# PRECODER DESIGN FOR MIMO BROADCAST CHANNELS WITH POWER LEAKAGE CONSTRAINTS

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## ABSTRACT

We investigate the design of MIMO linear precoders for multiuser broadcast channels under signal leakage constraints. This power leakage constraints reflect both an important security consideration and the need for interference suppression at receivers. Defining a convex optimization problem, we propose two sequential semi-definite program algorithms. In principle, our algorithms exploit both spatial diversity and multiuser diversity to enhance the overall system performance. The resulting MIMO precoded transmission demonstrates superior performance over time-division multiplexed multiuser transmission in conjunction with the traditional space-time transmission.

***Index Terms***— MIMO systems, power leakage, precoding

## 1. INTRODUCTION

As a promising technique for broadband wireless communication, multi-input-multi-output (MIMO) transceivers offer significant capacity gain over traditional single antenna systems, under limited bandwidth and transmission power. The advantage of MIMO systems is achieved for by more sophisticated signal processing and space-time coding in order to fully exploit the potentials of MIMO systems [1]. This is particularly true for multiuser MIMO systems. The well known Alamouti code, for example, provides an effective means to achieve maximum spatial diversity order, while offering no multiplexing gain. On the other hand, precoding designs of [2], [3], provide variable tradeoffs between diversity and multiplexing.

Linear precoder design in the multiuser MIMO systems has been actively investigated in recent years, e.g., [5], [6]. The general principle is to jointly exploit space diversity and multiuser multiplexing diversity simultaneously. Unlike single user MIMO systems or time-division multiplexing (TDM) MIMO systems, more than one user signals are transmitted in broadcast channels (BC). These approaches present an extra degree of flexibility as well as design complexity. They often are designed to maximize system capacity or signal to noise and interference ratio (SNIR). We note, however, that in addition to provide the necessary user information delievery to desired receivers, one major concern often neglected is the critical issue of leaking confidential user signal to unintended users. This additional concern against eavesdropping provides the motivation to our linear precoder designs.

Note that the when a user receives another user signal in addition to its desired signal, the ability to recover the interfering signal depends not necessarily on the SNIR of the desired user signal. Instead, the receiver is often capable of recovering the interfering signal by first recovering and cancelling the desired user signal from the received data if the SNIR is very high. In other words, if the desired user signal is perfectly recovered and cancelled from the received signal, then the remaining intererence may also be recovered as long as its signal power is strong enough against the channel noise. This analysis tells us that, in order to prevent eavesdropping, a well design precoder in multiuser should minimize the power leakage of unintended user signals at each of the MIMO receivers. Our work in this manuscript aims to maximize the desired user information capacity while minimizing the risk of eavesdropping by unintended users in a broadcast channel MIMO system.

We will present numerical algorithms to design linear MIMO multiuser precoders with maximum system capacity under a set of power leakage constraints. The power leakage constraints serve to limit interference and to protect overhearing by other MIMO users. We will develop design algorithms to maximize the sum capacity and, for fairness, to maximize the minimum of active user capacities, respectively. We formulate the precoder design problems as a sequential semi-definite programming problem. Central to our objective of limiting eavesdropping risk are a set of power leakage constraints, defined as thresholds to limit undesirable signal reception. We will also provide numerical results to illustrate the effect of our optimum linear precoders.

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## 2. SYSTEM MODEL

We assume $N_t$ antennas at the transmitter and $N$ receivers each equipped with $N_r$ antennas. We denote the signal vector for the $k$th user as $\mathbf{x}_k$ in the current broadcast time interval. Before transmission, each signal vector $x_k$ is processed by its corresponding precoder matrix $\mathbf{F}_k$. $N$ precoded user signals are transmitted simultaneous over the $N_t$ antennas as vector $\sum_{i=1}^{N} \mathbf{F}_i \mathbf{x}_i$. At a given receiver of user $k$ with MIMO channel matrix $\mathbf{H}_k$, the received baseband signal is

$$\mathbf{y}_k = \mathbf{H}_k \sum_{i=1}^{N} \mathbf{F}_i \mathbf{x}_i + \mathbf{v}_k \tag{1}$$

where $\mathbf{v}_k$ is the channel noise vector of reciever $k$ with independent identically distributed (i.i.d) circular complex Gaussian entries of zero mean and variance $\sigma^2$. We assume that the transmitter has perfect knowledge of all user channels $\{\mathbf{H}_k\}$.

Our objective is to find a group of linear precoders $\{\mathbf{F}_k\}$ to maximize system capacity. To be fair, we require sum power constraint $\sum_{k=1}^{N} \mathbf{F}_k \mathbf{F}_k^H \leq P$. We note that the capacity for the $k$th user is $C_k = \log\,\det\{\mathbf{I} + \mathbf{H}_k \mathbf{F}_k \mathbf{R}_k^{-1} \mathbf{F}_k^H \mathbf{H}_k^H\}$ where the (interference plus noise) covariance matrix is

$$\mathbf{R}_k = \sigma^2 \mathbf{I} + \sum_{i \neq k, i=1}^{N} \mathbf{H}_k \mathbf{F}_i \mathbf{F}_i^H \mathbf{H}_k^H \tag{2}$$

Define $\eta_{ki}$ as the power limit of the $i-$th interference received by the $k-$th user. Then, the user capacity can be bounded by

$$\begin{aligned} C_k &= \log\,\det\{\mathbf{I} + \mathbf{H}_k \mathbf{F}_k \mathbf{R}_k^{-1} \mathbf{F}^H \mathbf{H}_k^H\} \\ &\geq \log\det\left\{\mathbf{I} + \frac{\mathbf{H}_k \mathbf{F}_k \mathbf{F}_k^H \mathbf{H}_k^H}{(\sigma^2 + \mathrm{Tr}(\sum_{i\neq k, i=1}^{N} \mathbf{H}_k \mathbf{F}_i \mathbf{F}_i^H \mathbf{H}_k^H)}\right\} \\ &\geq \log\det\left\{\mathbf{I} + \frac{\mathbf{H}_k \mathbf{F}_k \mathbf{F}_k^H \mathbf{H}_k^H}{\sigma^2 + \sum_{i\neq k, i=1}^{N} \eta_{ki}}\right\}. \end{aligned}$$

We can now define a tight upperbound of the $k-$th user capacity as $\tilde{C}_k = \log\det\left\{\mathbf{I} + \frac{\mathbf{H}_k \mathbf{F}_k \mathbf{F}_k^H \mathbf{H}_k^H}{\sigma^2 + \sum_{i\neq k, i=1}^{N} \eta_{ki}}\right\}$. We will design precoders $\{\mathbf{F}_i\}$ to maximize the sum of the capacity bounds $\sum_i \tilde{C}_i$ in order to achieve high MIMO multiuser throughput.

Note that maximizing the sum capacity may still lead to some poor user performance despite the high sum capacity. For quality of service fairness, our alternative design strategy is to maximize the minimum user capacity $\min_i \tilde{C}_i$. Both the sum capacit maximization and max-min capacity designs will be discussed in our work.

## 3. CAPACITY MAXIMIZING PRECODING

As explained in the introduction, our precoder design must consider the risk of eavesdropper by unintended users who can first decode their own signals before applying signal cancellation to extract the undesired signal. Thus, our design of linear precoders must satisfy predetermined power leakage constraints which can serve to address both QoS and security concerns. For $N$ users, there are $N(N-1)$ power leakage constraints

$$\mathrm{Tr}\{\mathbf{H}_k \mathbf{F}_i \mathbf{F}_i^H \mathbf{H}_k^H\} \leq \eta_{ki},\ k \neq i.$$

These $N(N-1)$ constraints are linear with respect to unknown matrix products $\mathbf{F}_i \mathbf{F}_i^H$.

### 3.1. Maximizing the Sum Capacity

Define $\mathbf{A}_i = \mathbf{F}_i \mathbf{F}_i^H$. First, we present a linear precoder that maximizes the sum capacity bound. Because $-\log\det$ is a convex function, $\tilde{C}_k$ is a concave function with respect to matrix $\mathbf{A}_k$. Thus, we can summarize our precoder design problem for maximum sum capacity as the follow convex program:

$$\begin{aligned} \min \quad & -\sum_{i=1}^{N} \tilde{C}_k \\ \text{s.t.} \quad & \sum_{i=1} \mathrm{Tr}(\mathbf{A}_k) \leq P \\ & \mathbf{A}_k \succeq 0 \ \ (1 \leq k \leq N) \\ & \mathrm{Tr}\{\mathbf{H}_k \mathbf{A}_i \mathbf{H}_k^H\} \leq \eta_{ki} \ \ (k \neq i, 1 \leq k, i \leq N) \end{aligned}$$

where $\mathbf{A}_k \succeq 0$ denotes the positive semidefinite constraint.

Note the constraint $\mathbf{A}_k \succeq 0$ is not differentiable. Also note that, different from the standard SDP [4], the objective function in our problem is nonlinear. We proposed to solve this optimization problem by using a sequential Semi-Definite Programming (SDP) method. Our method is inspired by the sequential linear programming, which solves a nonlinear convex program by solving a sequence of linear programs with increasing complexity. We describe the solution as follows.

Let $\tilde{\mathbf{A}} = [\mathbf{A}_1, \mathbf{A}_2, \cdots, \mathbf{A}_N]$ . We use $f(\tilde{\mathbf{A}})$ to denote the objective function,

$$f(\tilde{\mathbf{A}}) = -\sum_{i=1}^{N} \tilde{C}_k.$$

By introducing a slack variable $t$, the optimization problem can formulated as

$$\begin{aligned} \min \quad & t \\ \text{s.t.} \quad & f(\tilde{\mathbf{A}}) \leq t \\ & \sum_{i=1} \mathrm{Tr}(\mathbf{A}_k) \leq P \\ & \mathbf{A}_k \succeq 0 \ \ (1 \leq k \leq N) \\ & \mathrm{Tr}\{\mathbf{H}_k \mathbf{A}_i \mathbf{H}_k^H\} \leq \eta_{ki} \ \ (k \neq i, 1 \leq k, i \leq N) \end{aligned}$$

Since $f$ is convex and differentiable, we have

$$f = \sup_{\tilde{\mathbf{A}}_k \in \Omega} \nabla f(\tilde{\mathbf{A}}_k)^T \mathrm{vec}(\tilde{\mathbf{A}} - \tilde{\mathbf{A}}_k) + f(\tilde{\mathbf{A}}_k), \qquad (3)$$

where $\Omega$ describes the feasible domain of the original optimization problem. Define $g_k(\tilde{\mathbf{A}})$ as

$$g_k(\tilde{\mathbf{A}}) = \nabla f(\tilde{\mathbf{A}}_k)^T \mathrm{vec}(\tilde{\mathbf{A}} - \tilde{\mathbf{A}}_k) + f(\tilde{\mathbf{A}}_k).$$

We can obtain the gradient of $f$ as

$$\nabla f = \mathrm{vec}\left[ \frac{(\mathbf{H}_1^H \mathbf{B}_1 \mathbf{H}_1)^T}{\sigma^2 + \sum_{i \neq 1, i=2}^{N} \eta_{1i}}, \ \cdots, \ \frac{(\mathbf{H}_N^H \mathbf{B}_N \mathbf{H}_N)^T}{\sigma^2 + \sum_{i \neq N, i=1}^{N} \eta_{Ni}} \right]$$

where $\mathbf{B}_k$ denotes

$$\mathbf{B}_k = \left( \mathbf{I} + \frac{\mathbf{H}_k \mathbf{A}_k \mathbf{H}_k^H}{(\sigma^2 + \sum_{i \neq k, i=1}^{N} \eta_{ki})} \right)^{-1}.$$

Therefore, we can approximate the constraint $f(\tilde{\mathbf{A}}) \leq t$ with a group of linear constraints $g_k(\tilde{\mathbf{A}}) \leq t$. This sequential SDP method can be implemented to find the optimum precoder matrices $\{\mathbf{F}_k\}$. We summarize the algorithm as follows

1. Set iteration index $l \leftarrow 0$ and a small positive number $\epsilon$ as threshold

2. Set initial solutions to $\tilde{\mathbf{A}}_0^* \leftarrow \frac{P}{N_t N}[\mathbf{I}, \cdots, \mathbf{I}]$.

3. Solve the following linear SDP problem

$$\begin{array}{ll} \min & t \\ \text{s.t.} & \mathbf{A}_k \geq 0 \ \ (1 \leq k \leq N) \\ & g_k(\tilde{\mathbf{A}}) \leq t \ \ (0 \leq k \leq l) \\ & \sum_{i=1}^{N} \mathrm{Tr}\left(\mathbf{A}_k\right) \leq P \\ & \mathrm{Tr}\{\mathbf{H}_k \mathbf{A}_i \mathbf{H}_k^H\} \leq \eta_{ki} \ \ (k \neq i, 1 \leq k, i \leq N) \end{array}$$

   Denote $t_{\min}$ as the minimum and $\tilde{\mathbf{A}}_{\min}$ as the solution.

4. If $f(\tilde{\mathbf{A}}_{\min}) - t_{\min} \leq \epsilon$, output $\tilde{\mathbf{A}}_{\min}$ as the optimum solution.

5. Otherwise, let $\tilde{\mathbf{A}}_{l+1} \leftarrow \tilde{\mathbf{A}}_{\min}$; form a new constraint

$$g_{l+1}(\tilde{\mathbf{A}}) = \nabla f(\tilde{\mathbf{A}}_{l+1})(\tilde{\mathbf{A}} - \tilde{\mathbf{A}}_{l+1}) + f(\tilde{\mathbf{A}}_{l+1}) \leq t,$$

   and let $l \leftarrow l + 1$, go back to step 3.

Note that $g_k \leq t$ serves as a relaxation of the convex constraints $f \leq t$. Thus, one always has $t \leq f(\tilde{\mathbf{F}}_{\mathrm{opt}})$.

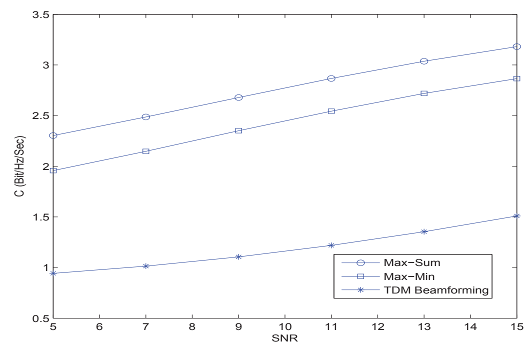


**Fig. 1**. One user has average SNR 15 dB, while the other has varying SNR. Power leakage constraint is $-10$ dB below the peak power

### 3.2. Maximizing minimum Capacity for Fairness

Alternatively, we develop a linear precoder to maximize the minimum capacity of $N$ users for QoS fairness. This objective function is a more conservative. By introducing a slack variable $t$, we can rearrange the problem as

$$\begin{array}{ll} \max & t \\ \text{s.t.} & -\tilde{C}_k \leq \ -t \\ & \sum_{i=1} \mathrm{Tr}(\mathbf{A}_k) \leq P \\ & \mathbf{A}_k \succeq 0 \ \ (1 \leq k \leq N) \\ & \mathrm{Tr}\{\mathbf{H}_k \mathbf{F}_i \mathbf{F}_i^H \mathbf{H}_k^H\} \leq \eta_{ki} \ \forall k \neq i, 1 \leq k, i \leq N \end{array}$$

$\tilde{C}_k$'s are concave, thus, $-\tilde{C}_k$'s are convex. As in the previous subsection, we approximate each $-\tilde{C}_k \leq \ -t$ with a group of linear constraints, and solve a sequence of linear SDP programs with increasing complexity. The details are similar to the sum-capacity algorithm and are omitted here.

## 4. SIMULATIONS

In our simulation, each element of the channel matrix is a zero-mean unit variance complex Gaussian random variable. Our benchmark is a TDM system using optimal beamformers for both users. The optimal beamformer of user $k$ is the eigenvector corresponding to the largest eigenvalue of $\mathbf{H}_k^H \mathbf{H}_k$. The transmission power is adjusted to satisfy the power leakage constraints.

We first test the sum capacity performance for a $2 \times 2$ BC under different SNR levels. In Figure 1, we show the capacity gain of the proposed method over conventional TDM. The average power of one user is set as 15 dB, while the other user SNR varies from 5 to 20 dB. For both TDM and our precoder transmissions, the power leakage constraint is set at 10 dB below the peak transmission power. The MIMO precoders maximizing both the sum capacity and the minimum capacity are shown to almost double the overall system capacity.

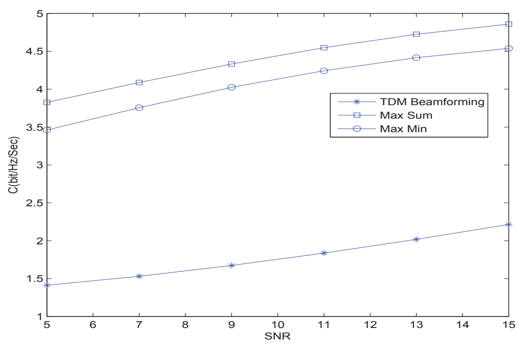


**Fig. 2**. One user has fix average SNR 15 dB, while the other varying receiving SNR. Power leakage constraints are $-10$ dB w.r.t. peak power

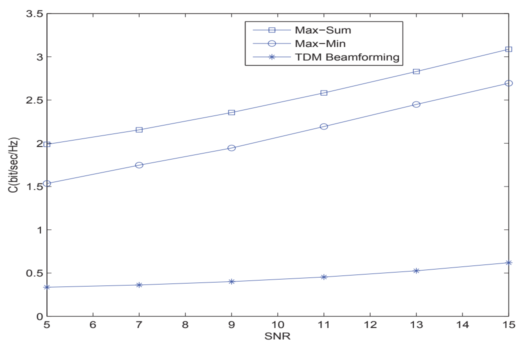


**Fig. 3**. One user has fix average SNR 15 dB, while the other varying receiving SNR. Power leakage constraints are $-20$ dB w.r.t. peak power

In our second set of simulations, we test $3 \times 3$ random MIMO channels. Again, the SNR is fixed for user one at 15 dB while the other user has varying SNR from 5 to 15 dB. The power leakage constraints are chosen to be 10 dB below the peak transmission power. It is clear from Figure 2 that our precoding transmissions designed for either maximum sum capacity or max-min capacity can nearly doubles the system performance in terms of sum capacity.

We also test the effect of power-leakage parameter on the MIMO system. Using a more conservative (tight) power leakage constraint at 20 dB below the peak transmission power, the same $3 \times 3$ MIMO system performance is shown in Figure 3. It is clear from Figure 3 that tighter leakage constraints can reduce the overall system capacity. This is fully expected as tighter eavesdropping concerns limit the transmit precoder's ability to deliver higher user capacity.

Finally, we illustrate the effect of the interference power leakage constraint in simulations. We consider a two user $3 \times 3$ MIMO system and test a number of random channels. The power constraint is chosen to be $-20$ dB below the peak power. Both users have SNR of 15 dB. Our precoder design can successfully enforce the leakage constraint. However, we also apply a standard linear precoder that neglects this constraint to simply maximize the system capacity. Testing 1000 random channel realizations, our results in Table 1 show the number of empirical cases in which the resulting MIMO system violates the leakage constraint. We use the first row to indicates the level of power leakage over the constraint in dB, while the second row provides the number of corresponding channels. This result clearly shows that traditional precoder designs without considering the inference leakage are prone to eavesdropping.

Table 1:

| 0-5 | 5-10 | 10-15 | 15-20 | 20-25 | 25-30 |
|---|---|---|---|---|---|
| 12 | 18 | 20 | 92 | 246 | 156 |

## 5. CONCLUSION

In this work, we study the design of linear transmission precoder in the multiuser BC systems. Our objective is to maximize user QoS while limiting the risk of eavesdropping by enforcing power leakage constraints. We develop efficient numerical algorithms to design precoders within the convex programming framework. Our results show significant performance gain over the conventional TDM.

## 6. REFERENCES


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