WEIGHTED HARMONIC MEAN SINR MAXIMIZATION FOR THE MIMO DOWNLINK

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ABSTRACT

Leakage-based methods which are based on the harmonic mean signal-to-interference ratio (SIR) maximization deliver a good trade-off between the sum rate and bit-error rate (BER). In this paper, we present the joint design of linear transmit and receive beamformers that maximize the weighted harmonic mean signal-to-interference-and-noise ratio (SINR) for the downlink of the multiuser MIMO channel. By exploiting the uplink-downlink SINR duality, the non-convex optimization problem is transformed into a series of simpler problems which can be solved by geometric programming. When the minimum mean square error (MMSE) receive beamformers are employed, it is shown that the harmonic mean SINR maximization becomes a close approximation to the sum MSE minimization problem in the high SINR region. The simulations show that the proposed scheme outperforms other leakage-based schemes in terms of harmonic mean SINR, sum MSE, BER and sum rate.

Index Terms— MIMO systems, broadcast channels, uplink-downlink duality, convex optimization, geometric programming.

1. INTRODUCTION

Recently, significant attention has been drawn towards leakage-based schemes that maximize the signal-to-leakage ratio (SLR) [1], signal-to-leakage-and-noise ratio (SLNR) [2] and signal-to-jamming-and-noise ratio (SJNR) [3]. The SLR maximization method designs the transmit beamforming vector in a way that the substream's signal power is maximized compared to its crosstalk power. SLNR and SJNR maximization schemes on the other hand, include the noise power into the denominator of the SLR during the maximization process. This process can be interpreted as adding onto the leakage channel correlation matrix an identity matrix which is scaled by the noise power, which can also be perceived as a form of diagonal loading or regularization, commonly practiced in robust beamforming methods [4]. A thorough literature survey shows that the leakage-based scheme can be traced back to the paper by Gerlach and Paulraj [5] where they looked into the harmonic mean signal-to-interference ratio (SIR) maximization, which is a problem involving the minimization of the sum of the inverse SIRs. The solution involves choosing the transmit beamformer as the generalized eigenvector of the signal's and leakage's channel correlation matrices that corresponds to the largest eigenvalue, which in turn maximizes the signal-to-leakage ratio. Diagonal loading was later proposed to reduce the amount of feedback and estimation needed, whereby the scaled identity matrix is approximated by the channel correlation matrices of the unidentified mobiles in nearby cells [4]. Gerlach's scheme can so far be understood as a subspace beamformer, which performs beamforming in a time-average fashion by relying on the long-term channel state information (CSI). For better performance but at the cost of an increased feedback, Gerlach's subspace beamformer can be directly adapted to work with the instantaneous CSI instead. These leakage-based schemes that perform beamforming in an instantaneous sense were based on the SJNR, SLR and SLNR criteria and were introduced by Wu, Tarighat and Sadek respectively. These schemes were later improved using an iterative algorithm to jointly design the transmit and receive beamformers such that the leakage-based criterion is maximized at the receiver beamformer outputs [6]. Extensions to support multiple substreams per user were also proposed in [7] and [2].

In our previous work [7], we pointed out that the SLR approach suffers from a degraded performance when the number of transmit antennas exceeds the sum of all the adjacent users' receive antennas. Under this condition, the instantaneous leakage channel correlation matrix becomes singular, resulting in multiple infinity-approaching eigenvalues. Although the generalized eigenvectors corresponding to these infinity-approaching eigenvalues would provide zero-leakage for the substream, choosing anyone of them randomly may result in a loss of performance because not all of them would maximize the substream's signal power. One of the solutions would be to carry out block diagonalization [8], while the other solution would be to avoid the occurrence of the infinity-approaching eigenvalues via diagonal loading. Generally, the SJNR and SLNR approaches that apply diagonal loading do not work particularly well in the high signal-to-noise ratio (SNR) region. However, SLR maximization (as the name suggests) does not involve noise reduction and thus has degraded performance in the low SNR region. These three schemes also assume an equal power allocation among all the substreams. Since these schemes optimize each user's ratio separately, the task of allocating power centrally is proven as a challenging task as the transmit beamformers are not necessarily orthogonal to each other.

In this paper, we explore the joint design of linear transmit and receive beamformers through the maximization of the weighted harmonic mean SINR given the instantaneous CSI. Instead of following Gerlach's approach in maximizing the harmonic mean SIR, we substituted the SINR for the SIR to consider the suppression of noise, critical in the low-SNR region. A weighted version of the criterion is introduced to distinguish between the different priorities assigned onto each substream. The proposed scheme is different from all of the previously mentioned schemes in the sense that power is allocated under the sum power constraint. Furthermore, it is well suited for a single-cell multiuser MIMO channel which supports an arbitrary number of substreams per scheduled user. We follow the general algorithm proposed by Codreanu *et al.* [9] (see Section III) but instead we have adapted the optimality criterion to maximize the weighted harmonic mean SINR. The algorithm exploits the uplink-downlink SINR duality to decompose the original non-convex optimization problem into a series of simpler optimization problems, which is then solvable via standard convex optimization tools.

2. SYSTEM MODEL

Consider a multiuser MIMO downlink system where the base station (BS), equipped with N transmit antennas, is communicating to K users, with M_k receive antennas at the kth user. The transmit antennas and the kth user's receive antennas are related through a flat fading channel, denoted by an $M_k \times N$ channel matrix $\mathbf{H}_k$, where its (m,n)-th element is a realisation of the random channel between the nth transmit and the mth receive antenna. We assume that all the channel matrices are perfectly known at the BS and we define the overall system model as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{s} + \mathbf{n}_k \tag{1}$$

where $\mathbf{y}_k \in \mathbb{C}^{M_k \times 1}$ is the received signal at the kth user, $\mathbf{s} \in \mathbb{C}^{N \times 1}$ is the transmitted signal and $\mathbf{n}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ models the additive noise experienced by the kth receiver, with $\mathcal{CN}(\mu, \mathbf{R})$ signifying a multivariate, complex, circular symmetric, Gaussian distribution with mean vector μ and covariance $\mathbf{R}$. Whitened noise with unity variance can be seen as a reasonable assumption since the colored noise component can be prefiltered, by pre-multiplying the channel matrix and the noise vector by a whitening matrix. The proposed system is capable of transmitting $S \leq N$ substreams and supports up to M_k substreams per kth user. Thus, $\mathbf{s}$ can be described as the superposition of the transmitted signals corresponding to these substreams, given by

$$\mathbf{s} = \mathbf{V} \mathrm{diag}(\mathbf{p})^{\frac{1}{2}} \mathbf{d} \tag{2}$$

where $\mathrm{diag}(\mathbf{x})$ is a diagonal matrix with the elements of $\mathbf{x}$ along its main diagonal, $\mathbf{V} = [\mathbf{v}_1, \ldots, \mathbf{v}_S] \in \mathbb{C}^{N \times S}$ is the normalized transmit beamforming matrix, $\mathbf{p} = [p_1, \ldots, p_S]^{\mathrm{T}}$ determines the downlink power allocation and $\mathbf{d} = [d_1, \ldots, d_S]^{\mathrm{T}} \in \mathbb{C}^{S \times 1}$ represents the data vector. We assume that the data vector and the transmit beamforming matrix are normalized such that $\mathrm{E}\{\mathbf{d}\mathbf{d}^{\mathrm{H}}\} = \mathbf{I}$ and $\|\mathbf{v}_s\| = 1$ for $s = 1, \ldots, S$ respectively. Let k_s denote the user index associated with the sth substream and let the receive beamformer of the sth substream, $\mathbf{u}_s \in \mathbb{C}^{M_{k_s} \times 1}, \|\mathbf{u}_s\| = 1, s = 1, \ldots, S$ be defined as the normalized minimum mean square error (MMSE) receiver given by

$$\mathbf{u}_s = \frac{\tilde{\mathbf{u}}_s}{\|\tilde{\mathbf{u}}_s\|}, \quad \tilde{\mathbf{u}}_s = \left(\sum_{j=1}^{S} p_j \mathbf{H}_{k_s} \mathbf{v}_j \mathbf{v}_j^{\mathrm{H}} \mathbf{H}_{k_s}^{\mathrm{H}} + \mathbf{I} \right)^{-1} \mathbf{H}_{k_s} \mathbf{v}_s p_s. \tag{3}$$

The data from substream s can subsequently be decoded by applying the normalized receive beamformer $\mathbf{u}_s$ on the received signal $\mathbf{y}_{k_s}$, i.e. $\hat{d}_s = \mathbf{u}_s^{\mathrm{H}} \mathbf{y}_{k_s}$. Thus, the downlink SINR of the sth substream can be shown as

$$\gamma_s^{\mathrm{dl}} = \frac{p_s |\mathbf{u}_s^{\mathrm{H}} \mathbf{H}_{k_s} \mathbf{v}_s|^2}{1 + \sum_{j=1, j \neq s}^{S} p_j |\mathbf{u}_s^{\mathrm{H}} \mathbf{H}_{k_s} \mathbf{v}_j|^2}. \tag{4}$$

By reversing the direction of transmission in the downlink channel, we could obtain a dual uplink channel where $\mathbf{u}_s$ becomes the k_sth user's transmit beamformer and $\mathbf{v}_s$ is the corresponding substream's receive beamformer at the BS. The uplink-downlink duality theorem shows that the same sets of SINRs achieved in the downlink can also be similarly achieved in the virtual uplink, as long as the sum of the powers are equal in both cases. Hence, by assigning the uplink power allocation vector as $\mathbf{q} = [q_1, \ldots, q_S]^{\mathrm{T}} \in \mathbb{C}^{S \times 1}$, the sth substream's uplink SINR can be expressed by

$$\gamma_s^{\mathrm{ul}} = \frac{q_s |\mathbf{v}_s^{\mathrm{H}} \mathbf{H}_{k_s} \mathbf{u}_s|^2}{1 + \sum_{j=1, j \neq s}^{S} q_j |\mathbf{v}_s^{\mathrm{H}} \mathbf{H}_{k_j} \mathbf{u}_j|^2}. \tag{5}$$

The normalized MMSE receiver for the sth substream of the dual uplink becomes

$$\mathbf{v}_s = \frac{\tilde{\mathbf{v}}_s}{\|\tilde{\mathbf{v}}_s\|}, \quad \tilde{\mathbf{v}}_s = \left(\sum_{j=1}^{S} q_j \mathbf{H}_{k_j} \mathbf{u}_j \mathbf{u}_j^{\mathrm{H}} \mathbf{H}_{k_j}^{\mathrm{H}} + \mathbf{I} \right)^{-1} \mathbf{H}_{k_s}^{\mathrm{H}} \mathbf{u}_s q_s. \tag{6}$$

3. GENERAL ITERATIVE OPTIMIZATION ALGORITHM

The joint beamformer and power allocation scheme proposed in [9] employs the uplink-downlink duality principle to break down the non-convex beamforming problem into a series of simpler power allocation problems which can be solved by using standard convex optimization tools. The algorithm proposed in [9] is briefly summarized below.

1. Initialize the power allocation $\mathbf{p}^{(0)} = \mathbf{1} P_{\mathrm{T}}/S$ and the transmit beamformers $\{\mathbf{v}_1^{(0)}, \ldots, \mathbf{v}_S^{(0)}\}$ for the downlink channel. Calculate the resulting receive beamformers $\{\mathbf{u}_1^{(0)}, \ldots, \mathbf{u}_S^{(0)}\}$ from (3) and downlink SINRs $\gamma_{\mathrm{dl}}^{(0)} = [\gamma_1^{\mathrm{dl}}, \ldots, \gamma_S^{\mathrm{dl}}]$ from (4). Let $i = 0$.
2. Convert to the dual uplink channel where $\{\mathbf{u}_1^{(i)}, \ldots, \mathbf{u}_S^{(i)}\}$ and $\{\mathbf{v}_1^{(i)}, \ldots, \mathbf{v}_S^{(i)}\}$ turn out to be the new transmit and receive beamformers respectively. Determine the uplink power allocation $\mathbf{q}^{(i+1)}$ that solves a particular uplink optimization problem:
$$\mathbf{q}^{(i+1)} = f_{\mathrm{ul}}(\{\mathbf{u}_s^{(i)}, \mathbf{v}_s^{(i)}\}_{s=1,\ldots,S}). \tag{7}$$
Calculate the corresponding receive beamformers $\{\mathbf{v}_1^{(i+1)}, \ldots, \mathbf{v}_S^{(i+1)}\}$ and uplink SINRs $\gamma_{\mathrm{ul}}^{(i+1)} = [\gamma_1^{\mathrm{ul}}, \ldots, \gamma_S^{\mathrm{ul}}]$ from (6) and (5) respectively.
3. Switch back to the downlink channel again where $\{\mathbf{v}_1^{(i+1)}, \ldots, \mathbf{v}_S^{(i+1)}\}$ and $\{\mathbf{u}_1^{(i)}, \ldots, \mathbf{u}_S^{(i)}\}$ become the new transmit and receive beamformers respectively. Solve the downlink optimization problem and update the downlink power allocation vectors $\mathbf{p}^{(i+1)}$:
$$\mathbf{p}^{(i+1)} = f_{\mathrm{dl}}(\{\mathbf{v}_s^{(i+1)}, \mathbf{u}_s^{(i)}\}_{s=1,\ldots,S}). \tag{8}$$
Compute the corresponding receive beamformers $\{\mathbf{u}_1^{(i+1)}, \ldots, \mathbf{u}_S^{(i+1)}\}$ and downlink SINRs $\gamma_{\mathrm{dl}}^{(i+1)} = [\gamma_1^{\mathrm{dl}}, \ldots, \gamma_S^{\mathrm{dl}}]$ from (3) and (4) respectively.
4. Test a stopping criterion. Terminate the algorithm if the criterion is satisfied. Otherwise, increment i by one, i.e. $i = i + 1$ and jump back to step (2).

The optimization problems (described by $f_{\mathrm{dl}}(\cdot)$ and $f_{\mathrm{ul}}(\cdot)$) can be designed to either maximize or minimize a certain performance criterion. Subject to the same sum-power constraint, the uplink-downlink duality allows the same set of SINRs to be achieved during the conversion to its dual channel. Therefore, by designing the optimization criterion for the maximization (minimization) problem to be a non-decreasing (non-increasing) function of the SINRs, it is obvious that the performance criterion will not decrease (increase) when switching between the channels. Since the normalized MMSE receiver is shown to be the optimal receiver that maximizes the SINRs, we can say that the performance criterion will improve further and this would ensure that the optimization criterion is monotonically increasing (decreasing) for a maximization (minimization) problem. In the next section, we look into the formulation of $f_{\mathrm{dl}}(\cdot)$

and $f_{\mathrm{ul}}(\cdot)$ to allow the power allocation vectors $\mathbf{p}^{(i)}$ and $\mathbf{q}^{(i)}$ to maximize the weighted harmonic mean SINR in the downlink and dual uplink channels.

4. WEIGHTED HARMONIC MEAN SINR MAXIMIZATION

The weighted harmonic mean SINR maximization is also equivalent to the minimization of the inverse SINRs' weighted sum. Hence, by defining $\mathbf{x}$, $g_{s,j}$ and γ_s, $s, j = 1, \ldots, S$ (below) differently for the downlink and uplink, it can be easily shown that $f_{\mathrm{dl}}(\cdot)$ and $f_{\mathrm{ul}}(\cdot)$ can be represented by the following power allocation problem:

$$\begin{aligned} \text{minimize} \quad & \sum_{s=1}^{S} w_s \frac{1+\sum_{j=1, j\neq s}^{S} x_j g_{s,j}}{x_s g_{s,s}} \\ \text{subject to} \quad & \sum_{s=1}^{S} w_s \frac{1+\sum_{j=1, j\neq s}^{S} x_j g_{s,j}}{x_s g_{s,s}} \leq \sum_{s=1}^{S} w_s \gamma_s^{-1} \\ & \mathbf{1}^{\mathrm{T}}\mathbf{x} \leq P_{\mathrm{T}} \\ & \mathbf{x} > \mathbf{0} \end{aligned} \tag{9}$$

where $\mathbf{x} = [x_1, \ldots, x_S]^{\mathrm{T}}$ is the vector consisting of optimization variables and $\mathbf{w} = [w_1, \ldots, w_S]^{\mathrm{T}}$, $\mathbf{w} \geq \mathbf{0}$ is the weight vector that determines the priority of each substream, which may depend on the QoS requirements and traffic conditions in the system. For solving $f_{\mathrm{ul}}(\cdot)$, we define $\mathbf{x}$, γ_s and $g_{s,j}$ as $\mathbf{q}^{(i)}$, $\gamma_{\mathrm{ul}}^{(i)}$ and $|\mathbf{v}_s^{(i)\mathrm{H}}\mathbf{H}_{k_j}^{\mathrm{H}}\mathbf{u}_j^{(i)}|^2$ respectively. On the other hand, we define $\mathbf{x}$, γ_s and $g_{s,j}$ respectively as $\mathbf{p}^{(i)}$, $\gamma_{\mathrm{dl}}^{(i)}$ and $|\mathbf{u}_s^{(i)\mathrm{H}}\mathbf{H}_{k_s}^{\mathrm{H}}\mathbf{v}_j^{(i+1)}|^2$ for solving $f_{\mathrm{dl}}(\cdot)$. By rewriting (9), the problem can be transformed into a geometric program (GP) whereby the objective and constraint functions consist of posynomial functions, appearing in the form $f(\mathbf{x}) = \sum_{j=1}^{m} c_j x_1^{a_{1,j}} x_2^{a_{2,j}} \ldots x_n^{a_{n,j}}$ where $c_j > 0, j = 1, \ldots, m$, i.e.

$$\begin{aligned} \text{minimize} \quad & \sum_{s=1}^{S} \left(w_s x_s^{-1} g_{s,s}^{-1} + w_s x_s^{-1} g_{s,s}^{-1} \textstyle\sum_{j=1, j\neq s}^{S} x_j g_{s,j} \right) \\ \text{subject to} \quad & \mathbf{1}^{\mathrm{T}}\mathbf{x} \leq P_{\mathrm{T}}. \end{aligned} \tag{10}$$

The first inequality constraint in (9) can be removed since it has been guaranteed by the uplink-downlink duality. Secondly, in GPs, the optimization variables are implicitly positive, i.e. $\mathbf{x} > \mathbf{0}$. Hence the final constraint in (9) can also be removed completely. Although non-convex in nature, GPs can be transformed to convex optimization problems, by a change of variables and a transformation of the objective and constraint functions, as shown in [10]. GPs are also directly supported by standard convex optimization tools such as CVX [11], which can be implemented in Matlab to allow disciplined convex programs to be analyzed and solved efficiently.

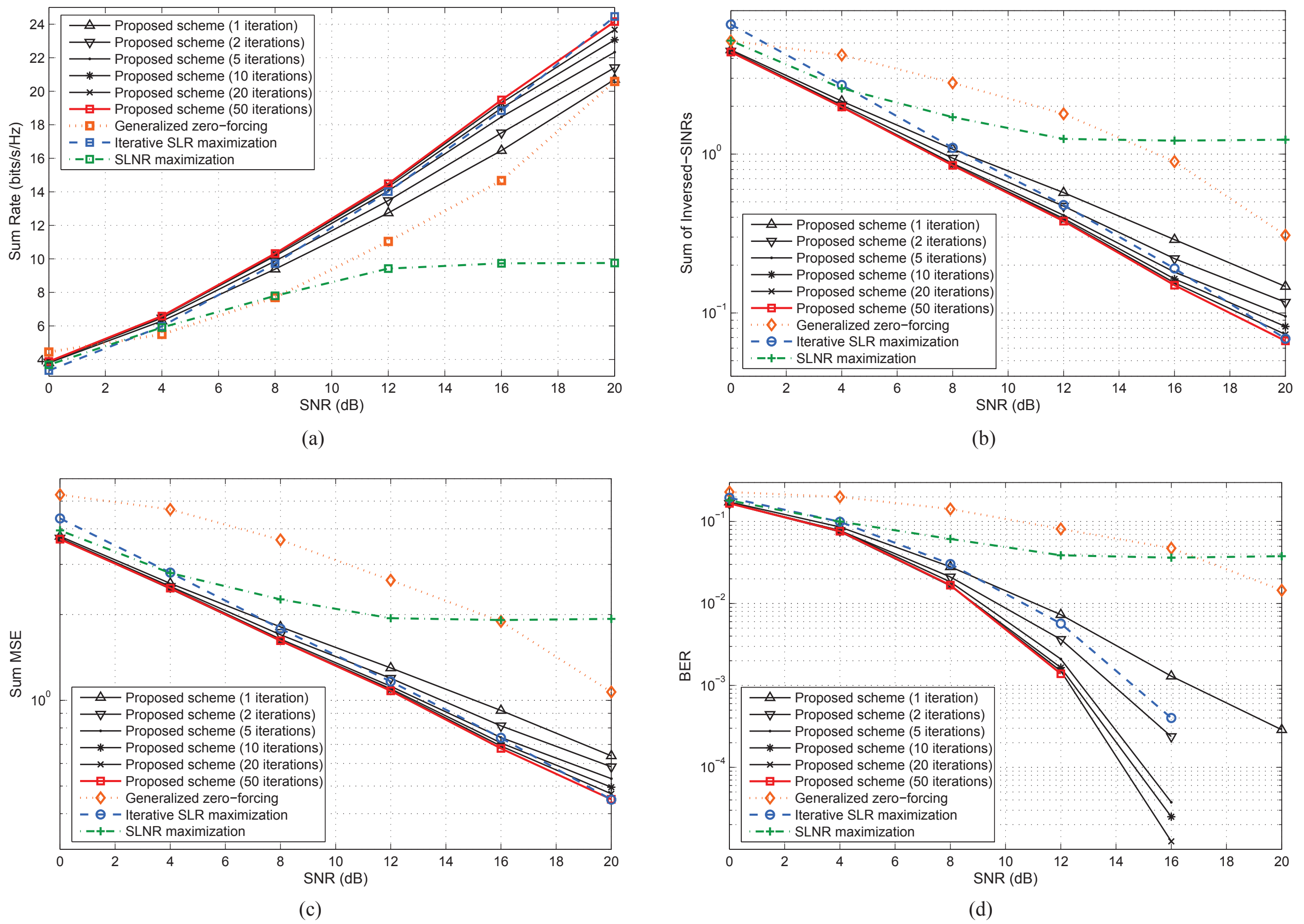
It has been pointed out in [9] and [12] that the weighted sum MSE at the output of the optimum MMSE receiver is given by $\sum_{s=1}^{S} w_s(1+\gamma_s)^{-1}$. This shows that at high SINRs, the sum MSE can be approximated by the sum of the weighted inverse SINRs, $\sum_{s=1}^{S} w_s \gamma_s^{-1}$. This explains why the leakage-based schemes, that are based on harmonic mean SIR maximization, actually achieve a decent performance when paired up with a MMSE receiver. In addition, we also want to mention that while Codreanu *et al.* [9] only formulated the weighted sum MSE minimization problem subject to individual SINR constraints, we instead relaxed both the objective function and constraints to support the weighted sum of the inverse SINRs.

5. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed system under a frequency flat Rayleigh fading channel with similar noise variance distributed on each receive antenna. We assume QPSK modulation throughout the simulations and let $[N, M_1(s_1), \ldots, M_K(s_K)]$ denote a K-user multiuser MIMO system with N transmit antennas at the BS and M_k receive antennas at the kth user who supports s_k substreams. Moreover, the kth user's transmit beamformers are initialized as the right singular vectors corresponding to the s_k largest singular values of $\mathbf{H}_k$. In Fig. 1, we compare the performance of the proposed method against the generalized zero-forcing (GZF) scheme with waterfilling power allocation [8], the iterative SLR maximization scheme [7] and the SLNR maximization scheme proposed in [2]. These schemes are simulated under the $[4, 2(1), 2(1), 2(1), 2(1)]$ system for the following performance criteria: sum rate, harmonic mean SINR, sum MSE and BER. First, we consider an equal allocation of weights across the substreams, i.e. $\mathbf{w} = [1, 1, 1, 1]^{\mathrm{T}}$, and we simulate the proposed scheme under 1, 2, 5, 10, 20 and 50 iterations respectively. Fig. 1(a) shows that the sum-rate of the proposed scheme improves as the number of iterations is increased. The improvements are quite rapid during the first few iterations but it eventually slows down and converges when it is close to 50 iterations. The same observations can also be made for the harmonic mean of the SINRs, sum MSE and BER. In Figs. 1(a), 1(b) and 1(c) the iterative SLR maximization scheme draws near the proposed scheme in the high SNR region. However, the gap between them starts to widen in the low SNR region due to the fact that the SLR maximization scheme is based only on the maximization of the harmonic mean SIRs, and does not take consideration of the noise component. The differences between these two schemes become more pronounced in terms of the BER (Fig. 1(d)). At BERs below 10^{-2}, the proposed scheme is shown to achieve an SNR gain of above 2dBs. The SLNR maximization scheme on the other hand, exhibits a lower inversed-SINR sum (Fig. 1(b)), sum MSE (Fig. 1(c)) and BER during low SNRs as compared to GZF and SLR maximization. Unfortunately, it reveals a quick saturation in the high SNR region and is soon overtaken by its counterparts. The GZF, which is optimized for sum rate, generally does not perform as well as the rest of the schemes, in terms of harmonic mean SINR, sum MSE and BER. It is worth pointing out that the close similarities between Fig. 1(b) and Fig. 1(c) indicate that the harmonic mean SINR well approximates the sum MSE. The proposed algorithm does not guarantee a global optimum solution although each power allocation problem is solved optimally, due to the non-convexity of the original problem.

6. CONCLUSION

In this paper, we first pointed out that leakage-based schemes were based upon the harmonic mean SIR maximization. We then investigated the weighed harmonic mean SINR maximization, by employing an iterative technique to jointly design the transmit and receive beamformers. This technique employs the MMSE receiver and exploits the uplink-downlink duality theorem to break down the non-convex problem into a series of smaller power allocation problems which can be efficiently solved using standard convex optimization tools. The simulations have shown that the harmonic mean SINRs well approximate the sum MSE and the proposed scheme outperforms other leakage-based schemes in terms of harmonic mean SINRs, sum MSE, BER and sum rate.



(a) (b) (c) (d)

Fig. 1. Performance comparison of the proposed method with GZF, SLR-maximization and SLNR-maximization schemes for a $[4, 2(1), 2(1), 2(1), 2(1)]$ system. The performances are simulated for the following criteria: (a) sum rate, (b) harmonic mean SINR (c) sum MSE and (d) BER.

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