbf{y}^m := \begin{bmatrix} \mathbf{y}_1^{mT} & \mathbf{y}_2^{mT} & \cdots & \mathbf{y}_{N_R}^m \end{bmatrix}^T$ . In the following, we assume that i) The length of the cyclic prefix appended to each OFDM symbol is longer than the maximum excess delay of the channel. ii) The channel can be considered time-invariant at least over each OFDM symbol. iii) The average power of the transmit symbol alphabet is normalized such that  $E[|x_1^m[k]|^2] = 1$ .

For MIMO-OFDM systems, the signal received at the *i*th receive antenna over the kth subcarrier and the mth OFDM symbol is given by

$$y_{i}^{m}[k] = \sqrt{\frac{E_{s}}{N_{T}}} \sum_{j=1}^{N_{T}} h_{i,j}^{m}[k] x_{j}^{m}[k] + n_{i}^{m}[k], \ i = 1, 2, \cdots, N_{R},$$
(1)

where  $E_s$  is the average energy evenly divided across the transmit antennas and allocated to the *k*th subcarrier,  $n_i^m[k]$  represents the zero mean circularly symmetric complex Gaussian (ZM-CSCG) noise at the *i*th receive antenna over the *k*th subcarrier and the *m*th OFDM symbol, and  $h_{i,j}^m[k]$  represents the equivalent frequency response between the rich receive antenna and the *j*th transmit antenna, over the *k*th subcarrier and the *m*th OFDM symbol. Let  $\mathbf{n}_i^m := [n_i^m[0] \ n_i^m[1] \ \cdots \ n_i^m[N_C - 1]]^T$  with  $\mathbf{n}^m := [\mathbf{n}_1^{mT} \ \mathbf{n}_2^{mT} \ \cdots \ \mathbf{n}_{N_R}^m]^T$ . Then the input-output relation of the MIMO-OFDM system may be expressed by

$$\mathbf{y}^{m} = \sqrt{\frac{E_{s}}{N_{T}}} \mathcal{H}^{m} \mathbf{x}^{m} + \mathbf{n}^{m}, \qquad (2)$$

$$\mathcal{H}^{m} = \begin{bmatrix} \mathcal{H}_{1,1}^{m} & \mathcal{H}_{1,2}^{m} & \cdots & \mathcal{H}_{1,N_{T}}^{m} \\ \mathcal{H}_{2,1}^{m} & \mathcal{H}_{2,2}^{m} & \cdots & \mathcal{H}_{2,N_{T}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{H}_{N_{R},1}^{m} & \mathcal{H}_{N_{R},2}^{m} & \cdots & \mathcal{H}_{N_{R},N_{T}}^{m} \end{bmatrix}, \qquad (3)$$

with  $\mathcal{H}_{i,j}^m := \text{diag}(h_{i,j}^m[0], h_{i,j}^m[1], \cdots, h_{i,j}^m[N_C - 1]).$ 

## 2.1. Subcarrier Grouping

Subspace-based blind estimators are in general not favoured when a fast time-varying channel is considered, since there may not be sufficient time samples to obtain the required statistics. However, by exploiting frequency correlation among adjacent subcarriers through the concept of subcarrier grouping, time-invariance requirement can be significantly relaxed [9][10]. In the following, assume that the frequency span of *P* adjacent subcarriers reside inside the coherence bandwidth of the wireless channel. Let  $S := \{0, 1, \dots, N_C - 1\}$  be the index set of the  $N_C$  subcarriers. *S* is partitioned into *P* disjoint subsets (assuming  $(N_C/P) = \zeta \in \mathbb{Z}^+$ ) with each subset denoted as  $S_p := \{s_{p,1}, s_{p,2}, \dots, s_{p,\zeta}\}$ , where  $s_{p,i} := p - 1 + (i - 1)P$ ,  $i = 1, 2, \dots, \zeta$  for  $p = 1, 2, \dots, P$ . Note that  $S_1 \cup S_2 \cup \dots \cup S_P =$ S, and  $S_i \cap S_j = \emptyset$ , where  $\emptyset$  denotes the empty set. Let  $\mathbf{x}_p^m := [\mathbf{x}_{1,p}^m T \ \mathbf{x}_{2,p}^m T \cdots \mathbf{x}_{N_T,p}^m T]^T$ ,  $\mathbf{y}_p^m := [\mathbf{y}_{1,p}^m T \ \mathbf{y}_{2,p}^m T \cdots \mathbf{y}_{N_R,p}^m T]^T$ , and  $\mathbf{n}_p^m := [\mathbf{n}_{1,p}^m T \ \mathbf{n}_{2,p}^m T \cdots \mathbf{n}_{N_R,p}^m T]^T$ , where

$$\begin{aligned} \mathbf{x}_{j,p}^{m} &:= & [ x_{j}^{m}[s_{p,1}] x_{j}^{m}[s_{p,2}] \cdots x_{j}^{m}[s_{p,\zeta}] ]^{T}, \\ \mathbf{y}_{i,p}^{m} &:= & [ y_{i}^{m}[s_{p,1}] y_{i}^{m}[s_{p,2}] \cdots y_{i}^{m}[s_{p,\zeta}] ]^{T}, \\ \mathbf{n}_{i,p}^{m} &:= & [ n_{i}^{m}[s_{p,1}] n_{i}^{m}[s_{p,2}] \cdots n_{i}^{m}[s_{p,\zeta}] ]^{T}. \end{aligned}$$

$$\end{aligned}$$

Define  $\mathcal{H}_{i,j,p}^m = \text{diag}(\tilde{\mathbf{h}}_{i,j,p}^m)$ , where  $\tilde{\mathbf{h}}_{i,j,p}^m := [h_{i,j}^m[s_{p,1}], h_{i,j}^m[s_{p,2}]$ ,  $\cdots$ ,  $h_{i,j}^m[s_{p,\zeta}]^T$ . Then (2) can be rewritten for the *p*th subset as

$$\mathbf{y}_p^m = \sqrt{\frac{E_s}{N_T}} \,\mathcal{H}_p^m \mathbf{x}_p^m + \mathbf{n}_p^m, \ p = 1, 2, \cdots, P, \tag{5}$$

$$\mathcal{H}_{p}^{m} = \begin{bmatrix} \mathcal{H}_{1,1,p}^{m} & \mathcal{H}_{1,2,p}^{m} & \cdots & \mathcal{H}_{1,N_{T},p}^{m} \\ \mathcal{H}_{2,1,p}^{m} & \mathcal{H}_{2,2,p}^{m} & \cdots & \mathcal{H}_{2,N_{T},p}^{m} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{H}_{N_{R},1,p}^{m} & \mathcal{H}_{N_{R},2,p}^{m} & \cdots & \mathcal{H}_{N_{R},N_{T},p}^{m} \end{bmatrix}.$$
 (6)

#### 2.2. Nonredundant Precoding

The price paid for exploiting frequency correlation to relax the timeinvariance requirement is a higher-dimensional ambiguity matrix, leading to a somewhat higher receiver complexity [9][10]. Although the dimension can be reduced when the ratio of the coherence bandwidth to the channel bandwidth is large, it can be simply reduced to the size of  $N_T \times N_T$  if precoding is applied at the transmitter side:

Suppose the input vector  $\mathbf{x}_{j,p}^m$  in (4) is precoded by the matrix  $\mathbf{\Upsilon} \in \mathbb{C}^{\zeta \times \zeta}$  (The choice of  $\mathbf{\Upsilon}$  is considered in Section 4). Then (5) can be updated accordingly by

$$\mathbf{y}_{p}^{m} = \sqrt{\frac{E_{s}}{N_{T}}} \,\mathcal{H}_{p}^{m} \left( \mathbf{I}_{N_{T}} \otimes \boldsymbol{\Upsilon} \right) \mathbf{x}_{p}^{m} + \mathbf{n}_{p}^{m}. \tag{7}$$

For simplicity in notation, let us temporarily drop the time-index m of all the channel related coefficients in the rest of this sub-section. The correlation matrix  $\mathbf{R}_{\mathbf{y}_p} := E[\mathbf{y}_p \mathbf{y}_p^H]$  can thus be rewritten by

$$\mathbf{R}_{\mathbf{y}_p} = \frac{\sigma_s^2 E_s}{N_T} \mathcal{H}_p \left( \mathbf{I}_{N_T} \otimes \Upsilon \Upsilon^H \right) \mathcal{H}_p^H + \sigma_n^2 \mathbf{I}, \qquad (8)$$

where we have assumed that  $\mathbf{n}_p$  and  $\mathbf{x}_p$  are uncorrelated,  $E[\mathbf{n}_p \mathbf{n}_p^H] = \sigma_n^2 \mathbf{I}$ , and  $E[\mathbf{x}_p \mathbf{x}_p^H] = \sigma_s^2 \mathbf{I}$ . By partitioning  $\mathbf{R}_{\mathbf{y}_p}$  into submatrices of size  $\zeta \times \zeta$  as

$$\mathbf{R}_{\mathbf{y}_{p}} = \begin{bmatrix} \mathbf{R}_{\mathbf{y}_{p},11} & \cdots & \mathbf{R}_{\mathbf{y}_{p},1N_{R}} \\ \mathbf{R}_{\mathbf{y}_{p},21} & \cdots & \mathbf{R}_{\mathbf{y}_{p},2N_{R}} \\ \vdots & \ddots & \vdots \\ \mathbf{R}_{\mathbf{y}_{p},N_{R}1} & \cdots & \mathbf{R}_{\mathbf{y}_{p},N_{R}N_{R}} \end{bmatrix},$$
(9)

we may similarly express the (u, v)th submatrix as

$$\mathbf{R}_{\mathbf{y}_{p},uv} = \frac{\sigma_{s}^{2} E_{s}}{N_{T}} \left( \sum_{j=1}^{N_{T}} \tilde{\mathbf{h}}_{u,j,p} \tilde{\mathbf{h}}_{v,j,p}^{H} \right) \odot \Upsilon \Upsilon^{H} + \delta(u-v) \sigma_{n}^{2} \mathbf{I},$$

where  $\odot$  represents the Hadamard product [11]. By re-arranging above equation and further defining

$$\mathbf{W}_{p,uv} = \left[\mathbf{R}_{\mathbf{y}_{p},uv} - \delta(u-v)\sigma_{n}^{2}\mathbf{I}\right] \oslash \mathbf{\Upsilon}^{H}, \qquad (10)$$

we can then construct

$$\mathbf{W}_{p} = \begin{bmatrix} \mathbf{W}_{p,11} & \cdots & \mathbf{W}_{p,1N_{R}} \\ \mathbf{W}_{p,21} & \cdots & \mathbf{W}_{p,2N_{R}} \\ \vdots & \ddots & \vdots \\ \mathbf{W}_{p,N_{R}1} & \cdots & \mathbf{W}_{p,N_{R}N_{R}} \end{bmatrix}$$
(11)

and arrive at

$$\mathbf{W}_{p} = \frac{\sigma_{s}^{2} E_{s}}{N_{T}} \tilde{\mathbf{H}}_{p} \tilde{\mathbf{H}}_{p}^{H}, \qquad (12)$$

where

$$\tilde{\mathbf{H}}_{p} := \begin{bmatrix} \tilde{\mathbf{h}}_{1,1,p} & \tilde{\mathbf{h}}_{1,2,p} & \cdots & \tilde{\mathbf{h}}_{1,N_{T},p} \\ \tilde{\mathbf{h}}_{2,1,p} & \tilde{\mathbf{h}}_{2,2,p} & \cdots & \tilde{\mathbf{h}}_{2,N_{T},p} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\mathbf{h}}_{N_{R},1,p} & \tilde{\mathbf{h}}_{N_{R},2,p} & \cdots & \tilde{\mathbf{h}}_{N_{R},N_{T},p} \end{bmatrix}.$$
(13)

By exploiting (12) and assuming  $\tilde{\mathbf{H}}_p \in \mathbb{C}^{(N_R \zeta) \times N_T}$  has full column rank, we can thus obtain the channel estimate by

$$\hat{\mathbf{H}}_p = \mathbf{U}_p \mathbf{A},\tag{14}$$

where  $\mathbf{U}_p \in \mathbb{C}^{(N_R \zeta) \times N_T}$  is constructed from the eigenvectors of the matrix  $\mathbf{W}_p$  corresponding to the  $N_T$  largest eigenvalues, and  $\mathbf{A}$  is an ambiguity matrix of size  $N_T \times N_T$ .

#### 2.3. Exploiting Frequency Correlation

Since coherence bandwidth is the range of frequencies over which the frequency response remains roughly the same [12], the subchannel matrices  $\mathcal{H}_p^m$ ,  $p = 1, 2, \dots, P$  can be *approximated* by denoting  $\bar{\mathcal{H}}^m = \mathcal{H}_1^m = \mathcal{H}_2^m = \dots = \mathcal{H}_P^{m,1}$  Therefore, an estimate of the correlation matrix in (8) can be obtained as

$$\hat{\mathbf{R}}_{\bar{\mathbf{y}}} = \sum_{m=1}^{T_{av}} \sum_{p=1}^{P} \mathbf{y}_p^m \mathbf{y}_p^{mH}, \qquad (15)$$

by exploiting the frequency correlation over the P subchannel matrices, where  $T_{av}$  denotes the number of time samples. By doing so, the number of time samples required can thus be significantly reduced since the order of the correlation matrix is reduced by a factor of P, and an averaging over P subsets, which is equivalent to the frequency averaging, is applied at each epoch.

## 3. SUBSPACE TRACKING

When the wireless channel is time-variant, we can employ a sliding window to estimate the correlation matrix by

$$\hat{\mathbf{R}}_{\bar{\mathbf{y}}}^{m} = \sum_{n=m-l+1}^{m} \sum_{p=1}^{P} \mathbf{y}_{p}^{n} \mathbf{y}_{p}^{nH}, \qquad (16)$$

with *l* denoting the window length. Note that  $\hat{\mathbf{R}}_{\bar{\mathbf{y}}}^{m}$  can also be recursively updated by

$$\hat{\mathbf{R}}_{\bar{\mathbf{y}}}^{m} = \hat{\mathbf{R}}_{\bar{\mathbf{y}}}^{m-1} + \sum_{p=1}^{P} \mathbf{y}_{p}^{m} \mathbf{y}_{p}^{mH} - \sum_{p=1}^{P} \mathbf{y}_{p}^{m-l} \mathbf{y}_{p}^{m-l}^{H}.$$
 (17)

By further defining

$$\hat{\mathbf{W}}^{m} = \begin{bmatrix} \hat{\mathbf{W}}_{11}^{m} \cdots & \hat{\mathbf{W}}_{1N_{R}}^{m} \\ \hat{\mathbf{W}}_{21}^{m} \cdots & \hat{\mathbf{W}}_{2N_{R}}^{m} \\ \vdots & \ddots & \vdots \\ \hat{\mathbf{W}}_{N_{R}1}^{m} \cdots & \hat{\mathbf{W}}_{N_{R}N_{R}}^{m} \end{bmatrix}, \qquad (18)$$

with the (u, v)th submatrix given as

$$\hat{\mathbf{W}}_{uv}^{m} = \begin{bmatrix} \hat{\mathbf{R}}_{\hat{\mathbf{y}},uv}^{m} - \delta(u-v)\hat{\sigma}_{n}^{2}\mathbf{I} \end{bmatrix} \oslash \Upsilon\Upsilon^{H},$$
(19)

we can recursively update the channel estimate  $\hat{\mathbf{H}}^m$  by employing the orthogonal iteration [13] to recursively update the principal eigenvectors of the matrix  $\hat{\mathbf{W}}^m$ . The resulting algorithm is given in Algorithm 1, where it is assumed that a noise variance estimate  $\hat{\sigma}_n^2$ can be obtained from additional estimation [14].

## 4. CHOICE OF THE PRECODER

Note that (19) can be rewritten as

$$\hat{\mathbf{W}}_{uv}^{m} = \left[ (\mathbf{R}_{\bar{\mathbf{y}},uv}^{m} + \Delta \mathbf{R}_{\bar{\mathbf{y}},uv}^{m}) - \delta(u - v)(\sigma_{n}^{2} + \Delta \sigma_{n}^{2})\mathbf{I} \right] \oslash \Upsilon\Upsilon^{H} \\
= \underbrace{\left[ \mathbf{R}_{\bar{\mathbf{y}},uv}^{m} - \delta(u - v)\sigma_{n}^{2}\mathbf{I} \right] \oslash \Upsilon\Upsilon^{H}}_{:=\mathbf{W}_{uv}^{m}} \\
+ \underbrace{\left[ \Delta \mathbf{R}_{\bar{\mathbf{y}},uv}^{m} - \delta(u - v)\Delta\sigma_{n}^{2}\mathbf{I} \right] \oslash \Upsilon\Upsilon^{H}}_{:=\Delta\mathbf{W}_{uv}^{m}}, \quad (20)$$

Algorithm 1 Tracking the channel estimate  $\hat{\mathbf{H}}^m$  by orthogonal iteration

Initialization: 
$$\mathbf{Q}^{l-1} = \mathbf{I}(:, 1:N_T), \hat{\mathbf{R}}_{\mathbf{y}}^0 = \mathbf{0}$$
  
Input vector:  $\mathbf{y}_p^m$   
for  $m = 1, 2, \cdots$  do  
if  $m < l$  then  
 $\hat{\mathbf{R}}_{\mathbf{y}}^m = \hat{\mathbf{R}}_{\mathbf{y}}^{m-1} + \sum_{p=1}^{P} \mathbf{y}_p^m \mathbf{y}_p^{mH}$   
else  
 $\hat{\mathbf{R}}_{\mathbf{y}}^m = \hat{\mathbf{R}}_{\mathbf{y}}^{m-1} + \sum_{p=1}^{P} \mathbf{y}_p^m \mathbf{y}_p^{mH} - \sum_{p=1}^{P} \mathbf{y}_p^{m-l} \mathbf{y}_p^{m-l}^H$  (17)  
 $\hat{\mathbf{W}}_{uv}^m = \left[\hat{\mathbf{R}}_{\mathbf{y},uv}^m - \delta(u-v)\hat{\sigma}_n^2\mathbf{I}\right] \oslash \Upsilon\Upsilon^H$  (18)-(19)  
 $\mathbf{U}^m = \hat{\mathbf{W}}^m \mathbf{Q}^{m-1}$   
 $\mathbf{Q}^m \mathbf{R}^m = \mathbf{U}^m$  (QR factorization on  $\mathbf{U}^m$ )  
 $\mathbf{A}^m = (\mathbf{U}^m)^{\dagger} \tilde{\mathbf{H}}_p^m$   
 $\hat{\mathbf{H}}^m = \mathbf{U}^m \mathbf{A}^m$  (14)  
end if  
end for

where  $\Delta \mathbf{W}_{uv}^m$  represents the perturbation from  $\mathbf{W}_{uv}^m$ . Therefore, as the channel estimate is constructed from the principal eigenvectors of  $\hat{\mathbf{W}}^m$ , the choice of the precoder  $\boldsymbol{\Upsilon}$  should focus on eliminating the effects of the perturbation matrix  $\Delta \mathbf{W}_{uv}^m$ , under the constraint that the entries of  $\boldsymbol{\Upsilon}\boldsymbol{\Upsilon}^H$  are not zero, i.e.,  $[\boldsymbol{\Upsilon}\boldsymbol{\Upsilon}^H]_{i,j} \neq 0, \forall i, j$ .

Due to limited space, we only discuss a simplified precoder design here, where the matrix  $\Upsilon$  only depends on a single coefficient, i.e.,

$$\mathbf{\Upsilon} := \frac{1}{\sqrt{p^2(\zeta - 1) + 1}} \begin{bmatrix} 1 & p & \cdots & p \\ p & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & p \\ p & \cdots & p & 1 \end{bmatrix}_{\zeta \times \zeta}$$
(21)

The structure also offers the advantage of reducing the number of unknowns to be optimized. Note that a similar structure is proposed in [11] for a *time-invariant* scenario.

#### 5. EXPERIMENT RESULTS

The best choice of the sliding window length l as well as the precoder coefficient p is determined by the statistics of the wireless channel under consideration. Here, for illustrative purpose, we consider a 3GPP-SCM suburban macro scenario [15], where the mobile station is allowed to travel in a random direction at a constant speed of 100km/h. The MIMO system consists of 2 transmit ( $N_T = 2$ ) and 3 receive antennas ( $N_R = 3$ ). The number of subcarriers used in the OFDM system is 256 ( $N_C = 256$ ) and P is chosen to be 64. For each time epoch, the incoming symbol streams are independent and identically distributed (i.i.d) QPSK symbols. In our OFDM system setup, the subcarrier spacing is chosen as 10.94kHz given the OFDM useful symbol duration is  $91.4\mu$ s and cyclic prefix length is 11.4 $\mu$ s. Since we consider  $N_C$  = 256, the channel bandwidth is approximately 2.5MHz. In the 3GPP-SCM setup, the carrier frequency is 2.5GHz. In other words, the maximum Doppler shift is 231.48Hz. Base station antenna spacing is  $10\lambda$  and MS antenna spacing is  $\lambda/2$ , where  $\lambda$  is the wavelength at the carrier frequency. Note that channel coefficients are generated according to the implementation in [16] and each simulation result is averaged over 200 runs.

<sup>&</sup>lt;sup>1</sup>The effects of the small variations over the subchannel matrices are considered and investigated analytically with perturbation analysis in [10].



Fig. 1. RMSE versus precoder coefficient p when MS speed is 100km/h (SNR=20dB).

Fig. 1 shows the root mean square error (RMSE) of the channel estimates when the number of iterations = 100 and the SNR = 20dB, evaluated over  $p \in [0.1, 10.0]$  with a step size of 0.1. The sliding window length *l* considered are 1, 5, 10, and 20. From the simulation results, we are not surprised to see that choosing the sliding window length l = 1 gives the best performance since the wireless channel is changing so rapidly in this case. We can also observe that the RMSE of the channel estimates reaches its optimum when p = 1.0 for the sliding window length l = 1.

Fig. 2 presents the RMSE of the channel estimates versus the number of OFDM symbols received (i.e., iterations) when p = 0.5, 0.8, 1.0, and 2.0. The result shows that the updating of the channel estimate over a fast fading channel by using orthogonal iteration can converge very quickly, reaching the RMSE less than the level of  $2 \times 10^{-2}$  in less than 5 OFDM symbols specifically when the precoder coefficient p = 1.

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Fig. 2. RMSE versus number of OFDM blocks when MS speed is 100 km/h (sliding window length = 1 and SNR = 20dB).

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