MINIMUM BER BEAMFORMING IN THE RF DOMAIN FOR OFDM TRANSMISSIONS AND LINEAR RECEIVERS

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ABSTRACT

In this paper, we study transmission schemes for a novel OFDM-based MIMO system which performs adaptive signal combining in radio-frequency (RF). Specifically, we consider the problem of selecting the linear precoder and the transmit and receive RF weights (or beamformers) for minimizing the bit error rate (BER) under the assumption of perfect channel knowledge and linear receivers. Firstly, it is shown that the optimal precoder amounts to uniformly distribute the overall mean square error (MSE) among the information symbols. Secondly, we propose a gradient search algorithm to obtain the optimal pair of beamformers. Interestingly, in the case of low signal to noise ratios (SNR), the proposed beamforming criterion is equivalent to the maximization of the received SNR. However, for moderate and high SNRs, part of the received SNR is sacrificed in order to improve the channel response of the worst subcarriers, which translates into significant advantages over other previously proposed approaches. Finally, the performance of the proposed scheme is illustrated by means of some numerical examples.

Index Terms— Analog combining, beamforming, OFDM.

1. INTRODUCTION

Conventional baseband implementations of multiple-input multiple-output (MIMO) systems must process simultaneously the signals transmitted and received by each antenna and, consequently, the hardware cost and power consumption associated to these full baseband MIMO schemes is typically very high. A lower cost alternative to full MIMO schemes is provided by the analog combing architecture shown in Fig. 1. As can be seen, with this scheme only one signal path is required for the second mixer/intermediate-frequency chain, the ADCs (or DACs at the transmitter side) and the baseband. Consequently, the system costs and size can be significantly reduced compared to a full baseband MIMO architecture, while still retaining some of the MIMO advantages such as better reliability (through spatial diversity) and better coverage (through array gain). Therefore, analog beamforming provides a very interesting alternative to conventional MIMO architectures, specially in some systems such as those operating around 60 Ghz.

From a signal processing point of view, the structure imposed by the adaptive antenna combining architecture gives rise to several

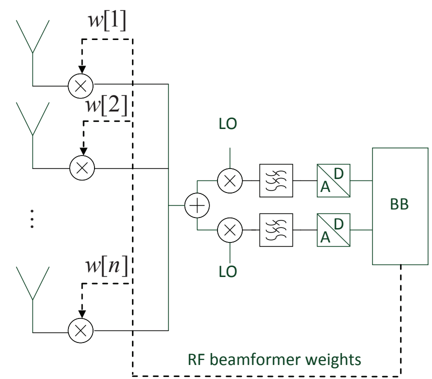

Fig. 1. Analog antenna combining in the RF path for MIMO communications systems. Exemplarily shown for a direct-conversion receiver.

challenging design problems. In this work, we address the problem of minimizing the bit-error rate (BER) of the linear receiver under OFDM-based transmissions and perfect channel state information. Unlike conventional OFDM-MIMO systems, the adaptive antenna combining architecture forces us to use a single pair of frequency-independent beamformers, which constitutes the main design challenge. In particular, we show that the optimal linear precoder amounts to uniformly distribute the overall mean square error (MSE) among the information symbols. Thus, the optimal RF weights (or beamformers) reduce to the minimization of the total MSE, and they can be obtained by means of a gradient search algorithm proposed in this paper. Interestingly, in the low signal to noise ratio (SNR) regime, the proposed beamforming criterion amounts to maximize the received SNR, and therefore it is equivalent to some previous related works [1–4]. However, it is shown that for moderate and high SNRs the proposed criterion sacrifices part of the received SNR in order to improve the channel for the worst subcarriers, which translates into significant improvements over the maximum SNR approaches.

Throughout this paper we will use bold-faced upper case letters to denote matrices, bold-faced lower case letters for column vector, and light-faced lower case letters for scalar quantities. Superscripts $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian, $\|\mathbf{A}\|$ denotes the Frobenius norm of matrix $\mathbf{A}$ and $E[\cdot]$ is the expectation operator. Finally, $\mathbf{I}$ denotes the identity matrix of the required dimensions.

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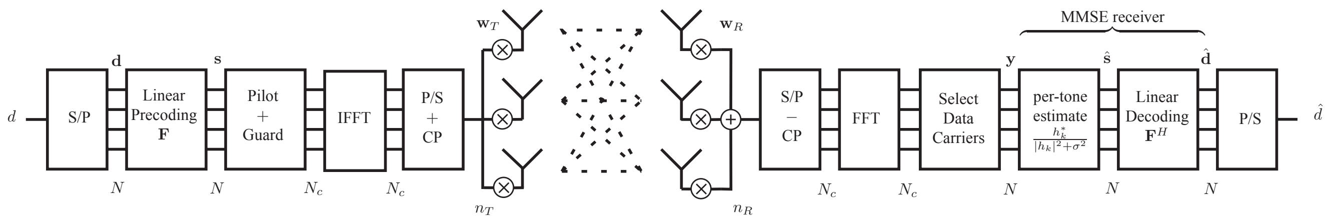

Fig. 2. Block diagram of the OFDM-based system with transmit and receive analog antenna combining.

2. MULTICARRIER BEAMFORMING WITH ANALOG COMBINING SCHEMES

Let us consider the OFDM-based $n_T \times n_R$ MIMO system shown in Fig. 2, where N out of N_c subcarriers contains linearly precoded information symbols. As can be seen, after insertion of pilot and null carriers, inverse fast Fourier transform (IFFT), and addition of the cyclic prefix, the OFDM symbol is transmitted through the n_T transmit antennas using the analog combining architecture shown in Fig. 1. Following an analogous processing at the receiver side, the data model after the FFT is

$$\mathbf{y} = \mathbf{\Lambda s} + \mathbf{n} = \mathbf{\Lambda F d} + \mathbf{n},$$

where $\mathbf{y} = [y_1, \ldots, y_N]^T$ are the observations, $\mathbf{s} = \mathbf{Fd}$ are the linearly precoded symbols, $\mathbf{d} = [d_1, \ldots, d_N]^T$ contains the N information symbols, $\mathbf{n} \in \mathbb{C}^{N\times 1}$ is the additive complex Gaussian noise with zero mean and variance σ^2, $\mathbf{F} \in \mathbb{C}^{N\times N}$ is the linear precoding matrix, and the equivalent single-input single-output (SISO) channel $\mathbf{\Lambda}$ is a diagonal matrix with elements

$$h_k = \mathbf{w}_R^H \mathbf{H}_k \mathbf{w}_T, \qquad k = 1, \ldots, N,$$

where $\mathbf{w}_R \in \mathbb{C}^{n_R\times 1}$ and $\mathbf{w}_T \in \mathbb{C}^{n_T\times 1}$ are the receive/transmit RF-beamformers, and $\mathbf{H}_k \in \mathbb{C}^{n_R\times n_T}$ represents the MIMO channel associated to the k-th data subcarrier.

Without loss of generality, we assume unit-power i.i.d. information symbols and unit-norm beamformers. Furthermore, in order to preserve the covariance matrix of the transmitted signals[1], we introduce the constraint

$$E[\mathbf{s}\mathbf{s}^H] = E[\mathbf{d}\mathbf{d}^H] = \mathbf{I}, \quad \Leftrightarrow \quad \mathbf{F}\mathbf{F}^H = \mathbf{I},$$

i.e., the precoding matrix $\mathbf{F}$ is assumed to be unitary. As we will see later, this assumption translates into an optimal precoding $\mathbf{F}$ independent of the channel and therefore, the necessary feedback can be reduced to the weights of the beamformer $\mathbf{w}_T$.

2.1. Linear MMSE Receiver

Under perfect channel knowledge at the receiver, the linear minimum mean-square error (MMSE) estimate of the information vector is

$$\hat{\mathbf{d}} = \mathbf{F}^H \hat{\mathbf{s}} = \mathbf{F}^H \mathbf{\Lambda}^H \left(\mathbf{\Lambda}^H \mathbf{\Lambda} + \sigma^2 \mathbf{I}\right)^{-1} \mathbf{y},$$

which yields the following MSE matrix

$$\mathbf{E} = E\left[(\hat{\mathbf{d}} - \mathbf{d})(\hat{\mathbf{d}} - \mathbf{d})^H\right] = \sigma^2 \left(\mathbf{F}^H \mathbf{\Lambda}^H \mathbf{\Lambda F} + \sigma^2 \mathbf{I}\right)^{-1}.$$

[1]The extension of the considered problem to non-unitary precoding matrices and/or adaptive power allocation schemes is a challenging task, which will be considered in future works.

Interestingly, due to the unitarity of the precoding matrix $\mathbf{F}$, the eigenvalues of $\mathbf{E}$ are

$$\text{MSE}_{s_k} = \frac{\sigma^2}{|h_k|^2 + \sigma^2}, \qquad k = 1, \ldots, N,$$

and they represent the MSE associated to the k-th data carrier. Thus, the total MSE, which does not depend on $\mathbf{F}$, is

$$\text{MSE}(\mathbf{w}_T, \mathbf{w}_R) = \sum_{k=1}^{N} \text{MSE}_{d_k} = \sum_{k=1}^{N} \frac{\sigma^2}{|h_k|^2 + \sigma^2},$$

where MSE_{d_k} are the elements along the diagonal of $\mathbf{E}$, i.e., the MSE in the estimate of the information symbols.

3. TRANSCEIVER DESIGN

In this section, we design the system parameters $\mathbf{F}$, $\mathbf{w}_T$ and $\mathbf{w}_R$ in order to minimize the averaged bit error rate (BER), i.e.,

$$\begin{aligned} \underset{\mathbf{F}, \mathbf{w}_T, \mathbf{w}_R}{\text{Minimize}} \quad & \text{BER}(\mathbf{F}, \mathbf{w}_T, \mathbf{w}_R), \\ \text{subject to} \quad & \|\mathbf{w}_T\| = \|\mathbf{w}_R\| = 1, \\ & \mathbf{F}\mathbf{F}^H = \mathbf{I}. \end{aligned} \tag{1}$$

3.1. Design of the Precoding Matrix F

The design of the linear precoding matrix $\mathbf{F}$ in OFDM systems has been addressed, under different criteria, in [5–7]. In the case of linear receivers, the basic idea consists in writing the averaged BER as a function of the MSE associated to the information symbols

$$\text{BER} = \frac{1}{N} \sum_{k=1}^{N} \text{BER}_k = \frac{1}{N} \sum_{k=1}^{N} g(\text{MSE}_{d_k}),$$

where $g(\cdot)$ is an increasing convex function[2], and BER_k represents the BER of the k-th information symbol d_k. Thus, the averaged BER is a Schur-convex function, i.e.,

$$\text{BER} = \frac{1}{N} \sum_{k=1}^{N} g(\text{MSE}_{d_k}) \geq g\left(\frac{1}{N} \sum_{k=1}^{N} \text{MSE}_{d_k}\right), \tag{2}$$

and the lower bound is achieved when all the MSE_{d_k} are equal [8]. Finally, it has been proved in [5, 8] that to ensure a uniform distribution of the total MSE among the information symbols, the optimal precoding matrix $\mathbf{F}$ must be unitary with constant modulus entries, such as the Fourier or Walsh-Hadamard matrices.

[2]The convexity of $g(\cdot)$ for practical BER values can be easily proven [8].

Estimate $\mathbf{H}$, σ^2 and initialize μ, $\mathbf{w}_R$, $\mathbf{w}_T$.
repeat
  Update of the receive beamformer
  Obtain the equivalent SIMO channels $\mathbf{h}_{\text{SIMO}_k}$.
  Update h_k, MSE_{s_k} and obtain $\mathbf{R}_{\text{SIMO}}$ with (5).
  Update $\mathbf{w}_R$ with (7) and normalize: $\mathbf{w}_R = \mathbf{w}_R/\|\mathbf{w}_R\|$.
  Update of the transmit beamformer
  Obtain the equivalent MISO channels $\mathbf{h}_{\text{MISO}_k}$.
  Update h_k, MSE_{s_k} and obtain $\mathbf{R}_{\text{MISO}}$ with (6).
  Update $\mathbf{w}_T$ with (7) and normalize: $\mathbf{w}_T = \mathbf{w}_T/\|\mathbf{w}_T\|$.
until Convergence

Algorithm 1: Proposed algorithm for minimum BER beamforming.

3.2. Design of the Beamformers

If the optimal precoding matrix $\mathbf{F}$ obtained in the previous subsection is used, the averaged BER is given by the rightmost term in (2) and the optimization problem (1) reduces to

$$\underset{\mathbf{w}_T,\mathbf{w}_R}{\text{Minimize}} \quad \sum_{k=1}^{N} \text{MSE}_{s_k}, \qquad \text{s. t.} \qquad \|\mathbf{w}_T\| = \|\mathbf{w}_R\| = 1. \quad (3)$$

Solving (3) with respect to $\mathbf{w}_R$ and $\mathbf{w}_T$ by means of the Lagrange multipliers method we obtain

$$\mathbf{R}_{\text{SIMO}}\mathbf{w}_R = \lambda \mathbf{w}_R, \qquad \mathbf{R}_{\text{MISO}}\mathbf{w}_T = \lambda \mathbf{w}_T, \quad (4)$$

where the products $\mathbf{R}_{\text{SIMO}}\mathbf{w}_R$ and $\mathbf{R}_{\text{MISO}}\mathbf{w}_T$ represent the negative gradient of $\text{MSE}(\mathbf{w}_T, \mathbf{w}_R)$ with respect to $\mathbf{w}_R$ and $\mathbf{w}_T$, λ is the Lagrange multiplier,

$$\mathbf{R}_{\text{SIMO}} = \sum_{k=1}^{N} \text{MSE}_{s_k}^2 \mathbf{h}_{\text{SIMO}_k} \mathbf{h}_{\text{SIMO}_k}^H, \quad (5)$$

$$\mathbf{R}_{\text{MISO}} = \sum_{k=1}^{N} \text{MSE}_{s_k}^2 \mathbf{h}_{\text{MISO}_k} \mathbf{h}_{\text{MISO}_k}^H, \quad (6)$$

and $\mathbf{h}_{\text{SIMO}_k} = \mathbf{H}_k \mathbf{w}_T$, (resp. $\mathbf{h}_{\text{MISO}_k} = \mathbf{H}_k^H \mathbf{w}_R$) are the equivalent SIMO (MISO) channels after fixing the transmit (receive) beamformer.

3.3. Optimization Algorithm

Unfortunately, the optimization problems in (4) are coupled through the matrices $\mathbf{R}_{\text{SIMO}}$ and $\mathbf{R}_{\text{MISO}}$, which depend on both $\mathbf{w}_T$ and $\mathbf{w}_R$, and this precludes obtaining a closed-form solution. Here, we propose a gradient search algorithm based on the following updating rules

$$\mathbf{w}_R = \mathbf{w}_R + \mu \mathbf{R}_{\text{SIMO}} \mathbf{w}_R, \qquad \mathbf{w}_T = \mathbf{w}_T + \mu \mathbf{R}_{\text{MISO}} \mathbf{w}_T, \quad (7)$$

where μ is a step-size (or smoothing factor). The overall method, which is summarized in Algorithm 1, can also be interpreted as a single iteration of a power method for the extraction of the eigenvectors of $\mathbf{I} + \mu \mathbf{R}_{\text{SIMO}}$ and $\mathbf{I} + \mu \mathbf{R}_{\text{MISO}}$.

Interestingly, in the low SNR regime the total MSE can be approximated by

$$\text{MSE}(\mathbf{w}_T, \mathbf{w}_R) = \sum_{k=1}^{N} \frac{\sigma^2}{|h_k|^2 + \sigma^2} \simeq 1 - \frac{1}{\sigma^2} \sum_{k=1}^{N} |h_k|^2,$$

and the weights $\text{MSE}_{s_k}^2$ in the definition of the matrices (5) and (6) are $\text{MSE}_{s_k}^2 \simeq 1$. Thus, problem (3) reduces to the maximization of the energy of the equivalent channel (or the received SNR): a criterion that has been addressed in previous works [1–4]. In particular, in the specific SIMO and MISO cases, the maximum SNR beamformers can be found in closed-form, whereas in the general MIMO case, the optimal solutions have to be found by means of iterative techniques such as that proposed in [3].

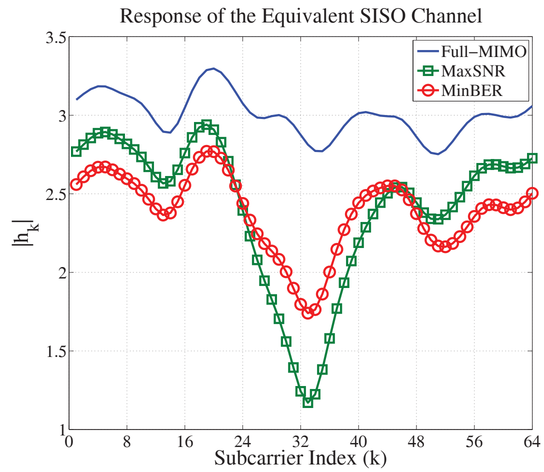

Fig. 3. Response of the equivalent SISO channel. SNR=10 dB.

On the other hand, for high or moderate SNRs, the mean square error MSE_{s_k} rapidly decreases with the energy of the k-th sub-channel $|h_k|^2$. Consequently, in this regime different values ($\text{MSE}_{s_k}^2$) are used to weight the equivalent SIMO and MISO matrices per subcarrier in (5) and (6), respectively.

Similarly to other gradient descent or alternating optimization techniques [3, 9], the method proposed in this paper could suffer from local minima. However, we have verified by means of exhaustive simulations that the local minima problems can be avoided if a proper initialization is used. In particular, we propose to use a closed-form approximation to the maximum SNR beamformers.[3] As it will be shown next, with this starting point the proposed algorithm converges in a small number of iterations, and it outperforms the maximum SNR approach.

4. SIMULATION RESULTS

The performance of the proposed technique, which we refer to as MinBER, is illustrated in this section by means of some Monte Carlo simulations. In all the experiments, a 4×4 Rayleigh MIMO channel with exponential power delay profile has been assumed. In particular, the total power associated to the l-th tap is $n_T n_R \rho^l (1-\rho)$, where we have selected $\rho = 0.36$. In all the experiments we have used $N_c = N = 64$ data subcarriers and QPSK information symbols, which are linearly precoded with the Fourier matrix. The proposed algorithm has been compared with the maximum SNR approach proposed in [3] (denoted as MaxSNR) and with a full MIMO scheme with dominant eigenmode transmission (DET) and maximum ratio combining (MRC) in each subcarrier (denoted as Full-MIMO), which can be seen as an upper bound for the performance of any system with antenna combining in the RF path.

[3]Details of this initialization technique can be found in [10]. They are not included in this paper due to the lack of space.

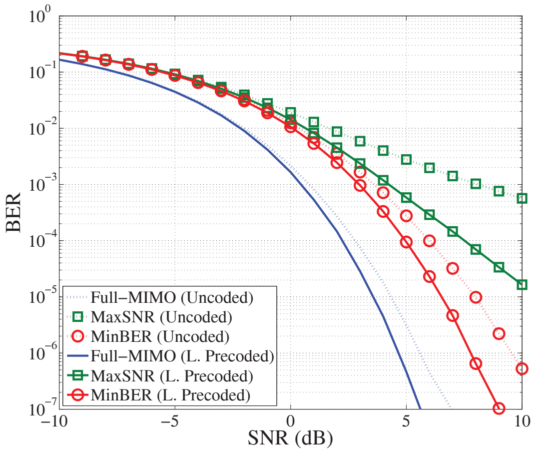

Fig. 4. BER performance of the evaluated schemes.

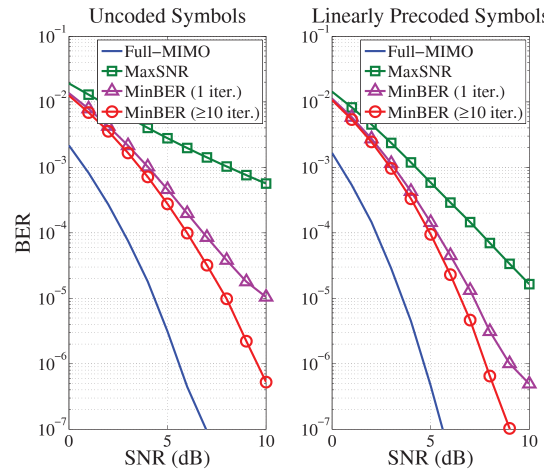

Fig. 5. Convergence of the MinBER algorithm.

In the first example, we analyze the equivalent channel after beamforming. Fig. 3 shows the frequency response of the equivalent channel for one random channel realization and a SNR equal to 10 dB. As expected, the best response is that of the Full-MIMO system, which applies a different pair of beamformers for each subcarrier. However, we can see that, unlike the MaxSNR approach, the MinBER criterion avoids deep nulls in the frequency response of the equivalent channel. Interestingly, this flattening of the equivalent channel suggests that the MinBER criterion will also outperform the MaxSNR approach when the information symbols are not linearly precoded (i.e., $\mathbf{F} = \mathbf{I}$), which is corroborated in the next example.

In the second experiment we have evaluated the BER performance of the three previous schemes. As can be seen in Fig. 4, the performance of the MinBER is again between those of the Full-MIMO and MaxSNR criteria. Furthermore, we can see that the advantage of the proposed criterion over the MaxSNR approach increases when the information symbols are not linearly precoded. This can be seen as a direct consequence of the fact that, for uncoded transmissions and high SNRs, the BER is dominated by the worst subcarrier, which in the MinBER case is better than that of the MaxSNR approach.

The convergence of the proposed beamforming technique is illustrated in the final example. In particular, we have limited the MinBER algorithm to 1, 10 and 20 iterations. The results obtained for the previous example are shown in Fig. 5, where we can see that the proposed algorithm converges very fast to the optimal solution. Furthermore, we can observe a significant improvement over the MaxSNR technique by means of only one iteration.

5. CONCLUSION

In this paper we have addressed the problem of adaptive antenna combining in the radio frequency (RF) path under multicarrier transmissions. Assuming linearly precoded transmissions and linear receivers, we have obtained the linear precoder and the pair of beamformers minimizing the averaged BER. Independently of the beamformers, the optimal precoder is given by the Fourier or Walsh-Hadamard matrices, which reduce the beamforming criterion to the minimization of the total MSE. Unlike other previous approaches, based on the maximization of the received signal to noise ratio (SNR), the proposed beamforming criterion takes into account the quality of all the flat-fading sub-channels, avoiding deep nulls which could have a strong impact over the average BER of the system. Finally, the performance of the proposed technique has been illustrated by means of some numerical examples.

6. REFERENCES

[1] M. Okada and S. Komaki, "Pre-DFT combining space diversity assisted COFDM," *IEEE Transactions on Vehicular Technology*, vol. 50, no. 2, pp. 487–496, Mar. 2001.

[2] S. Sandhu and M. Ho, "Analog combining of multiple receive antennas with OFDM," in *IEEE International Conference on Communications (ICC '03)*, May 2003, vol. 5, pp. 3428–3432.

[3] Defeng Huang and K. B. Letaief, "Symbol-based space diversity for coded OFDM systems," *IEEE Transactions on Wireless Communications*, vol. 3, no. 1, pp. 117–127, Jan. 2004.

[4] M.I. Rahman, K. Witrisal, S.S. Das, F.H.P. Fitzek, O. Olsen, and R. Prasad, "Optimum pre-DFT combining with cyclic delay diversity for OFDM based WLAN systems," in *IEEE Vehicular Tech. Conf.*, May 2004, vol. 4, pp. 1844–1848.

[5] S. Barbarossa, *Multiantenna Wireless Communication Systems*, Artech House Publishers, 2005.

[6] X. Ma and G. B. Giannakis, "Complex field coded MIMO systems: performance, rate, and trade-offs," *Wireless Comm. and Mobile Computing*, vol. 2, no. 7, pp. 693–717, 2002.

[7] Y. Xin, Z. Wang, and G.B. Giannakis, "Space-time diversity systems based on linear constellation precoding," *IEEE Trans. on Wireless Comm.*, vol. 2, no. 2, pp. 294–309, Mar. 2003.

[8] D. P. Palomar and Y. Jiang, "MIMO transceiver design via majorization theory," *Found. Trends Commun. Inf. Theory*, vol. 3, no. 4, pp. 331–551, 2006.

[9] D. P. Bertsekas, *Nonlinear Programming*, Athena Scientific, Belmont, MA, Sept. 1999.

[10] I. Santamaría, J. Vía, V. Elvira, J. Ibáñez, J. Pérez, R. Eickhoff, and U. Mayer, *Handbook of Smart Antennas for RFID Systems*, chapter Low cost and compact RF-MIMO transceivers, Wiley, 2009.