JOINT OPTIMIZATION OF THE REDUNDANCY OF MULTIPLE-DESCRIPTION CODERS FOR MULTICAST

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ABSTRACT

We consider the optimization of multicast over packet-switched communication networks with a non-zero packet-loss probability. For the system setup consisting of a number of multiple-description coders, we jointly optimize these coders. We propose an analytic solution, asymptotically optimal in the number of multiple-description coders. The analytic solution allows for fast system adaptation to changing network conditions. A locally optimal optimization algorithm that is useful when the number of multicast groups is small is derived. Simulations show that the utilization of the analytic solution incurs a low overhead on the performance when compared to the locally optimal solution, even for a small number of multiple-description coders.

***Index Terms*—** Multicast, multiple-description coding, optimization, packet-loss

1. INTRODUCTION

Communication of a single source to a large number of receivers over a packet-switched network poses an interesting set of problems. Considerations such as network utilization, receiver capacity, receiver fairness, receiver perceived distortion, and robustness to packet-loss define a range of system requirements that in most cases are contradictory. In our work, we consider communication systems that use multicast to minimize unnecessary network link load. Multicast holds most promise for communication of sources that are coded at a high bit-rate, e.g. video, making any redundancy on network links costly in terms of possible performance losses. Despite its positive characteristics, utilization of multicast incurs a penalty in terms of a limitation on the possibility to adapt to individual receiver capabilities [1]. This complicates the task of fulfilling requirements on receiver perceived distortion. The topic of this paper is the minimization of the receiver perceived distortion for multicast over packet-switched networks with a non-zero packet-loss probability.

A wide range of multicast systems have been proposed in the literature. Possible classification criteria for these are ability to adapt to changing network conditions such as receiver capacity, packet-loss, delay and jitter. In our recent work on multicast optimization [2], we addressed the problem of fast adaptation to changing receiver capacities. We provided analytical optimization results for stochastically modeled receiver capacities. In this paper, we address the problem of packet-loss robustness.

Work on optimization of multicast systems in which losses are considered include [3, 4]. In these papers, the common multicast systems replication and layered multicast, where respectively replicas and layers are sent on different multicast trees, is considered. Algorithms for rate optimization given a set of receiver capacities are presented. The rate optimization is constrained by loss tolerance, defined as the largest loss rate (the ratio between the rate of the lost data and the sent data) a receiver can tolerate. A drawback is the fact that the complexity of the algorithms cannot be neglected, making system adaptivity to changing network conditions slow. Further, as the loss rate alone is not enough to characterize perceived video quality [5, 6], these methods are not applicable to the most relevant case of multicast.

Utilization of forward error correction (FEC) as a means of robustness in multicast was proposed in [7, 8]. The main idea is to extend layered multicast by sending redundant FEC data on separate multicast trees. Receivers that experience packet-loss can then subscribe to multicast trees with redundant data. The setup has the obvious advantage of minimal introduction of redundancy on network links. However, the optimization of the setup remains a problem. Further, it has been argued in [9] that this setup can be costly in terms of overhead needed to support a large number of multicast trees.

Multiple-description coders have been used in the context of multicast in different system setups. Taal et al. [10, 11] proposed using one multicast tree for each description of a multiple-description coder. Besides the advantage that the receiver is able to subscribe to any combination of descriptions without loss of quality, it is obvious that the system is inherently robust to packet-loss. However, the system optimization in [10, 11] was performed only with respect to receiver capacities, assuming no packet-loss.

An extension of the idea behind replication to the case of packet-loss is the multicast setup where several multiple-description coders are used. The descriptions of each multiple-description coder are sent on one multicast tree and each receiver subscribes to the multicast tree that best suits the conditions of the receivers channel. For most existing multiple-description coders, e.g., [12], the optimization of one multiple-description coder needs to consider two parameters, the redundancy and the number of descriptions that are used by the coder. A third parameter, the rate, comes into play when optimizing several multiple-description coders. Hence, the general problem is that of jointly optimizing the mentioned three parameters of the multiple-description coders.

Chou and Ramchandran solved an interesting special case of the general problem in [9]. They optimized the redundancy of several symmetric multiple-description coders, each with an equal rate and an equal number of descriptions. The performance of the setup was optimized for a specific multiple-description coder [13], using an iterative locally optimal algorithm. In our work, we develop analytic results for the special case considered by Chou and Ramchandran. The analytic results are needed when fast adaptation to changing network conditions is required. Our results are not limited to any specific multiple-description coder.

The outline of the paper is as follows. In Section 2, we introduce notation and define the optimization problem. Section 3 is devoted to deriving an analytic solution that is asymptotically optimal in the number of multiple-description coders. We provide a locally optimal algorithm in Section 4. The presented theory is supported by simulation results in Section 5, where we show that the asymptotic solution provides good results even when the number of multiple description coders is small. Section 6 concludes the paper.

2. PROBLEM FORMULATION

Consider the communication of a source X from a sender to a receiver over a packet-switched network. The source is encoded using a multiple-description coder (MDC) with K descriptions. Each description is transmitted over the network in one packet. We assume that the MDC is symmetric, such that the rate of each description is R/K and the distortion perceived at the receiver only depends on the number of packets received. Let us denote the distortion obtained with $k \in \{0, \ldots, K\}$ descriptions by d_k.

Assuming that the packet transmission is performed over a stationary memoryless channel, the packet-loss probability of each packet is equal, say p, and independent of the possible loss of any other packet. For this type of channel, the expected distortion at the receiver side is given by $\sum_{k=0}^{K} \binom{K}{k} p^{K-k}(1-p)^k d_k$. The expected distortion depends on the redundancy between the descriptions. It is possible to optimize the redundancy of the MDC for a specific channel, i.e., design d_k such that the expected distortion is minimized. We note that the optimization is constrained by the total available rate R. Let us denote the distortions of an MDC optimized for packet-loss probability q, see for example [12], by $d_k(q)$. The expected distortion of this MDC applied to a channel with packet-loss probability p is written as

$$d(q,p) = \sum_{k=0}^{K} \binom{K}{k} p^{K-k}(1-p)^k d_k(q). \tag{1}$$

Consider now the communication of the source X from a sender to a number of receivers over a packet-switched network by means of multicast. The channels from the sender to the receivers are assumed to be stationary and memoryless, with packet-loss probabilities that are the i.i.d. realizations of an underlying stochastic variable P. P is characterized by the receiver packet-loss probability density function $f_P(p)$.

The source is encoded by I MDCs, indexed by the elements of the set $\mathbb{I} = \{1, \ldots, I\}$. The redundancy of each MDC is optimized for transmission of the descriptions over a stationary memoryless channel, with a packet-loss probability that is given by the mapping $\pi : \mathbb{I} \to \mathcal{I}$, where we defined the unit interval $\mathcal{I} = [0, 1]$.

Depending on the packet-loss probability of the channel of a specific receiver, that receiver is assigned to receive packets originating from one of the I available MDCs. This assignment is handled by the mapping $a : \mathcal{I} \to \mathbb{I}$. Hence, a receiver with a packet-loss probability p perceives the distortion $d(\pi(a(p)), p)$.

The problem that we are considering in this paper is that of joint optimization of the I multiple-description coders used for multicast. The objective is minimization of the expected weighted receiver distortion by optimization of the mappings a and π

$$J^* = \min_{a,\pi} E[w(P)d(\pi(a(P)), P)], \tag{2}$$

where the weighting function is denoted $w(p)$. The weighting function is a means for defining an optimization criterion that is fair and is further addressed in Section 5.

Reinterpretation of the variables involved in the problem formulation brings us to the conclusion that the problem considered is equivalent to a resolution-constrained scalar quantization problem. In particular, P can be interpreted as the source, a as the encoder mapping, π as the decoder mapping and $w(\cdot)d(\cdot,\cdot)$ as the weighted distortion measure. We use this interpretation of the problem in the following two sections and apply tools from quantization theory to obtain asymptotically and locally optimal solutions to the problem.

For notational purposes, let us introduce the ith cell as $\mathcal{I}_i = \{p : a(p) = i\}$ and the corresponding centroid $\pi(i)$. In this paper, we will make the common assumption that the cells are convex and that the centroids are contained within the cells, i.e., $\pi(i) \in \mathcal{I}_i$.

3. ASYMPTOTIC OPTIMALITY

In this section, we provide a solution that is asymptotically optimal in the number of multiple-description coders. The solution is based on the high-rate assumption, first introduced for quantization in [14].

Let us rewrite the distortion in (1) as the sum of the minimum distortion achievable over a channel with packet-loss probability p, called the nominal distortion, and the excess distortion incurred by usage of a multiple-description coder that is optimized for the incorrect packet-loss probability q

$$d(q,p) = d(p,p) + \delta(q,p). \tag{3}$$

Using (3), the optimization objective (2) can be expressed as

$$J^* = E[w(P)d(P,P)] + J^*_\delta, \tag{4}$$

where we defined the expected weighted excess distortion

$$J^*_\delta = \min_{a,\pi} \sum_{i=1}^{I} \int_{\mathcal{I}_i} w(p)\delta(\pi(i),p) f_P(p)\,\mathrm{d}p. \tag{5}$$

Asymptotically in the number of MDCs, the cell widths

$$\Delta_i = p_i^{\max} - p_i^{\min}, \quad i \in \mathbb{I} \tag{6}$$

where $p_i^{\max} = \max_{p\in\mathcal{I}_i} p$ and $p_i^{\min} = \min_{p\in\mathcal{I}_i} p$, will go to zero. It is then reasonable to assume that the probability density function $f_P(p)$ and the weighting function $w(p)$ are approximately constant within each cell

$$f_P(p) \approx f_P(\pi(i)), \quad p \in \mathcal{I}_i \tag{7}$$

$$w(p) \approx w(\pi(i)), \quad p \in \mathcal{I}_i \tag{8}$$

and that the excess distortion is well approximated by its second order Taylor expansion. Due to the definition of the excess distortion, only the second order term will be non-zero, yielding

$$\delta(\pi(i),p) \approx \frac{1}{2}(p-\pi(i))^2 \ddot{\delta}(\pi(i),\pi(i)), \tag{9}$$

where we introduced $\ddot{\delta}(\pi(i),\pi(i)) = \frac{\partial^2}{\partial p^2}\delta(\pi(i),p)\big|_{p=\pi(i)}$.

The introduced approximations (7), (8) and (9) allow us to rewrite the optimization objective function (5) to

$$J^*_\delta \approx \min_{\pi} \sum_{i=1}^{I} \frac{1}{2} w(\pi(i))\ddot{\delta}(\pi(i),\pi(i)) f_P(\pi(i)) \int_{\mathcal{I}_i} (p-\pi(i))^2\,\mathrm{d}p$$

$$= \min_{\pi} \sum_{i=1}^{I} \frac{1}{24} w(\pi(i))\ddot{\delta}(\pi(i),\pi(i)) f_P(\pi(i)) \Delta_i^3, \tag{10}$$

where we recognized that the asymptotically optimal decoder mapping is given by $\pi(i) = (p_i^{\max} - p_i^{\min})/2$.

In the limit of small cell widths, the sum in (10) becomes a Riemann sum, allowing for the approximation

$$J_\delta^* \approx \min_\pi \int_\mathcal{I} \frac{1}{24} w(p)\ddot{\delta}(p,p) f_P(p)\Delta^2(p)\, \mathrm{d}p, \qquad (11)$$

where the cell widths are assumed to lie on a continuous function of p, such that $\Delta_i = \Delta(\pi(i)) \approx \Delta(p)$ for all $p \in \mathcal{I}_i$, and it was used that $\ddot{\delta}(\pi(i), \pi(i)) \approx \ddot{\delta}(p,p)$ for all $p \in \mathcal{I}_i$.

The centroid density in a unit interval is given by $g_\Pi(p) = 1/\Delta(p)$, yielding the final optimization objective

$$J_\delta^* = \min_{g_\Pi(p)} \int_\mathcal{I} \frac{1}{24} w(p)\ddot{\delta}(p,p) f_P(p) g_\Pi^{-2}(p)\, \mathrm{d}p. \qquad (12)$$

The centroid density is constrained by the total number of cells, according to

$$\int_\mathcal{I} g_\Pi(p)\, \mathrm{d}p = I. \qquad (13)$$

Using the method of Lagrange multipliers, the constrained optimization problem defined in (12) and (13) can be solved. Forming the Lagrangian and solving the Euler-Lagrange equation yields the optimal centroid density

$$g_\Pi^*(p) = I \frac{(w(p)\ddot{\delta}(p,p) f_P(p))^{\frac{1}{3}}}{\int_\mathcal{I} (w(p)\ddot{\delta}(p,p) f_P(p))^{\frac{1}{3}}\, \mathrm{d}p}. \qquad (14)$$

The expected weighted excess distortion as a function of the number of multiple-description coders can be obtained by inserting (14) into (12)

$$J_\delta^* = \frac{1}{24I^2} \left[\int_\mathcal{I} (w(p)\ddot{\delta}(p,p) f_P(p))^{\frac{1}{3}}\, \mathrm{d}p \right]^3. \qquad (15)$$

The total expected weighted distortion is obtained from (15) and (4).

To obtain the actual centroids from the asymptotically optimal centroid density we use the companding approach [14]. We note that companding incurs no loss of optimality for the scalar case [15], as long as the compressor is chosen optimally. The derivation of the optimal compressor $h^*(p)$ is straightforward and is left out for reasons of brevity. The final result is given by

$$h^*(p) = I^{-1} \int_{-\infty}^{p} g_\Pi^*(p)\, \mathrm{d}p. \qquad (16)$$

4. LOCAL OPTIMALITY

The optimization problem considered is in general non-convex. In this section, we outline a locally optimal algorithm based on the Lloyd algorithm. The algorithm alternates between optimizing the mappings a and π, respectively. We note that a similar algorithm was proposed for joint optimization of the subset of multiple-description coders that are based on priority encoded transmission (PET) in [9]. In our formulation, we keep the notation general and applicable to any symmetric multiple-description coder.

We begin with keeping the decoder mapping π constant. The optimal encoder mapping a is then given by

$$a^* = \operatorname*{argmin}_a E[w(P)d(\pi(a(P)), P)] \qquad (17)$$

$$= \operatorname*{argmin}_a \int_\mathcal{I} w(p) d(\pi(a(p)), p) f_P(p)\, \mathrm{d}p. \qquad (18)$$

The integral in (18) is minimized by optimizing the mapping a such that the distortion $d(\pi(a(p)), p)$ is minimized for all $p \in \mathcal{I}$. We write the locally optimal encoder mapping as

$$\begin{aligned} a^*(p) = \{i :\ & d(\pi(i), p) \leq d(\pi(j), p), \quad \forall j > i \\ & d(\pi(i), p) < d(\pi(j), p), \quad \forall j < i\}, \end{aligned} \qquad (19)$$

where the two conditions make sure that the intersection of any two cells equals the empty set.

Next, we keep the encoder mapping a constant. The optimal decoder mapping π is then given by

$$\pi^* = \operatorname*{argmin}_\pi E[w(P)d(\pi(a(P)), P)] \qquad (20)$$

$$= \operatorname*{argmin}_\pi \sum_{i=1}^{I} \int_{\mathcal{I}_i} w(p) d(\pi(i), p) f_P(p)\, \mathrm{d}p. \qquad (21)$$

Since each term in the sum is independent of the other terms, the optimal centroid of each cell is given by

$$\pi^*(i) = \operatorname*{argmin}_q \int_{\mathcal{I}_i} w(p) d(q, p) f_P(p)\, \mathrm{d}p. \qquad (22)$$

The algorithm proposed is initiated with a decoder mapping π and then applies equations (19) and (22) iteratively. The algorithm terminates when the mappings π and a have converged or when the number of iterations has reached a predefined threshold.

5. SIMULATION RESULTS

In this section, we investigate the performance of our solutions. We show the improvements obtained when using more than one redundancy-optimized multiple-description coder for the asymptotically optimal solution, as well as the locally optimal algorithm. The optimization algorithm was initiated using the asymptotically optimal solution. Further, we quantify the overhead associated with using the asymptotically optimal solution when the number of multiple-description coders is small.

We considered a source X that produces independent identically distributed Gaussian variables with unit variance. The source symbols were assumed to be coded in vectors of dimension two, by I MDCs, whose performance is characterized by the analytical rate-distortion function—MSE distortion—derived by Ostergaard et al. in [12]. Hence, our simulations are valid for MDCs that are asymptotically optimal in the coding rate. We used $K = 3$ descriptions, each coded at $4/K$ bits/dimension.

Minimization of the expected distortion might be a criterion that is unfair, particularly to receivers on channels with low packet-loss probability. Taal et al. discussed in [10, 11] possible ways of designing a fair optimization criterion. In our work, we incorporate fairness by use of the weighting function. We chose the weighting function $w(p) = 1/d(p,p)$, such that we minimized the expected distortion normalized by the nominal distortion.

Following the example of Chou and Ramchandran, we used receiver packet-loss probability density functions in the form $\mathrm{Beta}(1, B)$, with different values of B. These distributions give high probability to low packet-loss rates in general. The expected receiver packet-loss probability is given by $E[P] = 1/(1 + B)$.

The expected weighted distortion J^* is in Figure 1 plotted for different I and different solutions. The expected receiver packet-loss probability was 10%. The expected weighted distortion is about 1.21 for one MDC and decreases with an increasing number of MDCs. We note that the minimum value achievable equals 1.

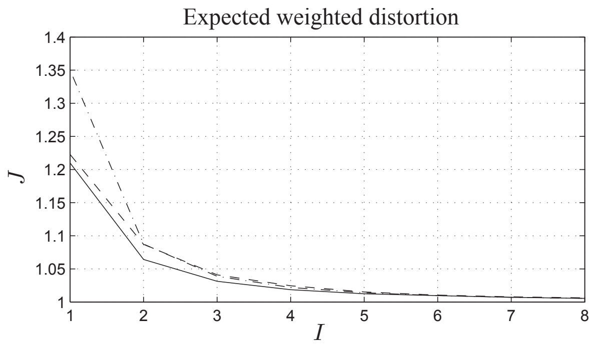

Fig. 1. The expected weighted distortion as a function of the number of MDCs, with $f_P(p) \sim \mathrm{Beta}(1, 9)$. The solid and the dashed curves represent the results for simulations using the locally optimal and the asymptotically optimal solutions, respectively. The dash-dotted curve represents the analytically predicted results, using (4) and (15).

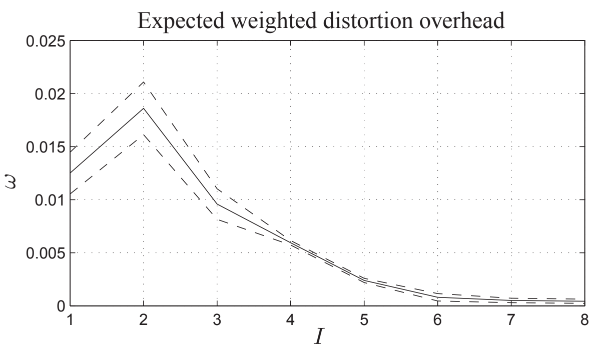

Fig. 2. The expected weighted distortion overhead as a function of the number of MDCs, using $f_P(p) = \mathrm{Beta}(1, B)$ and $B \sim \mathcal{U}[4, 9]$. The solid curve represents the average overhead of 100 realizations of B, while the dashed curves represent the interval of one standard deviation from the average.

Figure 2 illustrates the overhead associated with using the asymptotically optimal results for small values of I, averaged over 100 different distributions $f_P(p)$. The overhead is measured as the difference in the expected weighted distortion J^* for the asymptotically and locally optimal solutions, normalized by the expected weighted distortion of the locally optimal solution $\omega = (J_A^* - J_L^*)/J_L^*$. Here, the subscripts $_A$ and $_L$ denote the usage of the asymptotically and locally optimal solutions, respectively. The results show that the overhead is at most a few percent and that the overhead decreases with an increasing number of MDCs.

6. CONCLUSIONS

We have developed a theory for joint optimization of the redundancy of multiple-description coders for multicast. The main results are the asymptotically and the locally optimal solutions to the problem. Our simulations illustrate the low overhead of the asymptotically optimal solution, even for a small number of multiple-description coders. This makes the solution interesting for implementation in practical systems, where the complexity of locally optimal algorithms makes it difficult to adapt to changing network conditions. The optimization of the general system of jointly optimized multiple-description coders (with respect to the rates, the number of descriptions and the redundancy) remains an interesting problem. Our results provide a lower bound on the achievable performance of the general system.

7. REFERENCES

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