A PRACTICAL WALSH LAYERING SCHEME FOR RELIABLE TRANSMISSION

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ABSTRACT

Concerning the uncertainty of channels and peak power constraint, we give a new practical layering scheme to do reliable transmission. In our scheme, Walsh matrix is employed to do layer-time coding. Regarding columns of a layer-time coding matrix as layers and rows as time, after Walsh layer-time coding, interference among layers can be removed or diminished by adding rows up. When there are layers decoded successfully after the previous transmission, only not-yet-decoded layers will be retransmitted. Simulation results show that our Walsh layering scheme with hybrid automatic repeat request (HARQ) performs much better than the traditional single-layer ARQ sequential transmission with respect to average time delay.

***Index Terms*—** Multi-layer coding, automatic repeat request

1. INTRODUCTION

Nowadays, automatic repeat request (ARQ) is widely used in reliable transmission for combating the uncertainty of channels. In such transmission, how to reduce time delay is the key issue. Supposing there are a quantity of packets to be transmitted reliably, we could do certain kind of rateless coding for Gaussian channels: construct layers by packets, superimpose them and send; the receiver does successive interference cancelation (SIC) for decoding layers from top to bottom; if there are bottom layers undecoded, the receiver notifies the transmitter by hybrid ARQ (HARQ) signal and then the transmitter retransmits. In this strategy, how to cancel interference among layers is a critical problem.

In [1–3], Erez *et al.* studied rateless coding for Gaussian channels with respect to rate. For canceling interference among layers, they use random dithering to let all layered packets statistically independent to each other. Then, for each layer, assumed knowing SNRs of all transmissions, an MRC receiver can sum all individual SNRs up. For implementing their schemes, it is required that decoding at the receiver depends only on average SNR. However, in practice, for delay-limit transmission, a decoder depends on instantaneous SNR rather than average SNR, which makes the power allocation scheme and MRC receiver in [1–3] ineffective. Due to the uncertainty of channels, instantaneous SNRs are unknowable. Hence, the MRC receiver in [1–3] cannot give a sum of SNRs after several transmissions. The instantaneous SNR after such MRC would be even worse than that in single transmission. Assuming several single-symbol packets to be transmitted, our simulation shows that if we implement the power allocation scheme in [3] for layers and MRC receiver, the number of retransmissions may be very large.

In this paper, we refer to Erez *et al.*'s idea of layering packets and using SIC to decode layers, but not of random dithering and the MRC receiver. We employ Walsh layering to do rateless coding and then interference among packets can be removed or diminished by simple processing at the receiver. For the initial power allocation, we refer to the idea in [3] but change it a bit as we take peak power constraint and instantaneous SNR into account. For reducing interference and amplifying valid power in retransmission, we suggest to use a sort of hybrid ARQ signal that can tell the transmitter which layer has been successfully decoded and thus do not need not to be retransmitted. The simulation result will illustrate the good performance of our scheme.

2. SCHEME DESCRIPTION

In this section, for simplicity, first we will describe our practical scheme under assumption that what to be transmitted are single-symbol packets. Then, we will explain how to extend it for transmitting larger packets.

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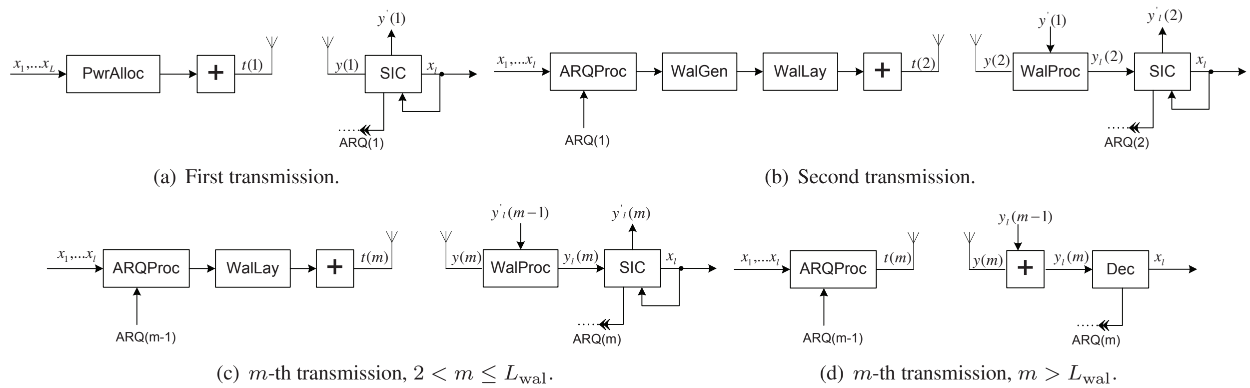

(a) First transmission.

(b) Second transmission.

(c) m-th transmission, $2 < m \le L_{\text{wal}}$.

(d) m-th transmission, $m > L_{\text{wal}}$.

Fig. 1. Block diagram of the practical scheme for rateless coding.

2.1. Walsh layer-time coding

Walsh matrices are the Hadamard matrices of dimension 2^k for $k \in \mathbb{N}$. They are given by the recursive formula

$$\begin{aligned} \mathbf{W}(2) &= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \\ \mathbf{W}(2^k) &= \begin{bmatrix} \mathbf{W}(2^{k-1}) & \mathbf{W}(2^{k-1}) \\ \mathbf{W}(2^{k-1}) & -\mathbf{W}(2^{k-1}) \end{bmatrix}. \end{aligned} \tag{1}$$

From the above formula, we can see that if we employ a Walsh matrix as a layer-time matrix whose columns are layers and rows are time, interference can be removed or diminished by adding rows up. Note that OVSF code matrix has the same property as it is a variation of Walsh matrix with changing the row order.

2.2. Walsh layering scheme for single-symbol packets

2.2.1. Channel model

Fig.1 gives the block diagram of our Walsh layering scheme for single-symbol packets. We suppose that for the m-th transmission, L unit-energy phase-modulated symbols $x_1, \cdots x_L$ compose a *transmit symbol* $t(m)$ via

$$t(m) = \mathbf{p}(m)\mathbf{x} \tag{2}$$

where the row vector $\mathbf{p}(m)$ is the layering code for the m-th transmission and the column vector $\mathbf{x}$ is composed of $x_1, \cdots x_L$. The peak power constraint of $t(m)$ is denoted by P. $\mathbf{p}(m)$ is relevant not only to a certain Walsh matrix but also to which layers have been decoded and how many transmissions have been done. A detailed description of $\mathbf{p}(m)$ will be given in subsections below.

For an AWGN slow-fading channel, the channel model for the m-th transmission can be represented by

$$y(m) = h\,\mathbf{p}(m)\mathbf{x} + n(m) \tag{3}$$

where $y(m)$ is the received signal, h is the channel coefficient constant for all transmissions in one block, and $n(m)$ is the white additive Gaussian noise for the m-th transmission.

2.2.2. First transmission

At the first transmission, the power allocation (PwrAlloc module in Fig.1) scheme for layers at the transmitter ensures that when the *instantaneous channel SNR* $|h|^2/|n(1)|^2$ is higher than a certain threshold, all layer symbols will be successfully decoded by successive interference cancelation (SIC) at the receiver.

Our power allocation scheme is similar to the one in [3] except some different considerations. Assume all layered symbols have the same SNR threshold $\underline{\rho}$ which ensures correct decoding and the received signal is processed by SIC in the order from the top layer x_L to the bottom layer x_1. Let P_l denote the allocated power for x_l. The instantaneous SNR of x_l at the first transmission,

$$\begin{aligned} \text{SNR}_l &= \frac{|h|^2 P_l}{|h \sum_{l'=1}^{l-1} \sqrt{P_{l'}} x_{l'} + n(1)|^2} \\ &\ge \frac{\rho_h(1) P_l}{L(\rho_h(1) \sum_{l'=1}^{l-1} P_{l'} + 1)} \end{aligned} \tag{4}$$

where $\rho_h(1)$ is the instantaneous channel SNR of the first transmission, $|h|^2/|n(1)|^2$.

The transmit power,

$$\begin{aligned} P_t(m) &= |\sum_{l=1}^{L} \sqrt{P_l} x_l|^2 \\ &\le L \sum_{l=1}^{L} P_l. \end{aligned} \tag{5}$$

By letting

$$\begin{aligned} &\frac{\rho_h(1) P_l}{L(\rho_h(1) \sum_{l'=1}^{l-1} P_{l'} + 1)} \ge \underline{\rho}, \quad l = 1, \ldots, L, \\ &L \sum_{l=1}^{L} P_l \le P, \end{aligned} \tag{6}$$

we can obtain the threshold of $\rho_h(1)$,

$$\underline{\rho_h}(1) = \frac{L\left\{\underline{\rho} + (\underline{\rho}+1)\left[(L\underline{\rho}+1)^{L-1} - 1\right]\right\}}{P}, \quad (7)$$

and the power allocation scheme,

$$\begin{aligned} P_1 &= \frac{\underline{\rho}}{\underline{\rho_h}(1)}, \\ P_l &= \frac{L\underline{\rho}(\underline{\rho}+1)(L\underline{\rho}+1)^{l-2}}{\underline{\rho_h}(1)}, \quad l = 2, \ldots, L \end{aligned} \quad (8)$$

which ensures that when $\rho_h(1) \geq \underline{\rho_h}(1)$, the receiver can successfully decode all layered packets under the peak power constraint P. Namely, when $\rho_h(1)$ is high enough, one transmission is enough.

Note that there are also cases that $\rho_h(1)$ is smaller than $\underline{\rho_h}(1)$ but all layered symbols can be decoded successfully at the receiver. In such cases, all layers and noise happen to interact positively.

If the receiver cannot decode $x_{L'}$ successfully, a HARQ signal will be fed back to the transmitter and notify that $x_1, \ldots, x_{L'}$ cannot be decoded and let the transmitter layer them and send again. For instance, such a HARQ signal could be a symbol composed of $\lceil \log_2(L+1) \rceil$ bits. In this paper, the HARQ signal from the m-th transmission is denoted by a L-length binary row vector $\mathbf{s}(m)$, where 0 represents decoded and 1 represents not-decoded.

It is seen that for the first transmission,

$$\mathbf{p}(1) = \{\sqrt{P_1}, \ldots, \sqrt{P_L}\}. \quad (9)$$

2.2.3. *m-th transmission,* $2 \leq m \leq L_{\text{wal}}$

If the transmitter receives the HARQ signal from the previous transmission and learns there are still $L^{'}(m)$ layers not-decoded, retransmission will start. Note that $L^{'}(1) = L$.

The power scaling coefficient for each retransmission is figured out by the module ARQProc,

$$a(m) = \sqrt{\frac{P}{\|\mathbf{s}(m-1) \otimes \mathbf{p}(1)\|^2}} \quad (10)$$

where $\otimes$ denotes Kronecker product. $a(m)$ is used to amplify powers of $L^{'}(m)$ not-decoded layers under the peak power constraint.

At the second transmission, a Walsh matrix $\mathbf{W}$ of size $L_{\text{wal}} = 2^{\lceil \log_2 L^{'}(2) \rceil}$ is generated by the module WalGen. Let $\mathbf{w}(m)$ denote the m-th row of $\mathbf{W}$. Then, the layering code for the m-th transmission, $2 \leq m \leq L_{\text{wal}}$,

$$\mathbf{p}(m) = a(m)\, \mathbf{p}(1) \otimes \mathbf{w}(m) \otimes \mathbf{s}(m-1). \quad (11)$$

Let $y_l(m)$ denote the processed received signal for the l-th layer at the m-th transmission. It is figured out as

$$y_l(m) = y_l^{'}(m-1) + \frac{w_{m,l}}{a(m)}\, y(m), \quad w_{m,l} = 1, -1 \quad (12)$$

where $y_l^{'}(m-1)$ is the processed received signal at the previous transmission for the l-th layer after removing all decoded layers, and $w_{m,l}$ is the l-th entry of $\mathbf{w}(m)$. Note that $y_l^{'}(1) = y^{'}(1)$ for all l. In Fig.1, this processing is done in the module WalProc.

$y_l(m)$ can also be represented as

$$y_l(m) = (\mathbf{c}_l(m) \otimes \mathbf{p}(1))\,\mathbf{x} + n_l(m) \quad (13)$$

where $n_l(m)$ is the equivalent additive noise for the l-th layer and $\mathbf{c}_l(m)$ is a row-vector composed of integer entries $\mathbf{c}_{l,l'}(m)$, $\mathbf{c}_{l,l'}(m) \in \mathbb{Z} \cap [-m, m]$.

In the module SIC, the receiver decodes $y_l(m)$ from the upper layer to the lower layer. For the l-th layer, only when all its upper layers ($l^{'} > l$) with non-zero coefficient $c_{l,l'}$ in $y_l(m)$ have been successfully decoded, the receiver could possibly decode it.

At the L_{wal}-th transmission, no matter how many layers have been decoded after previous transmissions, for the l-th layer, if it has not been decoded yet, we can always get

$$y_l(L_{\text{wal}}) = L_{\text{wal}} h \sqrt{P_l} x(l) + n_l(L_{\text{wal}}) \quad (14)$$

where there is no interference from other layers and the noise

$$n_l(L_{\text{wal}}) = n(1) + \sum_{m=2}^{L_{\text{wal}}} \frac{w_{m,l}\, n(m)}{a(m)}. \quad (15)$$

2.2.4. *m-th transmission, $m > L_{\text{wal}}$, switched to single-layer transmission*

From (14) and (15), we can see that the SNR in $y_l(L_{\text{wal}})$

$$\rho_l(L_{\text{wal}}) = \frac{L_{\text{wal}}^2 |h|^2 P_l}{|n_l(L_{\text{wal}})|^2} \quad (16)$$

If a layered packet x_l could not be decoded after L_{wal} transmissions, it may be because its initial power P_l is too small comparing to its lower layers. For solving this problem, single-layer sequential ARQ transmission is employed to transmit each not-yet-decoded packet alone at full peak power, received signals are summed up to do decision until the packet can be successfully decoded.

It is seen that if x_l is the top not-decoded packet and keeps to be transmitted till the m-th transmission ($m > L_{\text{wal}}$), the processed signal for decision will be

$$\begin{aligned} y_l(m) = {} & L_{\text{wal}} h \sqrt{P_l} x(l) + (m - L_{\text{wal}}) h \sqrt{P} x(l) + n_l(L_{\text{wal}}) \\ & + \sum_{m'=L_{\text{wal}}+1}^{m} n(m^{'}) \end{aligned} \quad (17)$$

whose instantaneous SNR is

$$\rho_l(m) = \frac{L_{\text{wal}}^2 |h|^2 P_l + (m - L_{\text{wal}})^2 |h|^2 P}{|n_l(L_{\text{wal}}) + \sum_{m'=L_{\text{wal}}+1}^{m} n(m)|^2}. \quad (18)$$

Namely, for $m > L_{\mathrm{wal}}$, the layering code $\mathbf{p}(m)$ is a row vector whose l-th entry is $\sqrt{P}$ and others are zeros.

This strategy is a compensation to the drawback of Walsh layering.

2.3. Further discussion

In our scheme, when m is not an even number, it is clear that no interference from any lower layer can be completely removed, however, it is diminished much.

Although, for simplicity, the above description of our Walsh layering scheme is given in the scenario of transmitting single-symbol packets, the scheme can be implemented to larger packet transmission easily. In that case, we just need to modify the block diagram in Fig.1 a bit: channel coding is done before power allocation and layer-time coding to build larger packets as layers; then each symbol of each packet is processed at the transmitter as a single-symbol packet; de-channel-coding is done before SIC; HARQ signal is fed back after whole packets are decoded and decided. We can see that in the implementation for larger packets, the overhead of HARQ is reduced.

In real situation, h may change during the whole procedure of the reliable transmission. Thus, the receiver is required to know h to do normalization, or the effect of reducing interference among layers would be worse.

3. SIMULATION RESULTS OF A QPSK EXAMPLE

In this section, our practical Walsh layering scheme is compared with the single-layer sequential scheme by a QPSK single-symbol packet example. The SNR threshold $\underline{\rho}$ for QPSK is figured out as 3 dB. Both schemes use maximum likelihood detectors for QPSK signals.

Suppose L QPSK symbols to be transmitted. The system is assumed to stop retransmitting till all single-symbol packets have been successfully decoded. The performance is measured by the average times of transmission per symbol over an AWGN slow-fading channel under the assumption that the channel coefficient $h \sim \mathcal{CN}(0,1)$ and the additive noise $n \sim \mathcal{CN}(0,1)$. The overhead of HARQ or ARQ feedback is not taken into account. We suppose the channel coefficient h remains constant for the whole procedure and the noise varies in every transmission.

Fig.2 indicates that comparing to single-layer sequential ARQ transmission, Walsh layering scheme reduces the delay of reliable transmission. From the plot, it is seen that for a certain range of SNR, there is a best number of layers. In our example, when peak power is between 5 dB and 11.5 dB, the layering scheme of $L = 2$ performs best; when peak power is between 11.5 dB and 28 dB, the layering scheme of $L = 3$ performs best; when peak power is between 28 dB and 30 dB, the layering scheme of $L = 4$ is the best.

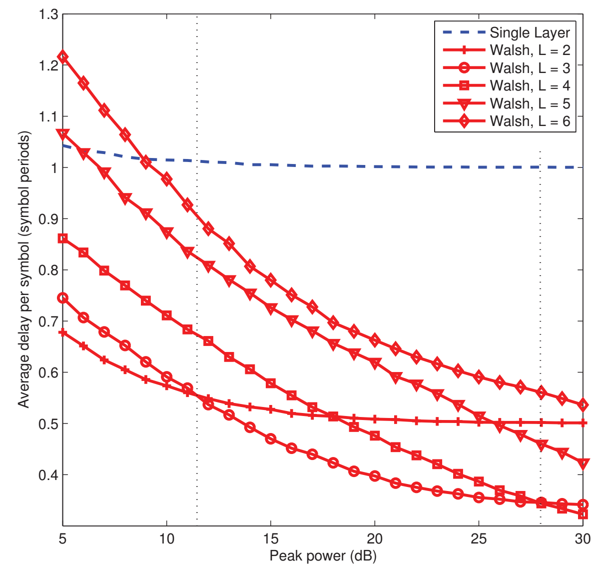

Fig. 2. Practical Walsh layering vs. single-layer: average delay for transmitting L QPSK symbols.

4. CONCLUSION

Taking the uncertainty of channels and peak power constraint into account, we have proposed a new practical layering scheme for implementing the idea of rateless coding for Gaussian channels. It is based on a feature of Walsh layer-time coding: interference from other lower layers can be removed or diminished by summing rows up. Nevertheless, when the times of retransmission exceeds over the size of the Walsh matrix, as it may be due to initial powers of not-yet-decoded packets are too small, we suggest to use the simple full power single-layer ARQ transmission subsequently. For illustrating the advantage of our scheme, we have given simulation results of a QPSK system. It shows that our Walsh layering scheme is a more efficient reliable transmission scheme than single-layer sequential ARQ transmission and we can always find a best number of layers for a certain range of SNR.

5. REFERENCES

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