MULTITERMINAL SOURCE CODING
OF BERNOULLI-GAUSSIAN CORRELATED SOURCES

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ABSTRACT

This paper presents a practical coding scheme for the direct symmetric multiterminal source coding problem with remote source, when the noise between the remote source and the observations is Gaussian-Bernoulli-Gaussian. The idea behind the design is to take advantage from the observed symbols being in the real field, in order to perform low-dimensional compressed sensing. The coding scheme is based on channel coding techniques involving BCH codes over the real field. Simulations with respect to the rate distortion performance are provided, along with insights about optimization issues. Robustness to variations of the probability of impulse is also investigated. Perspectives about the application to the CEO problem in presence of Gaussian-Bernoulli-Gaussian correlation noise between the remote source and the observations conclude the paper.

***Index Terms*—** Distributed source coding, multiterminal coding.

1. INTRODUCTION

Distributed source coding is a compression paradigm for correlated sources that rests on the principle that correlation can be exploited to reduce the cost of transmission, even if the sources are encoded independently.

The well-known Wyner-Ziv coding problem [1] is an example of *asymmetric* distributed source coding setup: given a pair of correlated sources, the goal is to compress the first under a fidelity criterion, assuming the second (the side information) directly available at the joint decoder. In the *symmetric* or *multiterminal* source coding setup all the sources are simultaneously subject to compression under a fidelity criterion. A variant is the CEO problem [2], where correlated sources are generated as noisy observations of some remote process, which the joint decoder aims to estimate.

This work focuses on the multiterminal setup, which is intended here to model the sensing process in a distributed sensor network. Each node in the network senses the same physical process X; its observation Y_i at the i-th sensor is affected by a small-variance noise component, the acquisition noise due to the physical equipment. It is assumed, moreover, that impulses of noise with variance comparable to the variance of X might appear (with probability p) to model occasional equipment malfunctioning or environmental perturbations. The joint decoder aims to estimate the observations of each node in the network.

In order to develop a coding solution for this general case the attention is limited, at first, to the problem of reconstruction of the observations when they are corrupted only by impulsive noise. The resulting correlation model between observations seized at different nodes inherits this impulsive nature. This setup can be regarded as a variant of the third joint sparsity (JSM-3) compressed sensing model considered in [3].

A considerable effort has been spent over the years towards the research of theoretical bounds for multiterminal source coding (see Section 3); yet the development of practical coding solutions is limited. An effective coding solution in presence of Gaussian correlation noise is based on Slepian-Wolf coded quantization [4], which exploits the performance of schemes tailored to the lossless compression problem.

The same approach can be adopted also in presence of impulsive correlation noise. In particular, one could employ the schemes proposed in [5] and [6] for the binary case. The observations need to be quantized, and a bit-plane representation of the quantized symbols is provided. Then the binary lossless encoding scheme can be applied to every bit plane. The scheme in [6] is well suited to solve the problem of the estimation of the correlation between different nodes in every bit plane; the rate-adaptation mechanism provides intrinsic resilience to the variation of the binary correlation parameter, and allows the same implementation on every bit plane. Nevertheless the rate adaptation mechanism imposes the presence of a feedback channel.

The idea behind the proposed coding solution is, instead, to take advantage of the fact that the observation samples belong to the real field, as already done for the case of asymmetric coding in [7], exploiting the concepts of DFT codes [8] and BCH codes on the real field [9]. As in the compressed sensing problem [3], in order to restrain the rate one can profit of the sparseness of the correlation noise sequence between the observations.

The paper is organized as follows. Section 2 formally states the problem; available theoretical results are discussed in Section 3. The practical coding scheme is presented in Section 4, along with simulation results in Section 5. Section 6 provides guidelines for the extension to the Gaussian-Bernoulli-Gaussian correlation model, and conclusions are briefly stated in Section 7.

2. PROBLEM STATEMENT

Consider the setup depicted in Figure 1. A source generates n Gaussian i.i.d. samples $X \sim \mathcal{N}(0, \sigma_x^2)$, which are observed by the m nodes of the sensing network. Let $W_i' = Y_i - X = B_i\, N_i$ be the impulsive noise between each source sample and its observation at the i-th node, with $i \in \mathcal{M} = \{1, 2, \dots, m\}$, where B_i is a Bernoulli random variable with $\Pr(B_i = 1) = p$, $\Pr(B_i = 0) = (1 - p)$, and $N_i \sim \mathcal{N}(0, \sigma_i^2)$. The noise W_i' is assumed independent on the source realization as well as on the other impulsive noise realizations. Assume, for simplicity, $\sigma_i^2 = \sigma^2$, $\forall i \in \mathcal{M}$. The resulting

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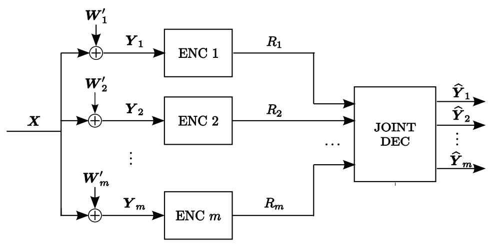

Fig. 1. Problem setup.

correlation noise $W_{i,j} = Y_i - Y_j$ between the i-th and the j-th observations is modeled as, for $i \neq j$,

$$W_{i,j} = \begin{cases} N & \text{with probability} \quad 2p\,(1-p) \\ N' & \text{with probability} \quad p^2 \\ 0 & \text{with probability} \quad (1-p)^2 \end{cases}, \tag{1}$$

with $N \sim \mathcal{N}(0, \sigma^2)$ and $N' \sim \mathcal{N}(0, 2\sigma^2)$. The i-th node encodes the sequence $\boldsymbol{Y}_i$ of length n and sends the coded stream with a rate of R_i bits per observed sample. The total rate spent in the system, normalized with respect to the total number of nodes, is $R = \frac{1}{m}\sum_{i=1}^{m} R_i$. The joint decoder outputs the estimated sequences $\hat{\boldsymbol{Y}}_i$, $\forall i \in \mathcal{M}$. Reconstruction of the observations is required within the same per symbol target distortion $\frac{1}{n}E[\| \hat{\boldsymbol{Y}}_i - \boldsymbol{Y}_i \|^2] \leq D$, $\forall i \in \mathcal{M}$.

3. THEORETICAL BOUNDS

The problem of lossless multiterminal source coding of correlated sources was introduced and theoretically characterized by Slepian and Wolf in [10]. The extension to the lossy case was first introduced by Wyner and Ziv in [1], where theoretical bounds for the case of asymmetric coding of two sources are derived. The theoretical rate region for symmetric lossy coding of multiple sources still remains unknown, though considerable work has allowed to determine inner and outer bounds for special cases (see [11–14] and references therein). For the setup considered in this paper, the theoretical limit performance can be defined in terms of the minimum total rate $R = \sum_{i \in \mathcal{M}} R_i$ per observed sample that allows to provide reconstructions within target distortion D. An asymptotically achievable bound can be obtained characterizing an ideal system based on vector quantization, followed by an ideal Slepian-Wolf coder operating on the limit of the theoretical Slepian-Wolf region. The ideal system works as follows. Each sequence $\boldsymbol{Y}_i$ is quantized using the best known lattice Λ in dimension n. As $n \longrightarrow \infty$ the induced per symbol distortion is $D' = \frac{V_0(\Lambda)^{\frac{2}{n}}}{2\pi e}$, where $V_0(\Lambda)$ is the fundamental volume of the Voronoi region of the lattice. The quantized sequences $\tilde{\boldsymbol{Y}}_i$ are Slepian-Wolf encoded, exploiting their mutual correlation, using rates that satisfy the Slepian-Wolf theorem for multiple sources [15]

$$\sum_{i=l}^{m} R_i \geq H(\tilde{Y}_m, \tilde{Y}_{m-1}, \ldots \tilde{Y}_l | \tilde{Y}_{l-1}, \ldots, \tilde{Y}_1), \tag{2}$$

with $l \in \mathcal{M}$. The asymptotically achievable rate per symbol for a distortion level D' is $R = H(\tilde{Y}_1, \tilde{Y}_2, \ldots, \tilde{Y}_m)$.

4. PRACTICAL SCHEME

The encoder for the i-th node of the sensing network and of the joint decoder are depicted in Figure 2 and 3 respectively.

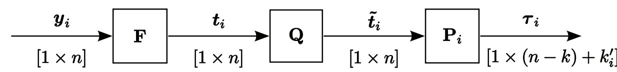

Fig. 2. Encoder, i-th node.

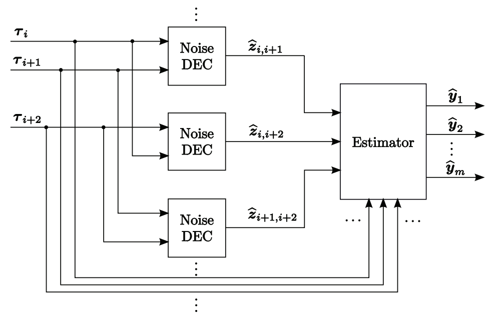

Fig. 3. Part of the decoder.

4.1. Encoding

An observation of n samples by the i-th node is denoted by $\boldsymbol{y}_i$. Let $\mathbf{F}$ be the n-dimensional Discrete Fourier Transform matrix. The first stage of the encoder outputs the sequence of the n transformed coefficients $\boldsymbol{t}_i = \mathbf{F}\boldsymbol{y}_i$. It is followed by a vector quantizer, whose output is $\tilde{\boldsymbol{t}}_i = \boldsymbol{t}_i + \boldsymbol{q}_i$; the quantization noise sequence $\boldsymbol{q}_i$ can be approximately assumed zero-mean white Gaussian i.i.d. for high rate regime [16, 17]. Assume that all the encoders operate with the same quantizer, at the same rate R_q bits per sample. Denote $\boldsymbol{\Gamma}_q = \sigma_q^2\,\mathbf{I}$ the covariance matrix of $\boldsymbol{q}_i$, $\forall i \in \mathcal{M}$.

Let $\mathbf{H}$ be the $(n-k) \times n$ matrix, with $k < n$, obtained choosing the rows of indexes in $\mathcal{R} = \{(k+1)/2+1, \ldots, n-(k-1)/2\}$ of the matrix $\mathbf{F}$, for k and n odd. $\mathbf{H}$ represents the parity check matrix of an (n, k) BCH code on the real field, in the sense of [9]. The sequence $\tilde{\boldsymbol{s}}_i = \mathbf{H}\boldsymbol{y}_i + \boldsymbol{q}_i$ is obtained by the encoder selecting the symbols of indexes in $\mathcal{R}$ of $\tilde{\boldsymbol{t}}_i$. It represents a noisy observation of the syndrome of $\boldsymbol{y}_i$, and is meant to be sent to the joint decoder. Further k'_i symbols from $\tilde{\boldsymbol{t}}_i$ are selected for transmission, under the constraint $\sum_{i=1}^{m} k'_i = k$, and such that the whole set of added coefficients is representative of every location, except those in $\mathcal{R}$. Let $\boldsymbol{\tau}_i = \mathbf{P}_i\tilde{\boldsymbol{t}}_i$ be the overall sequence of coefficients conveyed to the joint decoder by the i-th encoder, $\mathbf{P}_i$ being the matrix of dimension $[(n-k)+k'_i] \times n$ that performs the projection into the i-th transmission subspace. $\mathbf{H}$ is Hermitian symmetric by construction, so does the sequence of complex values $\tilde{\boldsymbol{s}}_i$; preserving the Hermitian symmetry by careful choice of the k'_i locations also allows to restrain the data sent by each node to $(n-k)+k'_i$ quantized symbols of $\boldsymbol{\tau}_i$. The transmission rate R_i per observed symbol is then

$$R_i = \frac{(n-k)+k'_i}{n} R_q \tag{3}$$

bits. The total rate per observed symbol is then

$$R = \left(1 - \frac{(m-1)}{m}\frac{k}{n}\right) R_q. \tag{4}$$

4.2. Joint decoding

The joint decoder attempts to reconstruct the sparse quantized correlation noise sequences $\boldsymbol{w}_{i,j}$ from the noisy observation of their syndromes $\tilde{\boldsymbol{s}}_{i,j}$. This information is used as a complement to the sequences $\boldsymbol{\tau}_i$, directly sent by each node, in order to produce the estimate $\hat{\boldsymbol{y}}_i$ of $\boldsymbol{y}_i$, $\forall i \in \mathcal{M}$. First, the decoder retrieves the syndromes

$\tilde{\boldsymbol{s}}_{i,j}$, $\forall(i,j) \in \mathcal{M} \times \mathcal{M}$, as

$$\tilde{\boldsymbol{s}}_{i,j} = \mathbf{H}\boldsymbol{w}_{i,j} + \mathbf{P}'(\boldsymbol{q}_i - \boldsymbol{q}_j) = \tilde{\boldsymbol{s}}_i - \tilde{\boldsymbol{s}}_j, \quad (5)$$

where $\mathbf{P}'$ is the $(n-k) \times n$ matrix that performs projection into the syndrome space. The estimation of $\boldsymbol{w}_{i,j}$ can be done employing modified versions of syndrome-decoding algorithms which are originally developed for BCH codes on finite fields, such as the modified Berlekamp-Massey algorithm [8] or the modified Peterson-Gorenstein-Zierler algorithm [9]. It is necessary to estimate ν, the number of impulses in the sequence. The estimated location indexes correspond to the ν powers of the n-th root of unity $e^{i\frac{2\pi}{n}}$ for which the norm of the locator polynomial is smaller. The amplitudes of the impulses are estimated in the ML sense. Let $\hat{\boldsymbol{w}}_{i,j}$ be the estimate of the correlation noise sequence between nodes i and j, and $\boldsymbol{e}_{i,j} = \hat{\boldsymbol{w}}_{i,j} - \boldsymbol{w}_{i,j}$ the estimation error. The covariance matrix $\mathbf{\Gamma}_{i,j}$ of the estimation error can be evaluated as in [9].

The decoder exploits all the available information (sequences $\boldsymbol{\tau}_i$, $\forall i \in \mathcal{M}$ directly sent by the encoders, and sequences $\hat{\boldsymbol{w}}_{i,j}$, $\forall(i,j) \in \mathcal{M} \times \mathcal{M}$, obtained through syndrome decoding) to jointly estimate $\boldsymbol{y} = [\boldsymbol{y}_1^T, \ldots, \boldsymbol{y}_m^T]$. The estimation is performed using the following equations (detailed for the case of $m = 3$):

$$\begin{bmatrix} \boldsymbol{\tau}_1 \\ \boldsymbol{\tau}_2 \\ \boldsymbol{\tau}_3 \\ \hat{\boldsymbol{w}}_{1,2} \\ \hat{\boldsymbol{w}}_{1,3} \\ \hat{\boldsymbol{w}}_{2,3} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \\ \boldsymbol{y}_3 \end{bmatrix} + \boldsymbol{v} \quad (6)$$

with

$$\mathbf{M} = \begin{bmatrix} \mathbf{P}_1\mathbf{F} & \mathbf{0}_1 & \mathbf{0}_1 \\ \mathbf{0}_2 & \mathbf{P}_2\mathbf{F} & \mathbf{0}_2 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{P}_3\mathbf{F} \\ \mathbf{I} & -\mathbf{I} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & -\mathbf{I} \\ \mathbf{0} & \mathbf{I} & -\mathbf{I} \end{bmatrix}, \quad \boldsymbol{v} = \begin{bmatrix} \mathbf{P}_1\boldsymbol{q}_1 \\ \mathbf{P}_2\boldsymbol{q}_2 \\ \mathbf{P}_3\boldsymbol{q}_3 \\ \boldsymbol{e}_{1,2} \\ \boldsymbol{e}_{1,3} \\ \boldsymbol{e}_{2,3} \end{bmatrix}, \quad (7)$$

where $\mathbf{I}$ is the n-dimensional identity matrix, $\mathbf{0}$ is the $n \times n$ null matrix, and $\mathbf{0}_i$ is the null matrix of dimensions $[(n-k)+k'_i] \times n$, $\forall i \in \mathcal{M}$. The covariance matrix $\mathbf{\Gamma}_v$ of $\boldsymbol{v}$ is block diagonal, in the form

$$\mathbf{\Gamma}_v = \text{diag}\left(\mathbf{P}_1\mathbf{\Gamma}_q\mathbf{P}_1^T\ ,\ \mathbf{P}_2\mathbf{\Gamma}_q\mathbf{P}_2^T\ ,\ \mathbf{P}_3\mathbf{\Gamma}_q\mathbf{P}_3^T\ ,\ \mathbf{\Gamma}_{1,2}\ ,\ \mathbf{\Gamma}_{1,3}\ ,\ \mathbf{\Gamma}_{2,3}\right).$$

In general, the total number of equations is $\left(m + \binom{m}{2}\right) n - (m-1)k$, to estimate $m \cdot n$ parameters. The estimation of $\boldsymbol{y}_i$, $\forall i \in \mathcal{M}$ is again performed in the ML sense. The covariance matrix $\mathbf{\Gamma}_y$ of the estimation noise $[\hat{\boldsymbol{y}}_1^T, \ldots, \hat{\boldsymbol{y}}_m^T] - \boldsymbol{y}$ is in the form

$$\mathbf{\Gamma}_y = \left(\mathbf{M}^H\mathbf{\Gamma}_v^{-1}\boldsymbol{M}\right)^{-1}. \quad (8)$$

4.3. Optimization

In order to optimize the proposed coding scheme for rate-distortion performance as described in Section 3, one needs to choose the (n,k) pair. Low values of the ratio k/n raise the error-correction capability of the BCH code, and thus allow to restrain the distortion on the reconstructions. Hence this also increases the total rate (4). The optimization problem cannot be solved analytically, due to the difficulty of the appraisal of the benefit on the overall estimation of an increased correction capability of the BCH code.

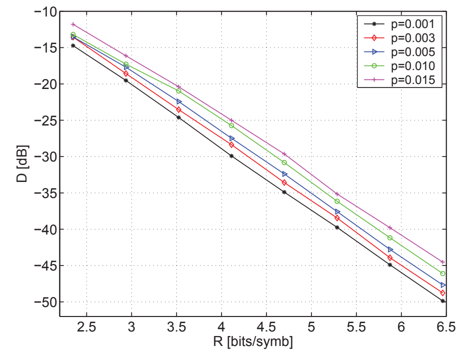

Fig. 4. Rate distortion performance for code $(21,13)$.

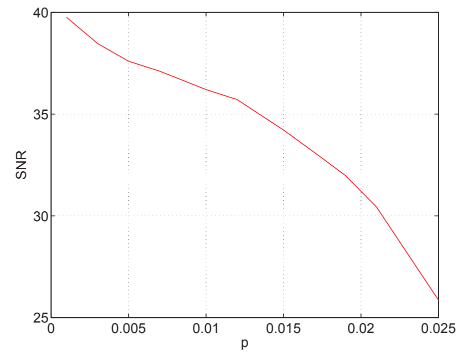

Fig. 5. SNR for varying impulse probability p, code $(21,13)$, rate $R = 5.28$ bits per sample.

5. SIMULATION RESULTS

The rate distortion performance (as defined in Section 3) is evaluated in the case of $m = 3$ sources, $\sigma_x^2 = 1$ and $\sigma^2 = 1$, for a $(21,13)$ DFT code. The number of simulated sequences for rate point is chosen, for every value of p, in order to ensure more than 150 impulsive events.

On the encoder side, trellis coded quantization (TCQ) [18] with a $1/2$ convolutional code feedback polynomial $(671, 1631)_o$ is performed. A Max-Lloyd partitioning of $2R_q$ codewords is used for TCQ. The performance of the quantization of a unit-variance zero-mean Gaussian source for 100 samples at rate $R_q = 4$ bits per sample, is 1.07 dB away from the theoretical limit.

The choice of k'_i, $i \in \{1,2,3\}$ is done in order to preserve symmetry on the rates, yielding $R_1 = 0.62\ R_q$, and $R_2 = R_3 = 0.57\ R_q$ bits per observed sample. At the decoder side detection of the impulses is performed by applying the modified Gorestein-Peterson-Zierler algorithm in [9] and [7].

Figure 4 depicts the rate distortion performance for values of p between $p = 0.001$ and $p = 0.015$. The probability p_f of observing an impulse between two sequences $\boldsymbol{y}_i$ and $\boldsymbol{y}_j$ is $p_f = 2p - p^2$, from equation (1). Denote p_{out} as the probability of occurrence of more than $(n-k)/2$ impulses between $\boldsymbol{y}_i$ and $\boldsymbol{y}_j$

$$p_{out} = \Pr\left(\nu > \frac{(n-k)}{2}\right) = \sum_{\nu=\frac{(n-k)}{2}+1}^{n} p_f^{\nu}(1-p_f)^{(n-\nu)}. \quad (9)$$

As p increases, the performance of the decoder degrades. Nevertheless the correction capability of the BCH code is not definitely compromised by a (small) increment of p_f. Furthermore, the redundancy of the number of equations in the ML estimation system (6) helps in mitigating the effects of the increased p_{out} in terms of misdetection of impulses.

The probability p of observing an impulse is now let vary between 0.001 and 0.025, and the distortion is evaluated at a fixed rate $R = 5.28$ bits per sample, corresponding to $R_q = 9$ bits. Figure 5 depicts the SNR as a function of p: graceful degradation of the performances is observed, with respect to small increase of the probability of impulse p. For $p = 0.020$, corresponding to $p_{out} = 1.1631 \cdot 10^{-3}$, the decoding process produces SNR which is about 8 dB worse with respect to $p = 0.001$. The SNR performance decreases more rapidly for $p > 0.020$, which can be then considered as the threshold probability of impulse above which the system with the considered DFT code is not able to operate.

6. ON THE GAUSSIAN-BERNOULLI-GAUSSIAN CORRELATION MODEL

Let the correlation noise between the remote source and the m observations be composed of a background noise component (Gaussian noise with 0 mean, and variance $\sigma_b^2 << \sigma_x^2$), and by an impulsive component; the resulting correlation model between the observations inherits the same structure.

The proposed coding scheme can be adapted to deal with indirect multiterminal source coding for the case of Gaussian-Bernoulli-Gaussian correlation as follows. Let the transformed observation sequences $\boldsymbol{t}_i$ be coarsely quantized at the encoder side, introducing a quantization noise level within σ_b^2. The encoding system detailed above can still be put at work, in order to provide coarse estimates $\hat{\boldsymbol{y}}_i$ of the observed sequences. The estimation noise on the sequences is bounded, provided that p_{out} remains sufficiently small, since symbols of background noise with relevant amplitude are detected as impulses by the decoding scheme. Every observation node sends some extra payload to the joint decoder, with rate R_i^{WZ} bits per symbol, along with sequence $\boldsymbol{\tau}_i$. This additional information is used for computing the refined estimates $\hat{\boldsymbol{y}}_i'$ of the observation sequences using a traditional Wyner-Ziv coding scheme, where $\hat{\boldsymbol{y}}_i$ acts as side information. Since the estimation noise on $\hat{\boldsymbol{y}}_i$ is bounded, the Wyner-Ziv coding scheme can be achieved by means of nested quantization. The decoder can exploit the knowledge of the detected locations of the impulses between the observations, along with the estimates $\hat{\boldsymbol{y}}_i'$, in order to provide the estimate $\hat{\boldsymbol{x}}$ of the remote source.

7. CONCLUSIONS

This work considers a practical multiterminal coding scheme for a remote source, with Bernoulli-Gaussian correlation model between the remote source and the observations. A practical coding scheme based on DFT codes has been proposed. Part of the allocated rate (the syndromes) is used first to estimate the sparse noise sequences between the observations. Then the reconstruction of each observation is evaluated by means of a joint ML estimator.

Simulation results show that the rate-distortion performance depends on the capability of the decoder of estimating the correct impulse locations. When the probability of impulse increases, however, the decoding performance degrades gracefully, allowing some robustness against time-varying characteristics of the correlation model. An extension of the scheme to the Gaussian-Bernoulli-Gaussian correlation model has been sketched. Future work must focus on rate distortion optimization, and on theoretically achievable performance of the scheme.

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