MULTIUSER MIMO-MAC CAPACITY IN THE LOW SNR REGIME: CHANNEL KNOWLEDGE AND DOUBLE-SCATTERING

Xiao Li[†]*, Shi Jin*[†,⋆]*, Matthew R. McKay*[‡]*, Xiqi Gao*[†]*, and Kai-Kit Wong*[⋆]

[†]National Mobile Communications Research Laboratory, Southeast University, Nanjing, China
[⋆]Adastral Park Campus, University College London, Martlesham Heath, UK
[‡]Department of Electronic and Computer Engineering,
Hong Kong University of Science and Technology, Hong Kong
E-mail: {li_xiao,jinshi,xqgao}@seu.edu.cn, eemckay@ust.hk, k.wong@adastral.ucl.ac.uk

ABSTRACT

We investigate the sum capacity of the uplink multiuser multiple-input multiple-output (MIMO) multiple-access channel (MAC) in the low signal-to-noise ratio (SNR) regime. For each user, the MIMO channel is modeled according to a general class of stochastic channel matrices, known as double-scattering. Assuming that each user knows only their own spatial correlation matrices and employs optimal statistical beamforming transmission, we present new analytical approximations for the sum capacity of the MIMO-MAC for low SNR values. Our approximations are accurate, and lead to key insights into the effect of correlation.

Index Terms— MIMO systems, Double-scattering, Multiple access channel.

1. INTRODUCTION

Using multiple-antenna arrays simultaneously at the transmitter and the receiver is widely recognized as an effective means of improving the performance and spectral efficiency of wireless communication systems [1–3]. Nevertheless, in practice, the performance of multiple-input multiple-output (MIMO) systems is degraded due to various physical channel phenomena such as rank deficiency and spatial correlation.

To embrace both the spatial correlation and rank-deficient aspects of the MIMO channel, a general class of stochastic channel matrices, known as *double-scattering*, has been proposed in [4]. For single-user systems, the capacity of this double-scattering model has been investigated in [5], assuming statistical channel state information information (CSI) at the transmitter. For multi-user systems, the capacity of the MIMO multiple-access channel (MAC) with double-scattering has been very recently considered in [6]. Specifically, a closed-form upper bound for the sum capacity was derived, under the assumption of statistical CSI at the transmitters, and used to propose various closed-form suboptimal power allocation policies. The optimal transmit directions and beamforming optimality conditions were also presented. The study in [6] focused on the capacity in terms of the per-symbol signal-to-noise ratio (SNR).

In [7], it was shown that in the low SNR (or "wideband" regime), it is often more appropriate to investigate the capacity in terms of the normalized energy per information bit, E_b/N_0, rather than the per-symbol SNR. Moreover, it was shown that the two key performance measures in the low-SNR regime are the minimum E_b/N_0 for reliable communications, and the wideband slope. In [8, 9], these parameters were studied in detail for various single-user single-scattering MIMO channel models.

In this paper, we investigate the low SNR capacity of the multi-user MIMO-MAC with double-scattering. In particular, we consider the uplink scenario where the receiver has access to perfect CSI, and each transmitter has only their own statistical CSI. As such, the optimal approach is for each user to employ statistical beamforming [6]. Our main technical contributions are new closed-form expressions which we derive for the minimum E_b/N_0 required for reliable communications, and the wideband slope. These key parameters lead directly to a simple and accurate closed-form approximation for the sum capacity of the double-scattering MIMO-MAC in the low SNR regime, which we employ to gain valuable insights into the effect of transmit, receive, and scatter correlation.

2. PRELIMINARIES

Consider a MIMO MAC model with K users. User k has m_k transmit antennas, and the receiver has n antennas. The $n \times 1$ receive signal vector is given by

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{x}_k + \mathbf{n}, \tag{1}$$

This work was supported by National Natural Science Foundation of China under Grants 60572072 and 60496311, the China High-Tech 863 Plan under Grant 2006AA01Z264, National Basic Research Program of China under Grant 2007CB310603, and the Doctor Subject Foundation of China under Grant 20060286016.

where $\mathbf{n}$ is an $n \times 1$ zero-mean complex additive white Gaussian noise (AWGN) vector containing statistically independent elements with power N_0, and $\mathbf{x}_k$ is the $m_k \times 1$ transmit signal vector of user k. Let $\mathbf{Q}_k = E\{\mathbf{x}_k \mathbf{x}_k^\dagger\}$ be the transmit covariance matrix of user k, satisfying the power constraint $\mathrm{tr}(\mathbf{Q}_k) = P_k$. The superscript $(\cdot)^\dagger$ indicates the matrix conjugate-transpose operation, $E\{\cdot\}$ represents expectation, and $\mathrm{tr}(\cdot)$ denotes the matrix trace operation. The matrix $\mathbf{H}_k$, of dimension $n \times m_k$, represents the channel between user k and the receiver.

2.1. Channel Model

For the double-scattering channel model, the MIMO channel matrix $\mathbf{H}_k$ can be factored according to [4]

$$\mathbf{H}_k = \frac{1}{\sqrt{s_k}} \mathbf{\Phi}_R^{1/2} \mathbf{H}_1^{(k)} \mathbf{\Phi}_S^{(k)\,1/2} \mathbf{H}_2^{(k)} \mathbf{\Phi}_T^{(k)\,1/2} \tag{2}$$

where $\mathbf{H}_1^{(k)} \in \mathbb{C}^{n \times s_k}$ and $\mathbf{H}_2^{(k)} \in \mathbb{C}^{s_k \times m_k}$, $k = 1, 2, \cdots, K$ are statistically independent matrices containing i.i.d. unit variance complex Gaussian entries, with s_k denoting the number of effective scatterers of user k on each of the transmit and receive sides. Also, $\mathbf{\Phi}_T^{(k)} \in \mathbb{C}^{m_k \times m_k}$, $\mathbf{\Phi}_S^{(k)} \in \mathbb{C}^{s_k \times s_k}$ and $\mathbf{\Phi}_R \in \mathbb{C}^{n \times n}$ are Hermitian non-negative definite transmit, scatter, and receive correlation matrices respectively, each with unit diagonal entries.

2.2. Sum Capacity

Throughout this paper we make the common assumptions that the receiver has access to perfect CSI for all users, while each transmitter knows their own transmit, scatter, and receive correlation matrices. Under these assumptions, the sum capacity is given as [10]

$$C_{sum} = \max_{\substack{\mathrm{tr}(\mathbf{Q}_k) = P_k \\ k=1,\cdots,K}} E\left\{\log_2 \left| \mathbf{I}_n + \frac{1}{N_0} \sum_{k=1}^{K} \mathbf{H}_k \mathbf{Q}_k \mathbf{H}_k^\dagger \right| \right\}, \tag{3}$$

where $|\mathbf{X}|$ denotes the determinant of matrix $\mathbf{X}$. It is convenient to introduce the following eigenvalue decompositions: $\mathbf{Q}_k = \mathbf{U}_Q^{(k)} \mathbf{\Lambda}_Q^{(k)} \mathbf{U}_Q^{(k)\dagger}$, $\mathbf{\Phi}_T^{(k)} = \mathbf{U}_T^{(k)} \mathbf{\Lambda}_T^{(k)} \mathbf{U}_T^{(k)\dagger}$, $\mathbf{\Phi}_S^{(k)} = \mathbf{U}_S^{(k)} \mathbf{\Lambda}_S^{(k)} \mathbf{U}_S^{(k)\dagger}$, $\mathbf{\Phi}_R = \mathbf{U}_R \mathbf{\Lambda}_R \mathbf{U}_R^\dagger$, where $\mathbf{U}_Q^{(k)}$, $\mathbf{U}_T^{(k)}$, $\mathbf{U}_S^{(k)}$ and $\mathbf{U}_R$ are unitary eigenvector matrices, and $\mathbf{\Lambda}_Q^{(k)} = \mathrm{diag}\{\lambda_{Q1}^{(k)}, \cdots, \lambda_{Qm_k}^{(k)}\}$, $\mathbf{\Lambda}_T^{(k)} = \mathrm{diag}\{\lambda_{T1}^{(k)}, \cdots, \lambda_{Tm_k}^{(k)}\}$, $\mathbf{\Lambda}_S^{(k)} = \mathrm{diag}\{\lambda_{S1}^{(k)}, \cdots, \lambda_{Ss_k}^{(k)}\}$ and $\mathbf{\Lambda}_R = \mathrm{diag}\{\lambda_{R1}, \cdots, \lambda_{Rn}\}$ are diagonal matrices, with diagonal elements pertaining to the descending ordered eigenvalues.

2.3. Low SNR Capacity

For low SNR, it is often appropriate to consider the capacity in terms of the normalized transmit energy per information bit, E_b/N_0, rather than per-symbol SNR. In this respect, the capacity (3) can be well-approximated by the following expression [7]

$$\mathsf{C}\left(\frac{E_b}{N_0}\right) \approx S_0 \log_2 \left(\frac{\frac{E_b}{N_0}}{\frac{E_b}{N_0}_{\min}} \right) \tag{4}$$

where $E_b/N_{0\min}$ is the minimum E_b/N_0 required to convey any positive rate reliably, and S_0 is the wideband slope. $\frac{E_b}{N_0}_{\min}$ and S_0 can be calculated from $C(\mathrm{SNR})$ via [7]

$$\frac{E_b}{N_0}_{\min} = \frac{\ln 2}{\dot{C}(0)}, \quad S_0 = -\frac{2\left[\dot{C}(0)\right]^2}{\ddot{C}(0)}, \tag{5}$$

where $\dot{C}$ and $\ddot{C}$ denote the first and second order derivative, respectively, of the function $C(\mathrm{SNR})$, computed in nats.

3. SUM CAPACITY IN THE LOW SNR REGIME

In this section, we investigate the sum capacity of the double-scattering fading MIMO MAC in the low SNR regime.

When each transmitter knows their own transmit, scatter, and receive correlation matrices, it was shown in [6] that the optimal transmission strategy is to employ independent complex Gaussian inputs along the eigenvectors of each user's transmit correlation matrix, such that $\mathbf{U}_Q^{(k)} = \mathbf{U}_T^{(k)}$. Moreover, at low SNR, the optimal capacity-achieving approach is to employ statistical beamforming, for which the input covariance is rank-1, with $\lambda_{Q1}^{(k)} = P_k, \lambda_{Q2}^{(k)} = \cdots = \lambda_{Qm_k}^{(k)} = 0, k = 1, \cdots, K$. In this case, the input covariance matrix is

$$\mathbf{Q}_k = P_k \mathbf{u}_{k,1} \mathbf{u}_{k,1}^\dagger, \; k = 1, \cdots, K, \tag{6}$$

where $\mathbf{u}_{k,1}$ is the eigenvector corresponding to the maximum eigenvalue of $\mathbf{\Phi}_T^{(k)}$; that is, the first column of $\mathbf{U}_T^{(k)}$. Therefore, the sum capacity at low SNR can be written as

$$C(\mathrm{SNR}) = E\left\{\log_2 \left| \mathbf{I}_n + \mathrm{SNR} \sum_{k=1}^{K} \mathbf{A}_k \right| \right\} \tag{7}$$

where $\mathrm{SNR} = \sum_{k=1}^{K} P_k \Big/ N_0$,

$$\mathbf{A}_k = \frac{\lambda_{T1}^{(k)} \mu_k}{s_k} \mathbf{\Lambda}_R^{\frac{1}{2}} \mathbf{H}_1^{(k)} \mathbf{\Lambda}_S^{(k)\,\frac{1}{2}} \mathbf{h}_2^{(k)} \mathbf{h}_2^{(k)\dagger} \mathbf{\Lambda}_S^{(k)\,\frac{1}{2}} \mathbf{H}_1^{(k)\dagger} \mathbf{\Lambda}_R^{\frac{1}{2}}, \tag{8}$$

$\mu_k = P_k \Big/ \sum_{l=1}^{K} P_l$, and $\mathbf{h}_2^{(k)}$ is the first column of $\mathbf{H}_2^{(k)}$.

We now present closed-form solutions for the minimum E_b/N_0 and S_0 of the double-scattering MIMO MAC.

Theorem 1 *For the double-scattering MIMO MAC with each transmitter knowing their own statistical CSI, the $E_b/N_{0\min}$ is given by*

$$\frac{E_b}{N_0}_{\min} = \frac{\ln 2}{n \sum_{k=1}^{K} \lambda_{T1}^{(k)} \mu_k} \tag{9}$$

and the wideband slope is given by (10) (see top of next page), where, for an $N \times N$ matrix $\mathbf{X}$,

$$\zeta(\mathbf{X}) = N \frac{\mathrm{tr}(\mathbf{X}^2)}{\mathrm{tr}^2(\mathbf{X})}. \tag{11}$$

Proof: See the Appendix. □

$$S_0 = \frac{2\left(\sum_{k=1}^{K}\lambda_{T1}^{(k)}\mu_k\right)^2}{\sum_{k=1}^{K}{\lambda_{T1}^{(k)}}^2\mu_k^2 + \frac{\zeta(\mathbf{\Phi}_R)}{n}\left(\sum_{k=1}^{K}\lambda_{T1}^{(k)}\mu_k\right)^2 + \left(1+\frac{\zeta(\mathbf{\Phi}_R)}{n}\right)\sum_{k=1}^{K}{\lambda_{T1}^{(k)}}^2\mu_k^2\frac{\zeta\left(\mathbf{\Phi}_S^{(k)}\right)}{s_k}} \tag{10}$$

We can make the following observations:

1) ***Impact of transmit correlation:*** We see that $E_b/N_{0_{\min}}$ depends on the transmit correlation for each user via the maximum eigenvalue $\lambda_{T1}^{(k)}$, weighted by the corresponding power ratio μ_k. Since $\lambda_{T1}^{(k)}$ varies monotonically with the level of transmit correlation, we see that increasing the correlation leads to a smaller $E_b/N_{0_{\min}}$.

To examine the impact of transmit correlation on S_0, we consider the special case where the scatter correlation matrices are all equal. In this case,

$$S_0 = \frac{2}{\omega + \frac{\zeta(\mathbf{\Phi}_R)}{n} + \omega\left(1+\frac{\zeta(\mathbf{\Phi}_R)}{n}\right)\frac{\zeta(\mathbf{\Phi}_S^{(1)})}{s_1}} \tag{12}$$

where $\omega = \sum_{k=1}^{K}{\lambda_{T1}^{(k)}}^2\mu_k^2 \Big/ \left(\sum_{k=1}^{K}\lambda_{T1}^{(k)}\mu_k\right)^2$. It can be easily seen that S_0 increases with increasing transmit correlation (ie. as $\lambda_{T1}^{(k)}$ increases, for any k).

Thus, we conclude that the presence of transmit correlation leads to an increase in sum capacity of the double-scattering MIMO MAC, by simultaneously decreasing the minimum required E_b/N_0 and increasing the wideband slope.

2) ***Impact of scatter/receive correlation:*** Interestingly, scatter and receive correlation have no effect on $E_b/N_{0_{\min}}$. Moreover, since $\zeta(\mathbf{\Phi}_R)$ and $\zeta(\mathbf{\Phi}_S^{(k)})$ increase with increasing levels of correlation, we can easily see that S_0 varies inversely with the receive/scatter correlation, satisfying

$$\frac{2}{1+3\omega} \le S_0 \le \frac{2}{\omega + \frac{1}{n} + \left(1+\frac{1}{n}\right)\sum_{k=1}^{K}\frac{\omega_k}{s_k}} \tag{13}$$

where $\omega_k = {\lambda_{T1}^{(k)}}^2\mu_k^2 \Big/ \left(\sum_{l=1}^{K}\lambda_{T1}^{(l)}\mu_l\right)^2$, with the lower-bound corresponding to the case of a fully-correlated receiver and fully correlated scatterers, and the upper-bound corresponding to an uncorrelated receiver and uncorrelated scatterers.

This result indicates that the presence of receive or scatter correlation, whilst not affecting the minimum required E_b/N_0, leads to a loss in sum capacity as reflected in a reduced S_0.

4. NUMERICAL RESULTS

In this section, we present numerical results to further investigate our analytical results. The spatial correlation matrices at the transmitters, scatterers, and receiver are assumed to have an exponential form. Based on this model, the (i, j)th elements of $\mathbf{\Phi}_T^{(k)}$, $\mathbf{\Phi}_S^{(k)}$ and $\mathbf{\Phi}_R$ are $\left[\mathbf{\Phi}_T^{(k)}\right]_{i,j} = \left(\alpha_t^{(k)}\right)^{|i-j|}$, $\left[\mathbf{\Phi}_S^{(k)}\right]_{i,j} = \left(\alpha_s^{(k)}\right)^{|i-j|}$ and $[\mathbf{\Phi}_R]_{i,j} = (\alpha_r)^{|i-j|}$ respectively, where $\alpha_t^{(k)}, \alpha_s^{(k)}, \alpha_r \in [0, 1]$ are the respective correlation coefficients. Moreover, we assume that $m_1 = \cdots = m_K = 2$, $s_1 = \cdots = s_K = 4$, $n = 3$, $\alpha_t^{(1)} = \cdots = \alpha_t^{(K)} = \alpha_t$, $\alpha_s^{(1)} = \cdots = \alpha_s^{(K)} = \alpha_s$, $\mu_1 = \cdots = \mu_K$.

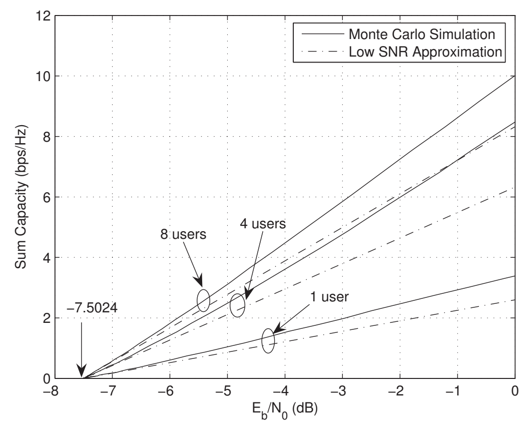

Fig. 1: Low SNR sum capacity of statistical beamforming in double-scattering MIMO MAC for different number of users; analytical approximations and Monte-Carlo simulations.

Fig. 1 illustrates the low SNR sum capacity of the double-scattering MIMO MAC. We compare Monte-Carlo simulations of the exact sum capacity, with the analytical low SNR approximation obtained by combining (4), (9), and (10). Results are shown for different numbers of users, with $\alpha_t = 0.3$, $\alpha_s = 0.5$, and $\alpha_r = 0.4$. We see that the low SNR approximations are accurate over a quite moderate range of E_b/N_0 values. Moreover, we observe that the minimum E_b/N_0 is the same, regardless of the number of users. This is due to the assumption that the correlation coefficients of all users are the same, and $\mu_1 = \cdots = \mu_K$.

Fig. 2 shows the results of Monte-Carlo simulations and the analytical low SNR approximation curves obtained by combining (4), (9), and (10), for a 4-user system with different correlation coefficients. We see that $E_b/N_{0_{\min}}$ depends on the transmit correlation, but not on the scatter and receive correlation matrices, and it decreases with increasing transmit correlation. The wideband slope decreases as the receive/scatter correlation increase. These results agree with our analytical conclusions in Section 3.

5. CONCLUSION

This paper has investigated the low SNR capacity of the double-scattering MIMO MAC, assuming perfect CSI at the receiver and statistical CSI at each transmitter. We derived the minimum E_b/N_0 and wideband slope, which were used to obtain an accurate closed-form approximation for the capacity in the

$$E\left\{\operatorname{tr}\left(\mathbf{A}_k^2\right)\right\} = \frac{\lambda_{T1}^{(k)^2}\mu_k^2}{s_k^2} E\left\{\left[\sum\nolimits_{l=1}^{n} \lambda_{Rl}\left(\sum\nolimits_{i=1}^{s_k}\sum\nolimits_{j=1}^{s_k} \lambda_{Si}^{(k)\frac{1}{2}}\lambda_{Sj}^{(k)\frac{1}{2}} h_{i,1}^{(k)*} h_{j,1}^{(k)} g_{l,i}^{(k)*} g_{l,j}^{(k)}\right)\right]^2\right\} \tag{19}$$

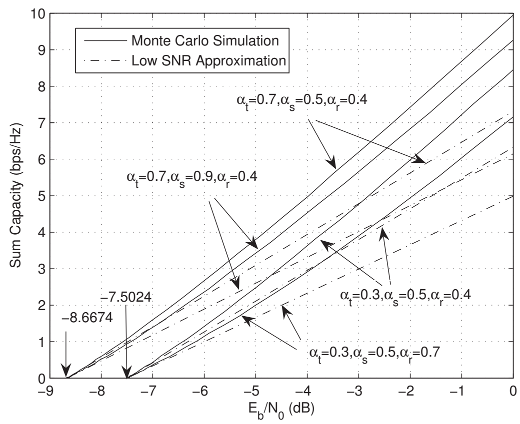

Fig. 2: Low SNR sum capacity of statistical beamforming in double-scattering MIMO MAC for different correlation coefficients; analytical approximations and Monte-Carlo simulations.

low SNR regime. Based on this result, we investigated the effect of transmit, receive, and scatter correlation on the capacity.

6. APPENDIX: PROOF OF THEOREM 1

The determinant of a square matrix $\mathbf{X}$ has the following properties [7]:

$$\frac{d}{du}\ln\left|\mathbf{I}+u\mathbf{X}\right|\Big|_{u=0} = \operatorname{tr}(\mathbf{X}), \ \frac{d^2}{du^2}\ln\left|\mathbf{I}+u\mathbf{X}\right|\Big|_{u=0} = -\operatorname{tr}\left(\mathbf{X}^2\right). \tag{14}$$

Using (5) and (14) yields

$$\frac{E_b}{N_0}_{\min} = \frac{\ln 2}{E\left\{\operatorname{tr}\left(\sum_{k=1}^K \mathbf{A}_k\right)\right\}}. \tag{15}$$

From (8), we can easily obtain

$$\frac{E_b}{N_0}_{\min} = \frac{\ln 2}{\sum_{k=1}^K \lambda_{T1}^{(k)}\mu_k \operatorname{tr}(\mathbf{\Phi}_R)} \tag{16}$$

to yield (9). Now consider S_0. From (5) and (14), we have

$$S_0 = \frac{2E^2\left\{\operatorname{tr}\left(\sum_{k=1}^K \mathbf{A}_k\right)\right\}}{E\left\{\operatorname{tr}\left(\left(\sum_{k=1}^K \mathbf{A}_k\right)^2\right)\right\}}. \tag{17}$$

The denominator in (17) can be written as follows

$$\sum_{k=1}^{K} E\left\{\operatorname{tr}\left(\mathbf{A}_k^2\right)\right\} + \sum_{k=1}^{K}\sum_{l=1, l\neq k}^{K} \operatorname{tr}\left(E\left\{\mathbf{A}_k\right\}E\left\{\mathbf{A}_l\right\}\right). \tag{18}$$

Considering the first term in (18), $E\left\{\operatorname{tr}\left(\mathbf{A}_k^2\right)\right\}$ can be written as in (19) (see top of this page), where $\mathbf{H}_1^{(k)} = \left[g_{i,j}^{(k)}\right]$, $\mathbf{H}_2^{(k)} = \left[h_{i,j}^{(k)}\right]$, and the superscript $(\cdot)^*$ indicates conjugate operation. Since $2\left|h_{i,j}^{(k)}\right|^2 \sim \chi_2^2$ and $2\left|g_{i,j}^{(k)}\right|^2 \sim \chi_2^2$, after some manipulations, (19) can be simplified as

$$E\left\{\operatorname{tr}\left(\mathbf{A}_k^2\right)\right\} = \frac{\lambda_{T1}^{(k)^2}\mu_k^2}{s_k^2}\left[s_k^2 + \operatorname{tr}\left(\mathbf{\Phi}_S^{(k)^2}\right)\right]\left[n^2 + \operatorname{tr}\left(\mathbf{\Phi}_R^2\right)\right]. \tag{20}$$

Now considering the second term in (18), it can be easily obtained that

$$\sum_{k=1}^{K}\sum_{\substack{l=1\\ l\neq k}}^{K} \operatorname{tr}\left(E\left\{\mathbf{A}_k\right\}E\left\{\mathbf{A}_l\right\}\right) = \sum_{k=1}^{K}\sum_{\substack{l=1\\ l\neq k}}^{K} \lambda_{T1}^{(k)}\lambda_{T1}^{(l)}\mu_k\mu_l \operatorname{tr}\left(\mathbf{\Phi}_R^2\right). \tag{21}$$

Substituting (20) and (21) into (17), and applying some manipulations, we obtain (10).

7. REFERENCES

[1] I. E. Telatar,"Capacity of multi-antenna Gaussian channels," *Euro. Trans. Telecom.*, vol 10, pp. 585–596. Nov.-Dec. 1999.

[2] M. R. McKay, and I. B. Collings,"General capacity bounds for spatially correlated Rician MIMO channels," *IEEE Trans. Inform. Theory*, vol. 51, no. 9, pp. 3121–3145, Sep. 2005.

[3] S. Jin, X. Q. Gao, and X. H. You,"On the ergodic capacity of rank-1 Ricean fading MIMO channels," *IEEE Trans. Inform. Theory*, vol. 53, no. 2, pp. 502–517, Feb. 2007.

[4] D. Gesbert, H. Bolcskei, D. A. Gore, and A. J. Paulraj,"Outdoor MIMO wireless channels: Models and performance prediction," *IEEE Trans. Commun.*, vol. 50, no. 12, pp. 1926–1934, Dec. 2002.

[5] X. Li, S. Jin, X. Q. Gao, M. R. McKay, and K. K. Wong, "Transmitter optimization and beamforming optimality conditions for double-scattering MIMO channels," *IEEE Trans. Wireless Communications*, vol. 7, no. 9, pp. 3647–3654, Sep. 2008.

[6] X. Li, S. Jin, X. Q. Gao, and M. R. McKay,"Transmitter optimization and power allocation in double-scattering MIMO multiple access channels," in Proc. *IEEE Int. Symp. Info. Theory (ISIT)*, Toronto, Canada, pp. 920–924, Jul. 2008.

[7] S. Verdú,"Spectral efficiency in the wideband regime," *IEEE Trans. Inform. Theory*, vol. 48, no. 6, pp. 1319–1343, Jun. 2002.

[8] A. Lozano, A. M. Tulino, and S. Verdú,"Multiple-antenna capacity in the low-power regime," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2527–2544, Oct. 2003.

[9] E. A. Jorswieck, and H. Boche,"Multiple-antenna capacity in the low-power regime: channel knowledge and correlation," in Proc. *IEEE Int. Conf. on Acous., Speech, and Sig. Proc. (ICASSP)*, Philadelphia, USA, pp. 385–388, 2005.

[10] A. Soysal, and S. Ulukus,"Asymptotic optimality of beamforming in multi-user MIMO-MAC with no or partial CSI at the transmitters," in Proc. *IEEE 61st Vehicular Technology Conference (VTC 2005-Spring)*, pp. 1619–1623, 2005.