OPTIMIZED OPPORTUNISTIC MULTICAST SCHEDULING (OMS) OVER HETEROGENEOUS CELLULAR NETWORKS

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ABSTRACT

Optimized opportunistic multicast scheduling (OMS) has been studied previously by the authors for homogeneous cellular networks, where the problem of efficiently transmitting a common set of data from a single base station to multiple users that have identical channel statistics was examined. It has been demonstrated that OMS can achieve significant performance improvement by exploiting the optimal tradeoff between multiuser diversity and multicast gain. In this work, we extend our studies to heterogeneous networks with users subject to different channel statistics. Specifically, we consider a single cell wireless network with users uniformly distributed in a circular region around the base station. Since users with low SNR are the ones that hinder system throughput, we argue that system performance may be predicted by the behavior of users in the outmost ring of the cell, which are approximately homogeneous. Using extreme value theory and results obtained from the homogeneous case, we determine the optimal user selection ratio for a homogeneous ring of users near the edge of the cell and then use it to derive the optimal selection ratio over the entire heterogeneous network. Simulations confirm theoretical results and illustrate the effectiveness of the proposed scheme.

Index Terms— opportunistic multicast scheduling (OMS), unicast, multiuser diversity gain, multicasting gain, extreme value theory.

1. INTRODUCTION

The ever-increasing demand for richer multimedia contents has driven the development of highly efficient content distribution technologies over wireless networks. In particular, opportunistic scheduling has recently emerged as one of the most promising techniques for content delivery by exploiting multiuser diversity inherent in wireless networks [1]. However, most existing work on opportunistic scheduling has focused on applications where the base station (BS) schedules different data to multiple users. In this work, we consider a broadcast application in a single-cell system where the BS intends to send a common set of information to multiple users that may be experiencing different instantaneous channel conditions. The design goal is to have all users successfully receive this common information with the shortest amount of time.

Two conventional scheduling schemes have been studied extensively for this scenario in the literature; namely, the unicast and the multicast scheduling schemes. The unicast scheme serves the best instantaneous user at the highest supportable data rate which exploits multiuser diversity. Since the unicast scheme serves only one user at a time, the transmission must be repeated multiple times until all users are served. On the other hand, the multicast scheduling scheme exploits the multicast gain by serving all users simultaneously. However, to prevent channel outage, the message has to be sent at a low data rate constrained by the user with the worst channel conditions, which in turn degrades the system performance.

Recent advances in Fountain codes [7] has inspired a new generation of opportunistic scheduling schemes. If data are encoded by Fountain codes, users can recover the full original content once a minimum set of encoded symbols is received, regardless of the specific sequence of encoded symbols. As a result, the BS is no longer handicapped by choosing either unicast or multicast scheduling. It has been shown that the BS can improve the system performance by optimizing the number of users served in each transmission. The pioneering work in [3,4] proposed a median opportunistic multicast scheduling (OMS) scheme in which the best 50% of users are served in each transmission. Despite its good performance, the optimality of choosing the best 50% users was not addressed in [3,4]. Recently, some initial steps have been taken by the authors to study the optimal selection ratio for homogeneous networks [2]. It was shown in [2] that the optimal multicast selection ratio is characterized by the average signal-to-noise ratio (SNR) for homogeneous networks.

Here, we extend the results of [2] to heterogeneous networks, where users uniformly distributed in a circular cell may experience varying degrees of path loss. Since users with low SNR are the ones that hinder system throughput, we argue that system performance may be predicted by the behavior of users in the outmost ring of the cell, which are approximately homogeneous. Using extreme value theory and results obtained from the homogeneous case, we determine the optimal user selection ratio for a homogeneous ring of users near the edge of the cell and use it to derive the optimal selection ratio over the entire heterogeneous network. Simulation results are presented to verify the accuracy of the asymptotic analysis for a finite user population and confirm the performance of the proposed opportunistic multicast scheme.

2. SYSTEM MODEL AND BACKGROUND

2.1. System model

Consider a single-antenna downlink cellular network where the BS is to transmit a common message to N active users. Let $h_n(k)$ be the instantaneous channel coefficient between the BS and the *n*-th user during the *k*-th transmission timeslot. We model $h_n(k)$ as a circularly symmetric complex Gaussian random variable with mean

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0 and variance 1, *i.e.* $h_n(k) \sim C\mathcal{N}(0, 1)$. The channel is assumed to be independent and identically distributed (i.i.d.) among users and over different time slots.

We examine a time-slotted system where the length of each time-slot is approximately equal to the channel coherence time T_c and, thus, the channel coefficient $(h_n(k))$ is assumed to be constant throughout each time-slot. The BS employs a fountain encoding scheme [7] where different fountain-encoded bits are transmitted over multiple timeslots until each user is able to receive at least S_m of them. That is, we assume that each user will be able to reliably decode the message once it receives S_m fountain encoded bits.

Suppose that the BS has the knowledge of the channel of all users at the beginning of each time slot. Based on the instantaneous channel state information (CSI), the BS will select a group of target users, whose index set is denoted by \mathcal{I}_k for slot k, and transmit at a rate, which is constrained by the user with the worst SNR among all selected users. Let

$$\gamma_n(k) = \frac{P \cdot |h_n(k)|^2}{N_0 \cdot L_p(D_n)}$$

be the SNR of user n in the k-th time slot, where P is the transmission power, N_0 is the noise variance, D_n is the distance between the BS and user n, and $L_p(D_n)$ is the path loss over distance D_n . Then, the maximum rate supportable by user n is $r_n(k) =$ $\log_2(1 + \gamma_n(k))$. When the group of users \mathcal{I}_k is chosen, the rate transmitted by the BS in slot k is equal to $\min_{n \in \mathcal{I}_k} r_n(k)$. As a result, all users in \mathcal{I}_k can successfully receive packets transmitted in slot k while other users will experience outage.

In the following discussion, we use K to denote the total number of timeslots required to successfully send the message to all users, *i.e.* the system delay, and $R_{sys} = \frac{NS_m}{KT_c}$ the corresponding system throughput.

2.2. OMS in homogeneous networks

We first review the opportunistic multicast scheduling scheme proposed in [2] for homogeneous networks. In the homogeneous case, we assume that all users experience the same average SNR ρ_0 (or for simplicity that $L_p(D_n) = 1$ for all n) and the CDF of the instantaneous SNR $\gamma_n(k)$ of the *n*-th user in the *k*-th timeslot can be found as $F_{\gamma_n(k)}(z) = 1 - \exp(-\frac{z}{\rho_0})$. We define the ordering scheme $\{\pi_k(i); i = 1, 2, \cdots, N\}$ such

that

$$\gamma_{\pi_k(1)}(k) \ge \ldots \ge \gamma_{\pi_k(N)}(k)$$

For sufficiently large N, the average throughput of the U-th user, where $U \approx \alpha N$, was found in [2] as

$$E\{r_{\pi(U)}\} \approx \log_2\left(1 - \rho_0 \ln\left(\frac{U}{N}\right)\right) \approx \log_2\left(1 - \rho_0 \ln\alpha\right).$$
(1)

In conventional scheduling schemes, the BS transmits at either $r_{\pi(1)}$ (unicast) or $r_{\pi(N)}$ (multicast). In contrast, the BS employing opportunistic multicast schemes transmits at a rate $r_{\pi(U)}$ where U = αN users are able to decode the transmission. The ratio $\alpha, 0 < \alpha \leq$ 1, is referred to as the user selection ratio. The selection ratio that optimizes the average delay is referred to as the optimal selection ratio α^* .

Following the treatment in [2], we consider a static strategy where the BS chooses a fixed number of users (i.e. U) in each time slot regardless of the number of bits still demanded by each user. Moreover, users served in the previous transmissions will still be considered for the current transmission. The minimum number of

BS transmissions needed before a user is able to collect S_m bits is given by

$$m_U \approx \frac{S_m}{E\{r_{\pi(U)}\}T_c},\tag{2}$$

which is achieved when the user is selected consecutively in the first m_U time slots. It was shown in [2] that, for homogeneous networks, the average delay for a chosen selection ratio α can be approximated by

$$E\{K_{\text{homo}}\} \approx \left[\frac{-\xi\sqrt{\alpha(1-\alpha)} + \sqrt{\xi^2\alpha(1-\alpha) + 4\alpha m_U}}{2\alpha}\right]^2,$$
(3)

where

$$\xi = (1 - \eta) \Phi^{-1} \left(\frac{1}{N}\right) + \eta \Phi^{-1} \left(\frac{1}{Ne}\right), \tag{4}$$

with η and Φ^{-1} being the Euler-Mascheroni constant (0.57721) and the inverse CDF of the standard normal distribution, respectively. The optimal multicast selection ratio is thus given by

$$\alpha^* = \operatorname*{arg\,min}_{\alpha} \left\{ E\{K_{\mathrm{homo}}\} \right\}. \tag{5}$$

3. OMS IN HETEROGENEOUS NETWORKS

In this section, we consider a heterogeneous system, where N active users are uniformly distributed around the BS in a circular cell of radius D_{max} . That is, the distance between the *n*-th user and the BS, denoted by D_n , has the following probability density function (PDF) [6]:

$$f_{D_n}(d) = \frac{2d}{D_{max}^2}, \quad 0 < d \le D_{max}.$$
 (6)

We model the effect of path loss between the BS and user n as

$$L_p(D_n) = \epsilon D_n^\beta,\tag{7}$$

where ϵ and β are the path loss constant and exponent, respectively. By taking into account the path loss, the average SNR experienced by the *n*-th user can be expressed as

$$\rho_n(D_n) = E\left[\gamma_n(k)\right] = \frac{\rho_0}{\epsilon D_n^\beta}.$$
(8)

Subsequently, the CDF of the instantaneous SNR of the n-th user during the k-th timeslot, denoted by $\gamma_n(k)$, can be expressed as

$$F_{\gamma_n(k)}(z) = 1 - \exp(-\frac{z}{\rho_n}).$$
 (9)

For heterogeneous networks, direct analysis on the system delay for a given selection ratio is intractable. Here, we will present an asymptotic analysis assuming that the number of users approaches infinity, *i.e.*, $N \to \infty$. We begin by observing that, in the asymptotic case, a heterogeneous cell can be considered as a composite of rings of homogeneous user groups as shown in Fig. 1. As a result, the overall delay performance of the network hinges on the group with the worst average SNR. From (8), it is easy to understand that such a group is composed of users at the edge of the cell and has an average SNR approximately equal to

$$\rho_{\text{edge}} = \frac{\rho_0}{\epsilon D_{max}^{\beta}},\tag{10}$$

which is referred to as the *edge group* in the sequel.



Fig. 1. Homogeneous rings of users in a heterogeneous cell.

Let α_{edge}^* be the optimized selection ratio for the edge group. Since the delay of the edge group will hinder the performance of the entire heterogeneous network, the set of users selected for transmission in the heterogeneous network should include exactly α_{edge}^* proportion of users in this group. Moreover, by treating edge users as a homogeneous group, the optimal user selection ratio for the edge group, α_{edge}^* , can be predicted by results obtained from homogeneous networks with their average SNR equal to ρ_{edge} . Based on this observation, we derive analytically the optimal selection ratio for heterogeneous networks, α_{hetero}^* . From results on central order statistics [5], the variance of the

From results on central order statistics [5], the variance of the instantaneous SNR of a user in any position U of the ordered SNR list asymptotically approaches zero, *i.e.*, $\sigma_{\gamma_{\pi}(U)}^2 \to 0$ as $N \to \infty$. Hence, for large networks, choosing a selection ratio is equivalent to setting an SNR threshold. To guarantee the selection ratio of α_{edge}^* for the edge group, the selection ratio taken over the entire heterogeneous network must yield the same equivalent SNR threshold as that predicted by the homogeneous edge group.

More specifically, let the asymptotic instantaneous SNR of the limiting user, or the equivalent SNR threshold, be γ_0 . For a user in the edge group with average SNR equal to ρ_{edge} , the probability of its instantaneous SNR exceeding the threshold γ_0 is given by

$$\Pr\{\gamma_n(D_{max}) > \gamma_0\} = 1 - \Pr\{\gamma_n(D_{max}) < \gamma_0\}$$

=
$$\exp(-\gamma_0/\rho_{\text{edge}}), \qquad (11)$$

where $\gamma_n(D_{max})$ is the instantaneous SNR of a user at the edge of the cell. For large N, this probability should be equal to α^*_{edge} , which yields

$$\gamma_0 \approx -\frac{\rho_0}{\epsilon D_{max}^{\beta}} \ln(\alpha_{edge}^*).$$
 (12)

Then, for the heterogeneous case, we should set the user selection ratio α_{hetero}^* such that the resulting SNR threshold is also approximately γ_0 . Similarly, from results of central order statistics, we know that this is achieved by choosing α_{hetero}^* equal to the probability that a user in the heterogeneous network exceeds threshold γ_0 . By averaging over the users' locations, this is given by

$$P_A = \frac{1}{\pi D_{max}^2} \int_0^{D_{max}} \int_0^{2\pi} \Pr\{\gamma_n(u) \ge \gamma_0\} u d\theta du$$
(13)

$$= \frac{2}{D_{max}^2} \int_0^{D_{max}} u \exp\left(\frac{\ln(\alpha_{edge}^*)}{D_{max}^\beta} u^\beta\right) du, \quad (14)$$

where $\gamma_n(u)$ is the instantaneous SNR of a user at distance u from the BS. As shown in the Appendix, this can be further simplified as

$$P_A = 1 + \sum_{m=1}^{\infty} \frac{(\ln(\alpha_{\text{edge}}^*))^m}{m! \left(\frac{m_2}{2} + 1\right)}.$$
 (15)

In summary, the BS first computes ρ_{edge} from (10). Replacing ρ_0 with ρ_{edge} in (1), we can derive α^*_{edge} by numerically solving (5). Upon obtaining α^*_{edge} , the BS can easily compute P_A from (15). Finally, BS sets the user selection ratio α^*_{hetero} to P_A , which ensures that the edge group has an effective selection ratio of α^*_{edge} , *i.e.*,

$$\alpha_{\text{hetero}}^* = P_A. \tag{16}$$

4. SIMULATION RESULTS

In this section, we study the performance of the proposed OMS scheme given in (16) via computer simulations. In our experiments, we set N = 100 so that a selection ratio of M% effectively selects M users. The users are distributed uniformly in a circular cell area of radius

$$D_{max} = \left(\frac{\rho_o}{\epsilon \rho_{\text{edge}}}\right)^{\frac{1}{\beta}},\tag{17}$$

with path-loss constant $\epsilon = 10^{3.15}$ and path-loss exponent $\beta = 3.5$.



Fig. 2. Optimal Selection Ratio as a function of ρ_{edge} .

First, we verify the validity of (15) by showing the optimal selection ratio as a function of the average SNR of the edge group, *i.e.* ρ_{edge} . Specifically, in Fig. 2, we compare the analytically optimized selection ratio given in (16) with the optimal selection ratio obtained via simulations. In the same figure, the analytically optimized selection ratios given in (5) for homogeneous networks is also compared with the selection probability of the furthest user in the simulation. As observed, the analytical result closely matches the optimal selection ratios for the range of ρ_{edge} under consideration.

Next, we compare the performance of the proposed OMS scheme against three other existing scheduling schemes. They are the conventional multicast, unicast, and the median-user scheme as proposed in [4]. In conventional multicast, the BS transmits at a rate such that all users can successfully receive in each time slot. For unicast and median-user schemes, we select users based on their normalized SNRs such that all users are guaranteed a fair chance to

be served. The unicast and median-user schemes that take the true SNR as the selection criterion may yield extremely large delay in the heterogeneous case since good users may be chosen repeatedly while bad users may never be chosen. When using the normalized SNR criterion, the rate is chosen such that (at least) users in the selected group can receive successfully. The actual number of users that are able to successfully receive in each time slot may be larger than the selected group size. The delay performance is shown in Fig. 3 and the corresponding throughput in Fig. 4.



Fig. 3. Comparison of delay performance in heterogeneous networks.



Fig. 4. Comparison of throughput performance in heterogeneous networks.

Inspection of Figs. 3 and 4 confirms the conventional wisdom. That is, when the cell size gets smaller (*i.e.* the edge SNR increases), the multicast gain becomes more dominant and the conventional multicast scheme achieves the best performance. As the cell size grows larger, the unicast scheme benefits from the increasing variations in users' instantaneous SNRs by effectively exploiting the multiuser diversity. Furthermore, the median-user scheme performs better than both conventional multicast and unicast schemes for $\rho_{\rm edge} < 8$ dB. The proposed OMS scheme achieves further improvement by

optimizing the user selection ratios. For a small cellular network, the proposed OMS scheme chooses to serve all users, which results in performance converging to that of the conventional multicast scheme. This also confirms the adaptability of the scheme to changing radio conditions.

5. CONCLUSION

In this work, we studied the opportunistic multicast scheduling (OMS) scheme with optimal user selection for heterogeneous cellular networks where users are uniformly distributed within a circular cell and are subject to different channel fading conditions. Using extreme value theory, we provided a theoretical analysis on the optimal user selection ratio that minimizes the multicast delay. It was shown that, even in the heterogeneous settings, the proposed OMS scheme achieves good delay performance by exploiting the optimal tradeoff between multiuser diversity and multicast gain regardless of the cell size.

6. APPENDIX

In this appendix, the derivation of (15) is highlighted.

P

$$\begin{aligned} \mathcal{P}_{A} &= \frac{2}{D_{max}^{2}} \int_{0}^{D_{max}} u \exp\left(\frac{\ln(\alpha_{edge}^{*})}{D_{max}^{\beta}}u^{\beta}\right) du \\ &= \frac{1}{D_{max}^{2}} \int_{0}^{D_{max}^{2}} \exp\left(\frac{\ln(\alpha_{edge}^{*})}{D_{max}^{\beta}}z^{\beta/2}\right) dz \\ &\stackrel{(a)}{=} \frac{1}{D_{max}^{2}} \int_{0}^{D_{max}^{2}} \left(1 + \sum_{m=1}^{\infty} \left[\left(\frac{\ln(\alpha_{edge}^{*})}{D_{max}^{\beta}}\right)^{m} \frac{z^{\frac{m\beta}{2}}}{m!}\right]\right) dz \\ &= 1 + \sum_{m=1}^{\infty} \frac{(\ln(\alpha_{edge}^{*}))^{m}}{m! \left(\frac{m2}{2} + 1\right)}, \end{aligned}$$
(18)

where (a) follows from the Taylor series expansion of the exponential function about z = 0.

7. REFERENCES

- P. Viswanath, D. N. C. Tse and R. Laroia, "Opportunistic beamforming using dumb antennas", *IEEE Trans. Info. Theory*, vol. 48, no. 6, pp. 1277-1294, June 2002.
- [2] T.P. Low, M.O. Pun and C.-C. Jay Kuo, "Optimized opportunistic multicast scheduling over cellular networks", to be presented at IEEE Globecom, New Orleans, LA, November 2008.
- [3] P.K. Gopala and H. El Gamal, "Opportunistic multicasting", in Proc. the Asilomar Conference on Signals, Systems and Computers, Pacific Grove, CA, November 2004.
- [4] P.K. Gopala and H. El Gamal, "On the throughput-delay tradeoff in cellular multicast", in *Proc. Int'l Conf. on Wireless Networks*, *Communications and Mobile Computing*, Maui, HI, June 2005.
- [5] E. Castillo, *Extreme Value Theory in Engineering*, Boston : Academic Press, 1988.
- [6] S. Y. Baek, H. Y. Hwang and D. K. Sung, "Performance analysis of scheduling-based systems in Rayleigh fading channels", in *Proc. IEEE Int'l Symposium on Personal, Indoor and Mobile Radio Communications*, Helsinki, Finland, September 2006.
- [7] D.J.C. MacKay, "Fountain codes communications", *IEE Proceedings*, Vol. 152, pp. 1062-1068, December 2005.