REDUCED-COMPLEXITY DELAY-DOPPLER CORRELATOR FOR TIME-FREQUENCY HOPPING SIGNALS

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ABSTRACT

The delay-Doppler correlator is most commonly used for target detection in a radar system. It is essentially a two-dimensional filter matched to the hypothesized delay and Doppler shifts of echoes reflected from the illuminating signal. More recently, it has been applied in communication to the detection of signature sequences generated from artificially introduced time-frequency shifts of a base sequence. In this paper, we derive a computationally efficient delay-Doppler correlator for a class of time-frequency hopping waveforms that can be represented by segments of equal-length sinusoids. These sequences can be found in applications such as continuous waveform radar and device identification in wireless communication. By applying sliding Discrete Fourier Transform and exploiting the structure of the waveform, the number of multiplications required for evaluating the entire delay-Doppler range can be reduced by a factor equaling the length of the sequence.

Index Terms— Radar, delay, Doppler, Correlator, Signature Sequence

1. INTRODUCTION

A delay-Doppler radar [1] measures an environment of interest by illuminating it with electromagnetic radiation. The illuminating field scattered by objects (or targets) in the environment is collected by a receiver. In most cases, each target incurs a time delay, a Doppler shift and a complex gain to the incoming signal. With properly designed illuminating signal, the processor at the receiver can determine the presence and scattering characteristics of the targets by correlating the received signal with time-frequency shifted hypotheses of the illuminating signal. A strong return at the output of this delay-Doppler correlator for a particular time-frequency shift indicates a high likelihood of a target's presence.

To be able to discriminate closely separated targets, an illuminating signal should in general be significantly different from any of its time-frequency shifts. The measure of such resemblance is the signal's ambiguity function commonly used to design radar waveforms.

The same principle has recently been applied to signature sequence design for wireless communication systems. To identify multiple devices, a base sequence with good ambiguity function is chosen first, just as in radar signal design. Each device is then assigned a unique circular time-frequency shift of the base sequence. The ideal ambiguity properties of the base sequence ensure that signals received from different transmitting devices can be detected and identified, similarly by a delay-Doppler correlator.

Among the many suitable signals (or sequences), a particular class can be expressed as a series of segments of equal-length sinusoids. The variation of the frequencies of these sinusoids constitutes a frequency hopping sequence as a function of time. The Costas [2] sequence is a time-frequency hopping sequence with ideal ambiguity function. It has a constant amplitude and is easy to synthesize. It can be generated by either algebraical construction or exhaustive search [3] for variable lengths of practical interests.

In this paper, we will further demonstrate that the delay-Doppler correlator for a time-frequency hopping sequence such as Costas sequence has a computationally efficient discrete implementation. The factor of improvement over a brute-force approach for an arbitrary sequence is equal to the sequence length. We will use the signature sequence identification problem in communication as our primary example. Extension to delay-Doppler radar, which involves non-cyclical implementation of the algorithm, is straightforward.

The remainder of this paper is organized as follows. Sec. 2 describes the discrete system model for the signature sequence identification problem. The Costas sequence is then briefly reviewed in Sec. 3, which is followed by a detailed derivation of its reduced complexity delay-Doppler correlator in Sec. 4. A brief summary is given in Sec. 5.

2. DISCRETE SYSTEM MODEL

The received signal corresponding to an arbitrary base sequence $s[n]$ of length N passing through a time-frequency selective channel can be expressed in discrete time as [4]

$$r[n] = \sum_{\tau=0}^{\tau_{\max}-1} \sum_{\nu=0}^{\nu_{\max}-1} h[\tau,\nu] s[n-\tau] e^{j\frac{2\pi\nu n}{N}} + z[n], \quad (1)$$

where $z[n]$ is the Additive White Gaussian Noise (AWGN) and $h[\tau,\nu]$ is the channel's delay-Doppler response with support $(0 \le \tau < \tau_{\max}, 0 \le \nu < \nu_{\max})$. Note here that unless otherwise specified, all the indexing in this paper is modulo N. Such circular operation at the receiver can be achieved in practice by introducing a cyclic prefix of appropriate length commonly seen in an Orthogonal Frequency Division Multiplexing (OFDM) system and by assuming that block-wise synchronization among the multiple devices and receiver has been achieved.

For the identification of multiple users in a wireless communication system, each device is assigned a signature sequence that is a unique circular time-frequency shift of the same base sequence $s[n]$:

$$s_{l,m}[n] = s[n - l\tau_d] e^{j\frac{2\pi m \nu_d n}{N}}, \quad (2)$$

where (τ_d, ν_d) is the minimum delay-Doppler separation between any pair of derived sequences and (l, m) is the unique identification index assigned to a user. For a properly chosen base sequence, multiple sequences can be distinguished even after passing through the channel as long as the minimum delay-Doppler separation of these artificially introduced shifts is greater then the maximum shift $(\tau_{\max}, \nu_{\max})$ introduced by the channel.

It has been shown [4] that the multiple sequences can be detected by a threshold test of a likelihood metric for all hypotheses of $[l, m]$:

$$\gamma[l,m] = \sum_{\tau=l\tau_d}^{l\tau_d+\tau_{\max}-1} \sum_{\nu=m\nu_d}^{m\nu_d+\nu_{\max}-1} |\mathcal{I}[\tau,\nu]|^2, \quad (3)$$

where

$$\mathcal{I}[\tau,\nu] = \sum_{n=0}^{N-1} r[n] s^*[n-\tau] e^{-\frac{j2\pi\nu n}{N}} \quad (4)$$

is the output of the delay-Doppler correlator at (τ, ν) in the delay-Doppler plane. When evaluated over the range of interest $0 \le \tau < N, 0 \le \nu < N$, the two-dimensional function $\mathcal{I}[\tau,\nu]$ is referred to as the delay-Doppler image of the scattering environment.

Substituting Eq. (1), which is the received signal corresponding to a device transmitting the sequence $s_{0,0}[n] = s[n]$, into Eq. (4) gives

$$\mathcal{I}[\tau,\nu] = \sum_{\tau'=0}^{N-1} \sum_{\nu'=0}^{N-1} e^{j2\pi(\nu-\nu')\tau'} h[\tau',\nu'] \chi_s[\tau-\tau', \nu-\nu'] + \chi_{s,z}[\tau,\nu], \quad (5)$$

where $\chi_{s,z}[\tau,\nu]$ is a noise term and

$$\chi_s[\tau,\nu] = \sum_{n=0}^{N-1} s[n] s^*[n-\tau] e^{-j\frac{2\pi\nu n}{N}} \quad (6)$$

is the (circular) ambiguity function [5] of the base signal $s[n]$. This function measures the resemblance of a signal to any of its time-frequency shifts. For distinguishing targets in a radar system or signature sequences in a wireless communication system, an ideal signal should then have an ambiguity function resembling a thumbtack with a sharp mainlobe and uniformly low sidelobes.

3. COSTAS SEQUENCE

Consider a sequence of length $N = LQ$ consisting of L segments of sinusoid, each of length Q:

$$s[n] = \sum_{l=0}^{L-1} p[n - lQ] e^{j\frac{2\pi\nu_l(n-lQ)}{Q}}, \quad (7)$$

where

$$p[n] = \begin{cases} 1, & \text{for } 0 \le n < Q \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

and ν_l may be an arbitrary hopping pattern ranging between 0 and $Q - 1$. The Costas sequence is a special case where ν_l is a unique permutation of the Q consecutive integers $\{0, \cdots, Q-1\}$ with an ideal non-periodic ambiguity function. It has been extended in [5] to the periodic case. An example for $Q = 6$ and $L = 7$ that will be used in the remainder of this paper is $\nu_l = \{0, 2, 1, 4, 5, 3, -\}$, where "–" indicates no transmission in that segment.

4. REDUCED COMPLEXITY DELAY-DOPPLER CORRELATOR

From Eq. (4) it is clear that the number of multiplications it takes to evaluate a single point $[\tau, \nu]$ in the delay-Doppler plane is $2N$ for a sequence of arbitrary length, or $2N^3$ for the entire plane. If N is a power of 2, for each Doppler index ν Eq. (4) can be evaluated by using a length-N FFT following the sample-by-sample multiplication between $r[n]$ and $s^*[n-\tau]$. For all points in the plane, the number of multiplication required is then $N(N + N\log_2 N)$.

The basic approach to reduce such complexity consists of two steps to be described in the following sub-sections. The first step is to partition the image space into sets of pixels on a grid so that they can be more efficiently computed as a group. The second step is to arrange the order in which these sets are evaluated so that past outcome can be reused subsequently.

4.1. Decomposition of the delay-Doppler Image

To exploit the sinusoidal structure of the sequence, we first decompose the delay-Doppler index into

$$[\tau, \nu] = [iQ + \delta_\tau, mL + \delta_\nu], \tag{9}$$

where $0 \leq i < L$, $0 \leq \delta_\tau < Q$, $0 \leq m < Q$ and $0 \leq \delta_\nu < L$. For a given $(\delta_\tau, \delta_\nu)$, the pixels spanned by the indices (i, m) form a grid shown in Fig. 1.

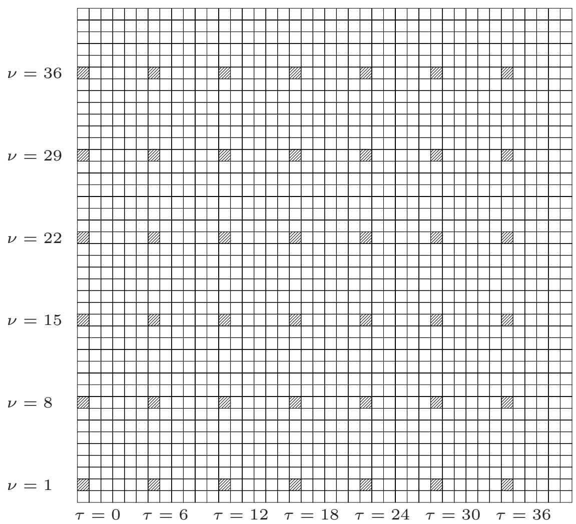

Fig. 1. $Q = 6, L = 7, \delta_\tau = 0, \delta_\nu = 1$

With such decomposition, the delay-Doppler image (4) is then given by

$$\begin{aligned} I[iQ + \delta_\tau, mL + \delta_\nu] &= \sum_{n=0}^{N-1} r[n] \cdot \\ &\quad s^*[n - (iQ + \delta_\tau)] e^{-j\frac{2\pi(mL+\delta_\nu)n}{N}} \\ &= e^{-j\frac{2\pi m \delta_\tau}{Q}} \sum_{l=0}^{L-1} \sum_{n=0}^{Q-1} r[n + (l+i)Q + \delta_\tau] \cdot \\ &\quad e^{-j\frac{2\pi\delta_\nu(n+(l+i)Q+\delta_\tau)}{N}} e^{-j\frac{2\pi(m+\nu_l)n}{Q}} \\ &= e^{-j\frac{2\pi m \delta_\tau}{Q}} \sum_{l=0}^{L-1} \sum_{n=0}^{Q-1} \tilde{r}[n, l+i, \delta_\tau, \delta_\nu] e^{-j\frac{2\pi(m+\nu_l)n}{Q}}, \end{aligned} \tag{10}$$

where

$$\tilde{r}[n, l, \delta_\tau, \delta_\nu] = r[n'] \, e^{-j\frac{2\pi\delta_\nu n'}{N}} \Big|_{n'=n+lQ+\delta_\tau} \tag{11}$$

is a sequence of the index n defined over $0 \leq n < Q$.

For a given $(\delta_\tau, \delta_\nu)$ we further define the length-Q DFT of $\tilde{r}[n, l, \delta_\tau, \delta_\nu]$ over the time index n as

$$\tilde{R}[k, l, \delta_\tau, \delta_\nu] = \sum_{n=0}^{Q-1} \tilde{r}[n, l, \delta_\tau, \delta_\nu] e^{-j\frac{2\pi kn}{Q}} \tag{12}$$

for all $0 \leq l < L$ and $0 \leq k < Q$. Eq. (10) then becomes

$$\begin{aligned} I[iQ + \delta_\tau, mL + \delta_\nu] &= e^{-j\frac{2\pi m \delta_\tau}{Q}} \cdot \\ &\sum_{l=0}^{L-1} \tilde{R}[(m + \nu_l) \bmod Q, (l+i) \bmod L, \delta_\tau, \delta_\nu]. \end{aligned} \tag{13}$$

From Eq. (13) it can be noted that the delay-Doppler correlation for all combinations of (i, m) conditioned on a given $(\delta_\tau, \delta_\nu)$ can be evaluated by selecting and summing the L metrics in $\tilde{R}$ that correspond to the circularly shifted time-frequency hopping pattern defined by $[l, \tau_l]$.

4.2. Sliding DFT

Furthermore, by carefully examining Eq. (12) it can be noted that the DFT is performed over a sliding window as the index δ_τ advances. Therefore, the complexity can be further reduced by using sliding DFT [6]:

$$\begin{aligned} \tilde{R}[k, l, \delta_\tau + 1, \delta_\nu] &= \sum_{n=0}^{Q-1} \tilde{r}[n, l, \delta_\tau + 1, \delta_\nu] e^{-j\frac{2\pi kn}{Q}} \\ &= \sum_{n=0}^{Q-1} \tilde{r}[n+1, l, \delta_\tau, \delta_\nu] e^{-j\frac{2\pi kn}{Q}} = e^{j\frac{2\pi k}{Q}} \cdot \\ &\left(\tilde{R}[k, l, \delta_\tau, \delta_\nu] + \tilde{r}[0, l+1, \delta_\tau, \delta_\nu] - \tilde{r}[0, l, \delta_\tau, \delta_\nu]\right). \end{aligned} \tag{14}$$

In other words, the DFT of a windowed segment can be derived from that of the previous overlapping segment with simple operations of addition and phase rotation. The computation is equivalent to performing N sliding DFT of length Q, each requiring Q multiplications, for all δ_ν. Additionally, N multiplications are needed to evaluate $\tilde{r}$ in Eq. (11) for all δ_ν. Therefore, the total number of multiplications required is $L(N \times Q + N)$ or approximately $L \times N \times Q = N^2$ multiplications. That is on average one multiplication for each point on the delay-Doppler plane.

4.3. Example

Fig. 2 shows an example of a delay-Doppler correlator for a time-frequency hopping pattern with $Q = 6$ and $L = 7$ at a given (but unspecified) index δ_ν. To evaluate the delay-Doppler image over its entire range, the same operation shown in this figure needs to be carried out for each of the $L = 7$ values of δ_ν ($\delta_\nu = 0, \cdots, 6$). In the beginning, the received samples $r[n]$ are first phase rotated by $-2\pi\delta_\nu n/N$. At $\delta_\tau = 0$, a length-6 DFT is performed for each of the 7 consecutive segments of length 6, as shown in the upper half of Fig. 2. The resulting array of frequency domain samples contain all the values required to evaluate the delay-Doppler image at all combination of (i, m) for the given $(\delta_\tau, \delta_\nu)$ according to Eq. (13). The same process is then executed for $\delta_\tau = 1$ by circularly sliding the DFT windows to the right by

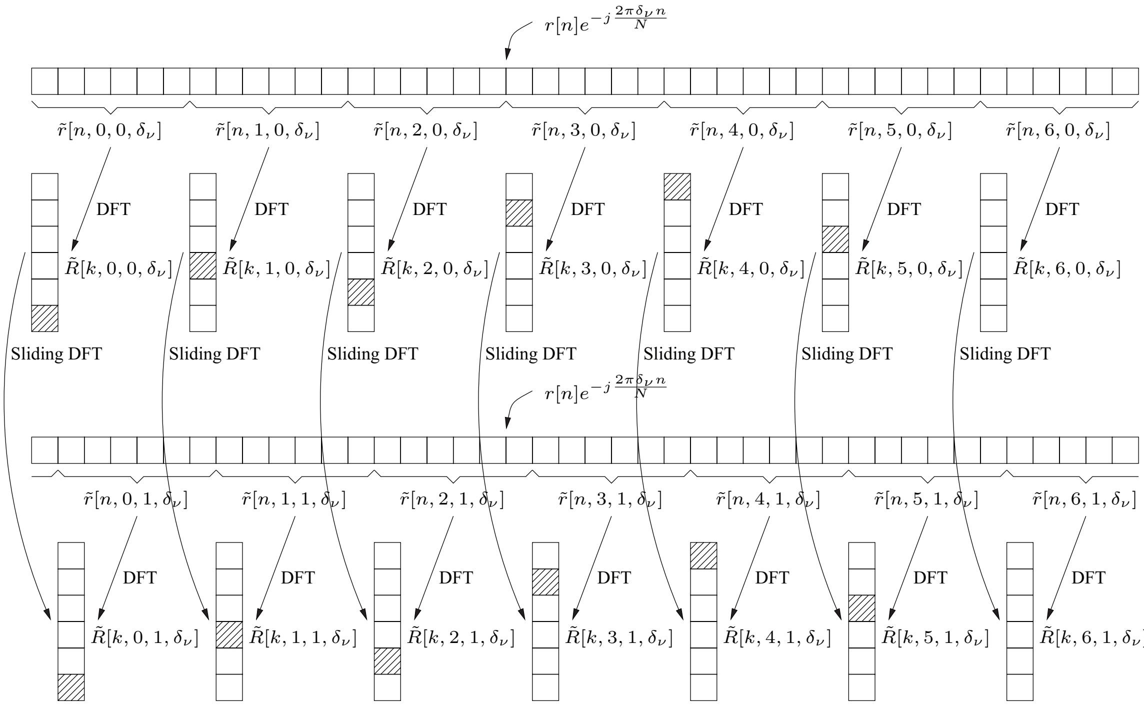

Fig. 2. delay-Doppler correlator for time-frequency hopping pattern at a given δ_ν

one sample, as shown in the lower half of Fig. 2. Although the DFT over the new window position can be calculated directly from the new samples, it is more efficient, as pointed out in Eq. (14), to derive it from the DFT of the previous window. Comparing with the straightforward computation for an arbitrary sequence of arbitrary length, the proposed method can reduce the number of multiplications by a factor of the sequence length.

5. CONCLUSION

In this paper, we have derived a reduced-complexity delay-Doppler correlator for time-frequency hopping sequences. The addressed problem can be found in applications such as continuous waveform delay-Doppler radar and signature sequence identification in wireless communication. The algorithm is summarized as follows:

1. Decompose the delay-Doppler planes into sets of grid formed by Eq. (9).
2. For a given $(\delta_\tau, \delta_\nu)$, a two-dimensional time-frequency array $\tilde{R}$ are calculated according to Eq. (12) using DFT.
3. The delay-Doppler image on the grid is calculated by summing the elements in the array matching the time-frequency hopping pattern according to Eq. (13).
4. For the same δ_ν, advance δ_τ by one and evaluate the two-dimensional array $\tilde{R}$ from the array corresponding to its preceding delay index using a sliding DFT according to Eq. (14) until all δ_τ exhausted.
5. Advance δ_ν and repeat the same procedure starting from Step 2 until all δ_ν exhausted.

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