OSCILLATOR PHASE NOISE COMPENSATION USING KALMAN TRACKING

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ABSTRACT

Phase Noise (PN) is a serious challenge in wireless transmission systems as it can cause significant degradation of the system performance. Recent publications propose iterative PN compensation algorithms for single or multicarrier systems. In this paper we will present an unscented Kalman filter PN tracking algorithm working in time domain, which is independent of the underlying system. Furthermore, we propose a reduced complexity tracking algorithm, where we perform an interpolation between the estimated PN samples. Simulation results for an OFDM setup show that by using this technique the system performance can be improved significantly.

Index Terms— Phase Noise, Kalman Tracking

1. INTRODUCTION

Phase noise (PN) in wireless communication describes a multiplicative phase distortion during the up/down conversion at the transmitter and receiver. This distortion is caused by RF imperfections such as imperfect oscillators. Previous work mainly focused on PN mitigation in either multicarrier systems [1] *or* single carrier systems. In multicarrier systems, PN is usually estimated in the frequency domain using a LMMSE estimation approach, which estimates higher order PN components [2]. In single carrier systems, PN is for example tracked using a PLL approach [3].

In this contribution we present an iterative PN compensation algorithm which is applicable for both setups. The algorithm tracks the PN samples using a Kalman filter. In its original version, the Kalman filter is designed for linear systems. However, the PN tracking problem is a nonlinear one (Sec. 3). For nonlinear systems the extended Kalman filter is a widely used estimation algorithm. But as stated in [4] the extended Kalman filter is not suitable for strong nonlinear systems as we have it in the case of PN. For such a nonlinear setup the authors in [4] propose an unscented transformation. In this work we adapt the unscented Kalman filtering idea to the problem of PN tracking. Due to limited space and since PN is a more severe problem in multicarrier systems [1], we will focus on OFDM as one possible example to show the effectiveness of the proposed scheme.

[*]Part of this work has been performed in the framework of the ICT project FP7 ICT-2007-2015533 FUTON, which is partly funded by the European Union.

The remaining paper is organized as follows. An OFDM system model disturbed by Wiener phase noise is introduced in Sec. 2. In Sec. 3 we give a detailed description of the PN compensation algorithm using an unscented Kalman filter approach. We also propose a reduced complexity version based on a linear interpolation. The performance for AWGN/HiperLan A channels is characterized in Sec. 4. We conclude the paper with a summary of our findings in Sec. 5.

2. SYSTEM MODEL

We consider a SISO (single-input, single-output) OFDM transmission system with N subcarriers. Fig.1 shows a simple transmission chain. Let $\mathbf{V}$ be a vector of information bits

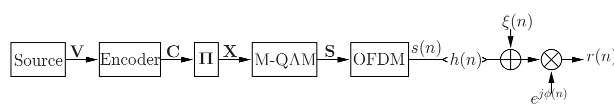

Fig. 1. OFDM transmission chain

which are first encoded by the outer encoder and interleaved. The resulting code bit stream is then partitioned into blocks $\mathbf{X}$ containing $N \cdot B$ independent binary digits. Out of these blocks, B bits are converted into one symbol, thus allowing to distinguish among $M = 2^B$ different constellation points (e.g. 64-QAM). As part of the transmission process, these symbols are mapped onto a $N \times 1$ complex vector of symbols $\mathbf{S} = [S_1, \cdots, S_N]^T$. The OFDM modulator takes this vector and performs an inverse Fast Fourier Transformation (iFFT) to obtain the time domain signals. A cyclic prefix (CP) of length N_g is also inserted before transmission to avoid possible intersymbol interference caused by frequency selective channels. Hence, the total number of time samples is $N_{tot} = N_g + N$

Phase noise (PN) is modeled by a Brownian motion or Wiener Process which has a Lorentzian power density spectrum. However, the presented PN compensation algorithm is also valid for other PN models (e.g. PLL). Since transmitter PN can be approximated by an effective receiver PN as presented in [5] we consider the case where PN occurs only at the receiver. The $(n)^{th}$ PN sample is related to the previous one as $\phi(n) = \phi(n-1) + \Delta\phi$, where $\Delta\phi$ is a Gaussian distributed random variable, with zero mean and variance $\sigma^2_{\Delta\phi} = 4\pi^2 f_c^2 c T_s$. In this notation, T_s describes the sample interval, c determines the oscillator quality and f_c is the carrier frequency. Related to the 3dB single side bandwidth Δf_{3dB} of the Lorentzian spectrum, c is given by $c = \Delta f_{\mathrm{3dB}}/(\pi f_c^2)$ [6]. With f_{sub} as the subcarrier spacing of an OFDM system, it is common to use the single relevant performance parameter δ_{3dB} as the relative oscillator linewidth with respect to the subcarrier spacing given by $\delta_{\mathrm{3dB}} = \Delta f_{\mathrm{3dB}}/f_{sub}$.

Assuming perfect frequency and timing synchronization, the received time domain signal can be expressed as $r(n) = (s(n) \star h(n))e^{j\phi(n)} + \xi(n)$, where $s(n)$ and $h(n)$ represent the samples of the transmitted signal and the channel impulse response, respectively. The term $\xi(n)$ represents the additive white Gaussian noise (AWGN) with variance σ^2_ξ.

3. PHASE NOISE COMPENSATION

Fig. 2 shows a general iterative setup for PN compensation. The main module is the PN compensation block. PN estimation can either be done in frequency domain [2] or in time domain. In the iterative process, soft information is exchanged between each module, see [7] for details. However, in both cases it is advantageous to bootstrap the iterative process by an initial common phase error (CPE) compensation [5]. After the CPE estimation and decoding a first estimation $\hat{s}(n)$ of the transmitted signal is available, which can be used to obtain a better estimation of the PN trajectory, using $\hat{a}(n) = (\hat{s}(n) \star h(n)) = a(n) + \Delta a(n) \approx a(n)$.

The error between the true and estimated noise-free versions of the received signal is given by $\Delta a(n)$. The variance of $\Delta a(n)$ can be determined using soft information provided by the decoder (see [7]). However, due to the coding gain we observed that the variance of $\Delta a(n)$ is much smaller than σ^2_ξ in the FER region of interest. Thus we will skip $\Delta a(n)$ from now on.

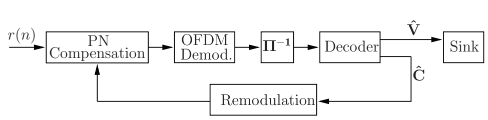

Fig. 2. OFDM receiver chain

Frequency domain PN estimation entails the problem that we try to approximate a piecewise nonperiodic signal (the Phase Noise sample) with a truncated Fourier series which implicitly assumes periodicity and hence leads to a high MSE at the beginning and the end of the time domain signal [7]. Therefore, we propose a Kalman tracking in time domain for every further iteration. The advantage of the Kalman filter is that it achieves a much lower estimation error and, as stated earlier, is also directly applicable for single carrier systems.

Figure 3 depicts the adapted scalar state and observation model, which is will be used to develop a Kalman filtering algorithm.

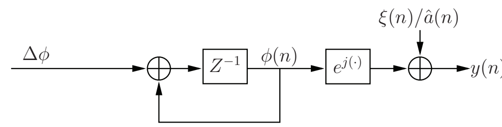

Fig. 3. Scalar state/observation model

As we have a deterministic input process, the first system equation describing the state transmission is given by:

$$\phi(n) = \phi(n-1) + \Delta\phi. \tag{1}$$

In Eq. (1) we used the fact that the PN increment ($\Delta\phi$) can be considered as process noise. The second system equation, describing the observation term which is feed to the Kalman filter, is given as follows:

$$y(n) = r(n)/\hat{a}(n) \approx \underbrace{\exp\left(j\phi(n)\right)}_{D(\cdot)} + \frac{\xi(n)}{\hat{a}(n)}. \tag{2}$$

Furthermore, the observation model is given by $e^{j(\cdot)}$, which is a nonlinear transformation. Due to the nonlinear structure the standard Kalman filtering operation is not applicable. Therefore, the unscented Kalman filter for nonlinear estimation is used as introduced in [8, 4]. Algorithm 1 describes the unscented Kalman filtering operation used for PN estimation in more detail, where we used the scaling parameters $\alpha^2 = 1 \cdot 10^{-3}, \beta = 2$ as proposed in [8]. The index notation (n, n) represents the a-posteriori knowledge and index $(n, n-1)$ defines the a-priori information. Typically the cyclic prefix is long enough for the Kalman filter to converge, thus the start PN sample is not important and will be set to "0". The estimated PN samples $\hat{\phi}(n)$ are used to correct the received signal samples in the time domain: $\hat{r}(n) = r(n)\exp(-j\hat{\phi}(n))$.

The complexity of the proposed algorithm can be reduced by decreasing the number of samples passed into the Kalman filter and reconstructing the missing PN samples using linear interpolation after the filter. Assume we have a code word length of N_{code} samples and assume further we sample every N_{step} points, then $N_{KF} = \lceil N_{code}/N_{step} \rceil$ samples are passed into the filter and $N_{interp} = N_{code} - N_{KF}$ interpolated points are constructed. Each sample passed into the Kalman filter requires f_{KF} flops and each linear interpolated point requires f_{interp} flops. Therefore, it requires a total of $f_{total} = f_{KF} \cdot N_{KF} + f_{interp} \cdot N_{interp}$ flops, which results in a saving in complexity if an interpolation is done of $1 - (f_{KF} + f_{interp}(N_{step} - 1))/(N_{step} f_{KF})$. As an example, we consider each addition as one flop and each multiplication as three flops for simplicity, thus $f_{KF} = 81$ flops and $f_{interp} = 16$ flops. For $N_{code} = 3 \cdot 80$ and $N_{step} = 4$, we could achieve a 60% reduction in complexity. However, interpolation results in a higher MSE, thus an increased error rate is expected. Therefore, depending on the application, one may trade SNR loss for efficiency by increasing N_{step}.

Input: $y_{(n)}$; Var$[y_{(n)}] = P_{\epsilon\epsilon} = \sigma_\xi^2/\|\hat{a}_{(n)}\|$; $\hat{\phi}_{(0,0)} = 0$;
$P_{\hat{\phi},\hat{\phi}(0,0)} =$ Var$[\Delta\phi]$; $\alpha^2 = 1 \cdot 10^{-3}, \beta = 2$;
$\lambda = \alpha^2 - 1$; $W_0^m = \lambda/(1+\lambda)$;
$W_0^c = \lambda/(1+\lambda) + (1 - \alpha^2 + \beta)$;
$W_{1,2}^m = W_{1,2}^c = 1/(2+2\lambda)$

Result: Phase Estimate $\hat{\phi}_{(n,n)}$

foreach $n = 1, \cdots, N_{code}$ **do**

A-priori Info

$\hat{\phi}_{(n,n-1)} = \hat{\phi}_{(n-1,n-1)}$

$P_{\hat{\phi},\hat{\phi}(n,n-1)} = P_{\hat{\phi},\hat{\phi}(n-1,n-1)}$ + Var$[\Delta\phi]$

Unscented Transformation (sigma points)

$\Phi_{0(n,n-1)} = \hat{\phi}_{(n,n-1)}$

$\Phi_{1,2(n,n-1)} = \hat{\phi}_{(n,n-1)} \pm \sqrt{(1+\lambda)P_{\hat{\phi},\hat{\phi}(n,n-1)}}$

Nonlinear observation function

$\nu_{i(n,n-1)} = \exp(j\Phi_{i(n,n-1)})$

Weighted samples

$\hat{y}_{(n,n-1)} \approx \sum_i W_i^m \nu_{i(n,n-1)}$

Covariance Matrix update

$P_{\hat{y},\hat{y}} = \sum_i W_i^c(\nu_{i(n,n-1)} - \hat{y}_{(n,n-1)}) \cdot (\nu_{i(n,n-1)} - \hat{y}_{(n,n-1)})^H$

$P_{\hat{\phi},\hat{y}} = \sum_i W_i^c(\Phi_{i(n,n-1)} - \hat{\phi}_{(n,n-1)}) \cdot (\nu_{i(n,n-1)} - \hat{y}_{(n,n-1)})^H$

Kalman Gain

$K_{(n)} = P_{\hat{\phi},\hat{y}}(P_{\hat{y},\hat{y}} + P_{\epsilon\epsilon})^{-1}$

A-posteriori Info

$\hat{\phi}_{(n,n)} = \hat{\phi}_{(n,n-1)} + K_{(n)}(y_{(n)} - \hat{y}_{(n,n-1)})$

$P_{\hat{\phi},\hat{\phi}(n,n)} = P_{\hat{\phi},\hat{\phi}(n,n-1)} - K_{(n)} P_{\hat{\phi},\hat{y}}^H$

end

Algorithm 1: Unscented Kalman Filtering

4. NUMERICAL RESULTS

The performance of Kalman tracking algorithm was studied with WLAN(802.11a) related parameters. A non-recursive rate 1/2 convolutional code with generator polynomial $G = [133, 171]_8$ and codeword length of 3 OFDM symbols was used ($N_{code} = 3 \cdot N_{tot} = 240$ samples). To be conform with the standard, out of $N = 64$ carriers 4 are reserved for pilots and 12 are zero carriers. For modulation 64-QAM was used.

We start the analysis by investigating the performance of the algorithm with an AWGN channel depicted in Figure 4. The relative oscillator linewidth δ_{3dB} was set to be 1% in this case and the step size is 1 (full complexity). The performance of our proposed algorithm is quite impressive since we are less than 1dB away from the no phase noise case at a target frame error rate (FER) of 10^{-2}. Furthermore, we were able to achieve a 1dB difference from the no phase noise case as early as the second iteration. Note, that an estimation algorithm working in the frequency domain, which only estimates a few PN harmonics [2] would result in an error floor at high SNR due to the non perfect cancelation of the intercarrier interference. However, the Kalman filter would perfectly track the PN at this region, as it does not assume any periodicity of the PN signal.

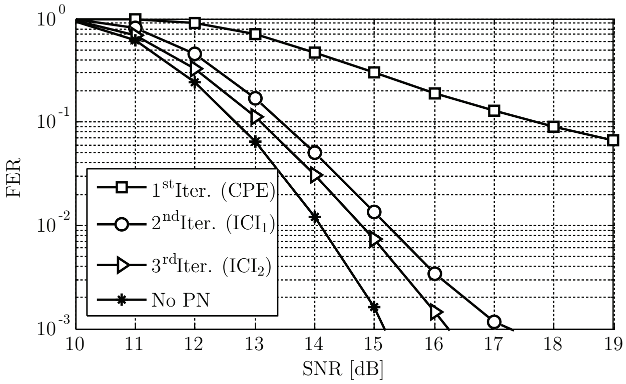

Fig. 4. FER, 64-QAM, AWGN channel, $\delta_{3dB} = 0.01$

From now on a transmission over an HiperLan A channel is considered. Figure 5 shows the performance results for both the frequency domain [7] and Kalman tracking algorithms. It is worth mentioning that we gain 2.5dB at the

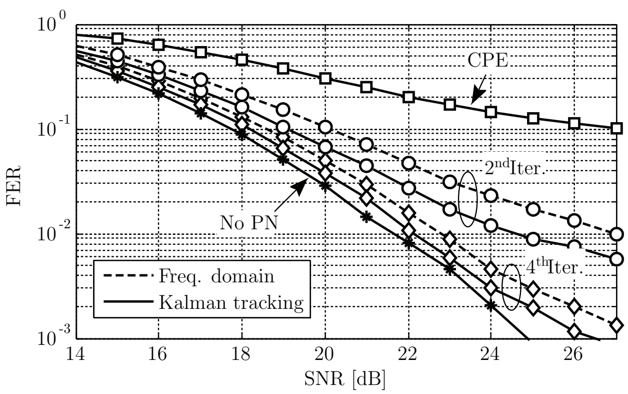

Fig. 5. FER 64-QAM, HiperLan A channel, $\delta_{3dB} = 0.01$

target FER of 10^{-2} at the second iteration using the Kalman tracking algorithm compared to the frequency domain one. The remaining error floor is mainly due to error propagation of the non perfectly estimated samples $\hat{s}(n)$, which are used in the Kalman tracking algorithm. Note, that in the case of perfectly known symbols (genie knowledge), already the second iteration ($Iter_2$) would not show an error floor behavior in contrast to the second iteration of the frequency domain estimation algorithm which only estimates the first harmonic of the PN trajectory.

Finally, we investigated how the algorithm performs as a function of different parameters. Figure 6 shows the SNR loss w.r.t. to no phase noise at FER = 10^{-2} as a function of the relative oscillator linewidth δ_{3dB}. We noticed that there is no SNR loss if δ_{3dB} is better than 0.005 and a 1dB loss at $\delta_{3dB} = 0.012$ for the fourth iteration. To reduce the algorithm's complexity, we increase the sampling step size N_{step} as discussed in Sec 3. Figure 7 shows the SNR at FER = 10^{-2} for different iterations as a function of N_{step}. For $N_{step} = 4$, we have a 1.4dB SNR loss in the fourth iteration while the complexity is reduced by 60%.

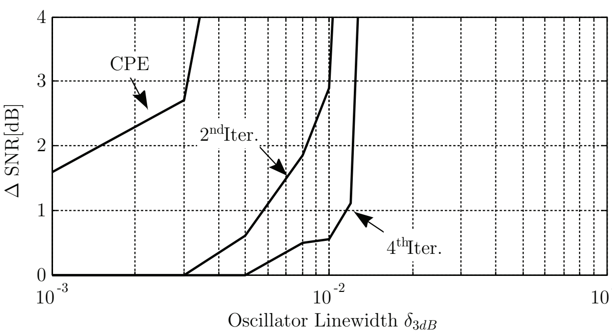

Fig. 6. SNR loss at FER 10^{-2} for different iterations versus δ_{3dB} (64-QAM, HiperLan A Channel)

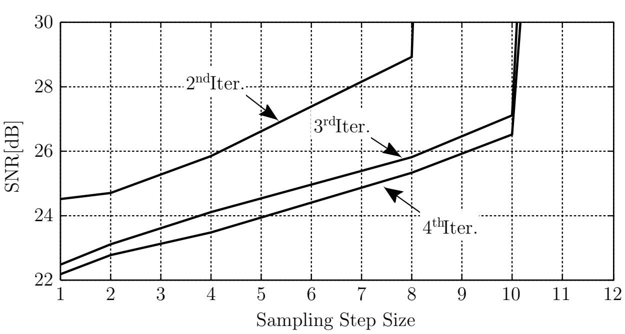

Fig. 7. SNR at FER 10^{-2} for different iterations versus N_{step} (64-QAM, HiperLan A Channel, $\delta_{3dB} = 0.01$)

5. CONCLUSIONS

Phase Noise describes an instantaneous phase fluctuation, which results in different phasors for each sample. For such a time variant system Kalman filtering is one way to track the PN samples. Due to the up/down conversion of the signal PN acts like a nonlinear distortion. Hence, a nonlinear estimation approach has to be used. In this work we adapted the unscented Kalman filtering to get an estimation of the PN trajectory. The proposed algorithm is suitable for single carrier as well as multicarrier systems. The performance of the given algorithm is evaluated for an OFDM system in terms of FER showing a significant performance improvement compared to known algorithms. Furthermore, we came up with a reduced complexity version allowing a considerable saving in complexity.

6. REFERENCES

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