MULTIUSER TRANSMIT BEAMFORMING FOR MIMO UPLINK WITH INDIVIDUAL SINR TARGETS

Songnan Xi and Michael D. Zoltowski

School of Electrical and Computer Engineering
Purdue University
{sxi, mikedz}@ecn.purdue.edu

ABSTRACT

For multiuser multiple-input multiple-output (MIMO) uplink communications where the signal-to-interference-plus-noise ratio (SINR) of the k-th user, denoted as SINR_k, is expected to achieve the target value γ_k, we first consider the ideal scenario where multiuser channel matrices are perfectly known to every user and we solve the problem of maximizing $\min_k(\text{SINR}_k/\gamma_k)$ over all possible beamformers under individual transmit power constraints via alternating optimization strategies and the generalized eigenvalue problem (GEVP) programming. Then, we study a more realistic situation where only a few number of bits are allowed to feed back to users. We propose to integrate the genetic algorithm into the Lloyd's vector quantization method to design the optimum codebook in terms of maximizing $\mathsf{E}(\min_k(\text{SINR}_k/\gamma_k))$ with expectation taken with respect to fading channels.

Index Terms— Beamforming optimization, GEVP, limited feedback, Lloyd's method, genetic algorithm

1. INTRODUCTION

The uplink of multiuser multiple-input multiple-output (MIMO) systems where beamforming is adopted at all users is considered. Each user is assigned a target signal to interference-plus-noise ratio (SINR) to be achieved at the output of its detector. Let SINR_k denote the k-th user's SINR and γ_k be the corresponding target value. We address the problem of optimizing the multiuser transmit beamformers to maximize the achievable SINR margin, i.e. to maximize $\min_k(\text{SINR}_k/\gamma_k)$, under individual transmit power constraints. The worst case optimization problem, that is to maximize $\min_{1\leq k\leq K} \text{SINR}_k$, is a special case where the same target SINR is set for all users. To our best knowledge, no one has addressed this problem before.

When single antenna is employed at all users, the uplink multiuser receive beamformer design is an easier problem than the downlink multiuser transmit beamfomer design, since the former is an un-coupled optimization problem while the latter, which involves the interdependence of different users' beamformers, is much more complicated. Thus the uplink-downlink duality has been exploited to solve the downlink problem via the corresponding uplink problem [1].

However, for the strict MIMO channels considered in this work, the uplink multiuser transmit beamformer design is also an coupled and thus quite complicated problem as the downlink multiuser transmit beamformer design. The authors of [2] propose to replace maximizing SINR with maximizing signal-to-jamming plus noise ratio (SJIR) to circumvent the interdependence among multiuser beamforming vectors, and still keep the benefits of exploiting channel knowledge of all users. Unfortunately, when applying the same method to uplink, the resulting design is simply the well-known eigen-beamforming (Eigen-BF), where one can not take advantage of extra channel knowledge from interfering users.

We propose to apply alternating optimization strategies and the generalized eigenvalue problem (GEVP) programming [3] to solve for the optimum beamformers assuming that channel knowledge is perfectly known to all users. Then we further consider the channels with limited feedback and integrate the genetic algorithm (GA) into the Lloyd's method [4, 5] to design the codebook of multiuser beamformers.

2. OPTIMUM MULTIUSER UPLINK TRANSMIT BEAMFORMING SOLUTION

Consider the uplink of multiuser MIMO systems, where there are K simultaneous users and a base station. Each user is equipped with N_T antennas to perform beamforming and the base station is equipped with N_R antennas. The channel is assumed to be frequency non-selective Rayleigh fading. The received signal vector, denoted as $\mathbf{r}$, can be expressed as

$$\mathbf{r} = \sum\nolimits_{k=1}^{K} \sqrt{\frac{P_k}{E_s}} \mathbf{H}_k \mathbf{c}_k s_k + \mathbf{w}_N, \tag{1}$$

where s_k is the modulated symbol of average energy E_s transmitted by the k-th user; $\mathbf{c}_k$ is the beamforming weight vector, or beamfomer; $\mathbf{H}_k$ is the channel matrix from the k-th user to the receiver; $\mathbf{w}_N$ is complex additive white Gaussian noise

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(AWGN) vector with independent and identically distributed (i.i.d.) elements of zero mean and variance N_0. For Rayleigh fading spatially independent MIMO channels, the elements in $\mathbf{H}_k$ are assumed to be independent $\mathcal{CN}(0,1)$. P_k is the maximum allowable transmission power for the k-th user and it is easy to derive the constraints that $\|\mathbf{c}_k\| \leq 1$. In this section, it is assumed that a realization of the full multiuser channel state information

$$\mathcal{H} = \{\mathbf{H}_k, k = 1, \cdots, K\} \tag{2}$$

is given and perfectly known to all users as well as the base station.

Let $\boldsymbol{\ell}_k$ denote the linear transformation for the k-th user and the detector output for the k-th user is $\boldsymbol{\ell}_k^H \mathbf{r}$. Suppose the target SINR of the k-th user is γ_k. Given $\mathcal{H}$, as pointed out in [1], a sufficient and necessary condition for all targets to be achieved simultaneously, i.e. $\min_{1\leq k\leq K}(\text{SINR}_k/\gamma_k) \geq 1$, is obtained by solving the optimization problem:

$$\begin{array}{ll} \max\limits_{\{\mathbf{c}_k,\boldsymbol{\ell}_k\}_{k=1}^K} \min\limits_{1\leq k\leq K} \frac{\text{SINR}_k}{\gamma_k} & \\ s.t. \quad \|\mathbf{c}_k\| \leq 1 \quad k = 1, \cdots, K \end{array} \tag{3}$$

where $\{\mathbf{c}_k, \boldsymbol{\ell}_k\}_{k=1}^K$ stands for the set of all users' beamformers and linear detectors and SINR_k is expressed as

$$\text{SINR}_k = \frac{\rho_k |\boldsymbol{\ell}_k^H \mathbf{H}_k \mathbf{c}_k|^2}{\sum_{\substack{i=1\\ i\neq k}}^K \rho_i |\boldsymbol{\ell}_k^H \mathbf{H}_i \mathbf{c}_i|^2 + \|\boldsymbol{\ell}_k\|^2}, \tag{4}$$

where $\rho_k = P_k/N_0, 1 \leq k \leq K$ is the maximum transmit SNR of k-th user.

It is well known that for a given transmitter, the linear minimum mean squared error (LMMSE) detector maximizes the output SINR [6]. So, it can be easily derived that for any given multiuser beamformers, the linear detectors that maximize $\min_{1\leq k\leq K}(\text{SINR}_k/\gamma_k)$ are

$$\boldsymbol{\ell}_k^H = \left[\left(\mathbf{H}_{c\rho}^H \mathbf{H}_{c\rho} + \mathbf{I}\right)^{-1} \mathbf{H}_{c\rho}^H\right]_{(k,:)}, \tag{5}$$

where $\mathbf{H}_{c\rho} = [\sqrt{\rho_1}\mathbf{H}_1\mathbf{c}_1, \cdots, \sqrt{\rho_K}\mathbf{H}_K\mathbf{c}_K]$ and $[\cdot]_{(k,:)}$ denotes the k-th row of a matrix. And the resulting SINR's are expressed as

$$\text{SINR}_k = \frac{1}{\left[\left(\mathbf{H}_{c\rho}^H \mathbf{H}_{c\rho} + \mathbf{I}\right)^{-1}\right]_{(k,k)}} - 1. \tag{6}$$

The equation (5) suggests that the optimum detectors are functions of the beamformers. At this point,one tends to plug (5) into (3) and reduce the original optimization problem to maximizing over $\{\mathbf{c}_k\}_{k=1}^K$, a single optimization variable set. However the resulting reduced optimization problem is a coupled problem and is too complicated to be solved efficiently.

We propose to consider the original representation of the optimization problem as given in (3) and apply alternating optimization strategies for solution by considering $\{\boldsymbol{\ell}_k\}_{k=1}^K$ and $\{\mathbf{c}_k\}_{k=1}^K$ as two independent sets of optimization variables. In other words, we maximize $\min_{1\leq k\leq K}(\text{SINR}_k/\gamma_k)$ over $\{\boldsymbol{\ell}_k\}_{k=1}^K$ for given beamformers and over $\{\mathbf{c}_k\}_{k=1}^K$ for given linear detectors alternatively. The former sub-problem is denoted as $\mathcal{P}(\mathbf{l}|\mathbf{c})$ and the latter one is denoted as $\mathcal{P}(\mathbf{c}|\mathbf{l})$.

The sub-problem $\mathcal{P}(\mathbf{l}|\mathbf{c})$ has already been solved as given by (5) and (6).

Following the approach in [3], we can express the sub-problem $\mathcal{P}(\mathbf{c}|\mathbf{l})$ into a standard generalized eigenvalue problem (GEVP) form as

$$\begin{array}{l} \min\limits_{\{\mathbf{c}_k\}_{k=1}^K,\beta} \quad \beta = \sqrt{1/\gamma_0} \\ s.t. \\ \left\{ \begin{array}{l} 0 \preceq \begin{bmatrix} 1 & \mathbf{c}_k^H \\ \mathbf{c}_k & \mathbf{I}_{N_T} \end{bmatrix} \\ 0 \leq \boldsymbol{\ell}_k^H \mathbf{H}_k \mathbf{c}_k \\ \begin{bmatrix} 0 & -\begin{bmatrix} \tilde{\mathbf{z}}_k(\{\mathbf{c}_k\})^H & \|\boldsymbol{\ell}_k\| \end{bmatrix} \\ -\begin{bmatrix} \tilde{\mathbf{z}}_k(\{\mathbf{c}_k\}) \\ \|\boldsymbol{\ell}_k\| \end{bmatrix} & \mathbf{0}_K \end{bmatrix} \\ \preceq \beta \sqrt{\frac{\rho_k}{\gamma_k}} \boldsymbol{\ell}_k^H \mathbf{H}_k \mathbf{c}_k \mathbf{I}_{(K+1)} \end{array} \right. , \\ k = 1, \cdots, K \end{array} \tag{7}$$

where $\gamma_0 \leq \min_{1\leq k\leq K}(\text{SINR}_k/\gamma_k)$ is a real valued slack variable , and

$$\tilde{\mathbf{z}}_k(\{\mathbf{c}_k\}) = \begin{bmatrix} \sqrt{\rho_1}\boldsymbol{\ell}_k^H \mathbf{H}_1 \mathbf{c}_1 \\ \vdots \\ \sqrt{\rho_{(k-1)}}\boldsymbol{\ell}_k^H \mathbf{H}_{(k-1)} \mathbf{c}_{(k-1)} \\ \sqrt{\rho_{(k+1)}}\boldsymbol{\ell}_k^H \mathbf{H}_{(k+1)} \mathbf{c}_{(k+1)} \\ \vdots \\ \sqrt{\rho_K}\boldsymbol{\ell}_k^H \mathbf{H}_K \mathbf{c}_K \end{bmatrix}. \tag{8}$$

$\tilde{\mathbf{z}}_k(\{\mathbf{c}_k\})$ represents the interference to the k-th user from any other user, and can be considered as a linear function of optimization variables $\{\mathbf{c}_k\}$, as noted by " $(\{\mathbf{c}_k\})$".

In case of $\gamma_k = \gamma, \forall k$, the original optimization problem (3) is reduced to a worst SINR maximization problem, which can be formulated into an alternative GEVP form as

$$\begin{array}{l} \min\limits_{\{\mathbf{c}_k\}_{k=1}^K,\beta} \quad \beta = \sqrt{1 + 1/\gamma_0} \\ s.t. \\ \left\{ \begin{array}{l} 0 \preceq \begin{bmatrix} 1 & \mathbf{c}_k^H \\ \mathbf{c}_k & \mathbf{I}_{N_T} \end{bmatrix} \\ 0 \leq \boldsymbol{\ell}_k^H \mathbf{H}_k \mathbf{c}_k \\ \begin{bmatrix} 0 & -\begin{bmatrix} \mathbf{z}_k(\{\mathbf{c}_k\})^H & \|\boldsymbol{\ell}_k\| \end{bmatrix} \\ -\begin{bmatrix} \mathbf{z}_k(\{\mathbf{c}_k\}) \\ \|\boldsymbol{\ell}_k\| \end{bmatrix} & \mathbf{0}_{(K+1)} \end{bmatrix} \\ \preceq \beta \sqrt{\rho_k} \boldsymbol{\ell}_k^H \mathbf{H}_k \mathbf{c}_k \mathbf{I}_{(K+2)} \end{array} \right. , \\ k = 1, \cdots, K \end{array} \tag{9}$$

where the slack variable $\gamma_0 \leq \min_{1\leq k\leq K} \text{SINR}_k$ and

$$\tilde{\mathbf{z}}_k(\{\mathbf{c}_k\}) = \begin{bmatrix} \sqrt{\rho_1}\boldsymbol{\ell}_k^H\mathbf{H}_1\mathbf{c}_1 \\ \vdots \\ \sqrt{\rho_K}\boldsymbol{\ell}_k^H\mathbf{H}_K\mathbf{c}_K \end{bmatrix}. \quad (10)$$

The GEVP is a standard conic program and can be solved using existing softwares, such as the *gevp* function in Matlab linear matrix inequality (LMI) control toolbox [7].

3. LLOYD-GENETIC TRANSMIT BEAMFORMER CODEBOOK

For MIMO channels with limited feedback where only a small number N of bits are allowed for feedback , we can design a codebook $\mathcal{C}$ of size $|\mathcal{C}| = 2^N$, which contains a finite number of codes as multiuser beamformers $\mathcal{C} = \{\mathbf{C}_1, \cdots, \mathbf{C}_{|\mathcal{C}|}\}$, and let the receiver choose the best one for each channel realization and then feedback the index of the best code to all the users. Thus, only the receiver needs the full multiuser channel state information. Each code provides beamformers of all users, that is,

$$\mathbf{C}_i = [\mathbf{c}_{1,i}, \mathbf{c}_{2,i}, \cdots, \mathbf{c}_{K,i}], \quad (11)$$

where $\mathbf{c}_{k,i}$ is the ith code for k-th user's beamformer. The codebook is designed off line and restored at all users and the receiver. To isolate the effects of beamformer quantization on performance, we also assume that the feedback channel is error free so the users know the accurate code index.

As discussed in the section 2, $\min_{1\leq k\leq K}(\text{SINR}_k/\gamma_k)$ is essentially a function of channel matrices and the beamformers. When the beamformers are taken as a code $\mathbf{C}$ in the codebook $\mathcal{C}$, the function can be denoted as $\gamma(\mathcal{H}, \mathbf{C}) = \min_{1\leq k\leq K}(\text{SINR}_k/\gamma_k)$. For the optimization problem interested in this work, the performance metric of a codebook $\mathcal{C}$ is defined as

$$\begin{aligned} \bar{\gamma}(\mathcal{C}) &= \mathsf{E}_{\mathcal{H}}\big(\min_{1\leq k\leq K}(\text{SINR}_k/\gamma_k)\big) \\ &= \mathsf{E}_{\mathcal{H}}\big(\gamma(\mathcal{H}, \mathbf{C}); \mathbf{C} \in \mathcal{C}\big) \end{aligned}. \quad (12)$$

Given a set of channel samples of size M

$$\mathcal{S}_{\mathcal{H}} = \{\mathcal{H}_j; j = 1, \cdots, M\}, \quad (13)$$

where $\mathcal{H}$ is defined in (2). The two fundamental steps in Lloyd's method is firstly to find the best code for each of the $\mathcal{H}$ sample, resulting the best partitions of $\mathcal{S}$ for current codes, and secondly, to find the new best code for each partition of $\mathcal{S}$ obtained from previous step. The first step is quite simple and achieved by assigning the code with maximum $\gamma(\mathcal{H}, \mathbf{C})$ to each $\mathcal{H}$ in $\mathcal{S}$. The second step is much harder. It is expected that the solution can be obtained by modifying the algorithm developed for the situation as in the section 2 where the optimization is carried over all possible N_T-vectors and does not involve expectation operation. But due to the fact that $\mathsf{E}(X/Y)$ is generally not equal to $\mathsf{E}(X)/\mathsf{E}(Y)$, the GEVP formulation (7) or (9) is not readily to be modified for codebook design. We solved the second step via genetic algorithm (GA), whose running time is not going to be an issue since the codebook is designed off-line. The basics of genetic algorithm can be found in [8]. The resulting codebook is referred to as *Lloyd-GA* codebook

The presence of transmit SNR's in the objective function (see (4)) suggests that infinite number of codebooks should be computed and stored to handle all possible SNR values. This is an impossible mission. One way to solve this is that we partition the range of possible actual transmit SNR values into several sections and use the codebook corresponding to the center transmit SNR of a section for the whole section. Then only a limited number of codebooks are needed. However this still requires more than one single codebook to be computed and stored. In this work, we propose to take the distribution of transmit SNR into consideration and extend the channel sample set $\mathcal{S}_{\mathcal{H}}$ so as to also include the samples of transmit SNR, i.e.

$$\mathcal{S}_{\mathcal{H},\rho} = \big\{(\mathcal{H}, \{\rho_k\}_{k=1}^K)_j; j = 1, \cdots, M\big\} \quad (14)$$

Thus only one codebook is needed for all possible values of transmit SNR in a system. The numerical results presented in the next section validated such a solution. To distinguish from the actual transmit SNR ρ_k, the pre-assigned transmit SNR based on which a codebook is designed is denoted as $\rho_{\mathcal{C}_k}$.

4. NUMERICAL RESULTS

For purpose of illustration, without loss of generality, it is assumed that $P_k = P, \forall k$, and thus $\rho_k = P/N_0 \equiv \rho, \forall k$ and $\rho_{\mathcal{C}_k} \equiv \rho_{\mathcal{C}}$. It is also assumed that $\gamma_k = \gamma, \forall k$.

As for the optimum multiuser beamforming solution presented in the section 2, the convergence speed in terms of the average number of iterations needed for reaching convergence is affected by the initial beamformers. Three different types of initial values, which are uniformly generated beamformers, the eigen-beamforming (*Eigen-BF*) and the quasi SINR-Max cooperative beamforming (*q-SCBF*) given in our previous work [9] are compared in Fig. 1. The initial beamformers obtained via q-SCBF obviously reduces iteration number. The average minimum SINR and bit-error-rate (BER) are shown in Figs 2 and 3 respectively.

The performances of the Lloyd-GA codebook when $\rho_{\mathcal{C}} = \rho$ or $\rho_{\mathcal{C}}$ is assumed to be uniformly distributed over a valid range $[a, b]$, denoted as $\rho_{\mathcal{C}} \sim U(a, b)$, are compared and illustrated in Figs 2 and 3. The *Eigen-BF* codebook, is also included for comparison. It can be seen that the performance of single codebook solution by taking the distribution of transmit SNR into consideration overlaps with that of multiple codebooks for multiple SNR's, which validates our solution.

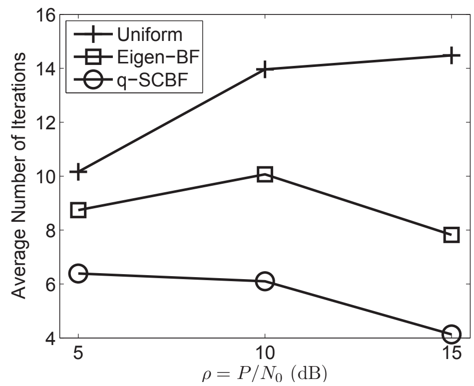

Fig. 1. Convergence speed of optimum multiuser beamforming solution with different initial beamformers

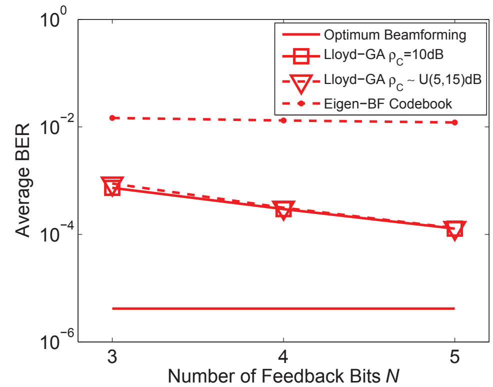

Fig. 3. Average BER over MIMO channels with limited feedback at $\rho = 10$dB

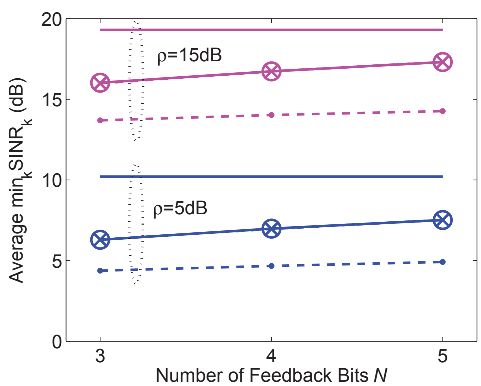

Fig. 2. solid line: optimum beamformers (ideal feedback); solid 'o' line: Lloyd-GA codebook ($\rho_{\mathcal{C}} = \rho$); dashed '×' line :Lloyd-GA Codebook ($\rho_{\mathcal{C}} \sim U(5, 15)$dB); dashed '·' line :Eigen-BF Codebook

5. CONCLUSION

We optimize multiuser transmit beamformers to maximize the achievable SINR margin by alternatively optimizing over the beamformers and the linear detectors. For fixed detectors, the problem can be formulated into standard GEVP form and solved using appropriate softwares. For channels with limited feedback, we integrate GA into the Lloyd's method to design the beamformer codebook.

6. REFERENCES

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