A GENERALIZED ITERATIVE WATER-FILLING ALGORITHM FOR DISTRIBUTED POWER CONTROL IN THE PRESENCE OF A JAMMER

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ABSTRACT

Consider a scenario in which K users and a jammer have a limited power budget and share a common spectrum of N orthogonal tones. The goal of each user is to allocate its power across the N tones in such a way that maximizes the total sum rate that he/she can achieve, while treating the interference of other users and the jammer's signal as additive Gaussian noise. The jammer, on the other hand, wishes to allocate its power in such a way that minimizes the utility of the whole system; that being the total sum of the rates communicated over the network. For this non-cooperative game, we propose a generalized version of the existing iterative water-filling algorithm whereby the users and the jammer update their power allocations in a greedy manner. We study conditions under which the generalized iterative water-filling algorithm converges to a Nash equilibrium of the game. The conditions that we derive in this paper depend only on the system parameters, and hence can be checked *a priori*.

Index Terms— Open-spectrum communications, jamming, non-cooperative games, Nash equilibrium, contraction mapping

1. INTRODUCTION

In open-spectrum communication systems the spectrum is typically partitioned into N narrowband orthogonal tones and all users are allowed to use all the tones simultaneously. In comparison with fixed tone-assignment policies, this setup offers significantly greater freedom in utilizing the spectrum. However, this freedom comes at the expense of a number of challenges that ought to be taken into consideration by the system designer. In particular, the inherent spectral overlap in these systems gives rise to the so-called multi-user interference, which is a limiting factor for multi-user communication systems. To mitigate the effect of multi-user interference, the users may employ a distributed power allocation mechanism whereby each user measures the interference level on each tone [1] and allocates its power dynamically across tones in such a way that maximizes its total achievable rate.

Open-spectrum communication systems typically operate over unlicensed spectral bands [2] whereby multiple users can access the shared spectrum simultaneously and freely. This feature renders these systems susceptible to antagonistic behaviour of potential jammers, who may be interested in reducing the utility of the entire system.[1] For example, a jammer may be able to 'listen' to the users' transmissions, and subsequently updates its power allocation across tones in order to reduce the total sum rate communicated over the network. As such, the procedure of both the users and the jammer can be represented as a non-cooperative game [3] in which players are interested in maximizing their individual utilities in a selfish fashion. Since the impact of the jammer's signal can be deleterious to the overall system performance, our goal in this paper is to study Nash equilibrium of this sum rate game and subsequently the jammer's effect on the achievable system utility. We also wish to analyze the convergence behavior of a generalized version of the iterative water-filling algorithm (IWFA) for this non-cooperative game whereby users and the jammer sequentially update their power allocations in a greedy manner to maximize their respective utilities.

In this paper we consider a communication system in which K users and a jammer share N orthogonal tones. Both the users and the jammer have limited power budgets. The goal of each user is to allocate its power across the N tones in such a way that maximizes the total sum rate that he/she can reliably communicate. The jammer, on the other hand, wishes to allocate its power in such a way that minimizes the utility of the whole system; that being the total sum of the rates communicated over the network. This scenario is analogous to a zero-sum non-cooperative game. In this paper we show that at least one Nash equilibrium exists for this non-cooperative game. Moreover, we consider a generalized version of the iterative water-filling algorithm (GIWFA) whereby users and the jammer update their power allocations in a greedy manner to maximize their respective utilities. For the case in which the users and the jammer update their power loads sequentially according to some prescribed order we derive sufficient conditions under which GIWFA converges to a unique Nash equilibrium of this non-cooperative game. In [4] these conditions are extended to the case in which the users and the jammer update these loads in a totally asynchronous fashion at arbitrary time instants and using possibly outdated information about the interference from other users. We present numerical results that illustrate the impact of the jammer on the system utility and on the convergence of the users' iterates. In particular we show that the presence of a strong jammer can not only reduce the total utility of the system, but also cause the, otherwise convergent, IWFA algo-

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[1]In this paper, the sum rate of each user across tones will be referred to as the utility of the user, and the sum of utilities of all users will be referred to as the system utility.

rithm to oscillate.

2. SYSTEM MODEL AND DEFINITIONS

Consider a communication system in which N tones are shared by K user pairs and one jammer. (We refer to a transmitter-receiver pair as one user.) Let each user have one transmit and one receive antenna and let h_{jk}^n denote the gain between the transmitter of User j and the receiver of User k at the n-th tones, for $j, k \in \mathcal{K}$ and $n \in \mathcal{N}$, where $\mathcal{K} \triangleq \{1, \ldots, K\}$ and $\mathcal{N} \triangleq \{1, \ldots, N\}$. Furthermore, let s_k^n and s_0^n be the power allocated by User k and the jammer to the n-th tone, respectively. (Throughout this paper the jammer will be denoted as User 0.) If both the users and the jammer transmit Gaussian signals, then the rate that can be achieved by User $k \in \mathcal{K}$ on the n-th tone is given by [5]

$$R_k^n(s_1^n, \ldots, s_K^n) = \log\Big(1 + \tfrac{|h_{kk}^n|^2 s_k^n}{N_k^n + \sum_{j \neq k} |h_{jk}^n|^2 s_j^n + |h_{0k}^n|^2 s_0^n}\Big), \quad (1)$$

where N_k^n denotes the noise variance observed by User k on the n-th tone. By dividing both the numerator and the denominator by $|h_{kk}^n|^2$, the expression in (1) can be expressed as

$$R_k^n(s_1^n, \ldots, s_K^n) = \log\Big(1 + \tfrac{s_k^n}{\sigma_k^n + \sum_{j \neq k} \alpha_{jk}^n s_j^n + \alpha_{0k}^n s_0^n}\Big), \quad (2)$$

where we define $\alpha_{0k}^n = |h_{0k}^n|^2/|h_{kk}^n|^2 \geq 0$, $\alpha_{jk}^n = |h_{jk}^n|^2/|h_{kk}^n|^2 \geq 0$, and $\sigma_k^n = N_k^n/|h_{kk}^n|^2 > 0$, for $j, k \in \mathcal{K}$, $n \in \mathcal{N}$. Suppose that User $k \in \mathcal{K}$ ($k \neq 0$), is interested in maximizing its own sum-rate, so its utility is given by

$$\begin{aligned} U_k(\mathbf{s}_0, \mathbf{s}_1, \cdots, \mathbf{s}_K) &= \sum_{n=1}^{N} R_k^n(s_1^n, \ldots, s_K^n) \\ &= \sum_{n=1}^{N} \log\Big(1 + \tfrac{s_k^n}{\sigma_k^n + \sum_{j \neq k} \alpha_{jk}^n s_j^n + \alpha_{0k}^n s_0^n}\Big), \quad (3) \end{aligned}$$

while the utility of the jammer is

$$\begin{aligned} U_0(\mathbf{s}_0, \mathbf{s}_1, \cdots, \mathbf{s}_K) &= -\sum_{k=1}^{K} U_k \\ &= -\sum_{k=1}^{K} \sum_{n=1}^{N} \log\Big(1 + \tfrac{s_k^n}{\sigma_k^n + \sum_{j \neq k} \alpha_{jk}^n s_j^n + \alpha_{0k}^n s_0^n}\Big), \quad (4) \end{aligned}$$

where we use $\mathbf{s}_k$ to denote the vector $[s_k^1, \cdots, s_k^N]^T$.

Given a limited power budget, and a maximum power constraint on each tone, the goal of User k, is to maximize U_k; that is, User k wishes to solve the following optimization problem,

$$\begin{aligned} \max \quad & U_k(\mathbf{s}_0, \mathbf{s}_1, \cdots, \mathbf{s}_K), \\ \text{subject to} \quad & \sum_{n=1}^{N} s_k^n \leq P_k, && (5a) \\ & 0 \leq s_k^n \leq S_{\max,k}^n, && (5b) \end{aligned}$$

where, P_k denotes the total power budget of User k, $S_{\max,k}^n$ denotes the maximum signal power that User k can use on the n-th tone, and in order for (5a) not to be redundant, we assume that $P_k \leq \sum_{n=1}^{N} S_{\max,k}^n$. We will denote the feasible set of User k as $\mathcal{P}_k$; i.e.,

$$\mathcal{P}_k \triangleq \Big\{\mathbf{s}_k = [s_k^1, \cdots, s_k^N]^T \Big| \sum_{n=1}^{N} s_k^n \leq P_k, s_k^n \in [0, S_{\max,k}^n]\Big\}. \quad (6)$$

Since individual users do not collaborate among themselves nor do they collaborate with the jammer, and both users and the jammer selfishly maximizes their own utilities, this communication scenario can be modelled as a non-cooperative game [3]. In this game individual users and the jammer are non-cooperative players, and the power allocations of any User k, including the jammer, that lie in $\mathcal{P}_k$ (cf., (6)) represent the set of admissible strategies of this user. A Nash equilibrium of this game [3] is a $(K+1)$-tuple of power strategies $\{\mathbf{s}_k^*\}_{k=0}^K$, such that for any $k \in \{0\} \cup \mathcal{K}$

$$\begin{aligned} & U_k(\mathbf{s}_0^*, \mathbf{s}_1^*, \cdots, \mathbf{s}_{k-1}^*, \mathbf{s}_k^*, \mathbf{s}_{k+1}^*, \cdots, \mathbf{s}_K^*) \geq \\ & \quad U_k(\mathbf{s}_0^*, \mathbf{s}_1^*, \cdots, \mathbf{s}_{k-1}^*, \mathbf{s}_k, \mathbf{s}_{k+1}^*, \cdots, \mathbf{s}_K^*), \quad \forall \mathbf{s}_k \in \mathcal{P}_k. \quad (7) \end{aligned}$$

In other words, a Nash equilibrium of the game is a locally optimal strategy for each player that no player has an incentive to unilaterally change [3]. In the next section, we will propose a decentralized algorithm for updating the jammer and the users' power allocations. By analyzing the convergence of this algorithm, we derive sufficient conditions under which the Nash equilibrium is unique.

3. EXISTENCE AND UNIQUENESS OF A NASH EQUILIBRIUM

Since, for every $k \in \mathcal{K}$, $U_k(s_0, s_1, \cdots, s_{k-1}, \bullet, s_{k+1}, s_K)$ is a continuously differentiable concave function, and so is $U_0(\bullet, s_1, \cdots, s_K)$, and since each $\mathcal{P}_k$ is a compact convex set, it follows readily from [6, Proposition 2.2.9] that a Nash equilibrium exists. Such an equilibrium can be found using a standard fixed-point algorithm, an instance of which is given in the next section.

3.1. A generalized iterative water-filling algorithm (GIWFA)

A simple distributed algorithm for the users and the jammer to update their power allocation is the following generalized iterative water-filling algorithm (GIWFA). Let $s_k^{n,\nu}$ be the power allocation of User k on the n-th tone at iteration ν, and $\mathbf{s}_k^\nu$ be the vector $[s_k^{1,\nu}, \cdots, s_k^{N,\nu}]^T$. Consider the case in which the users update their power allocations sequentially.[2] Assume, without loss of generality, that the users are ordered so that User 1 updates its power allocation first then User 2 and so on, and that the jammer (User 0) updates its power allocation last. Hence, in each iteration User $k \in \mathcal{K}$ updates its power allocations to solve

$$\begin{aligned} \mathbf{s}_k^{\nu+1} = \Big[\mathbf{s}_k^{\nu+1} + \nabla_{\mathbf{s}_k} U_k(\mathbf{s}_0^\nu, \mathbf{s}_1^{\nu+1}, \cdots, \\ \mathbf{s}_{k-1}^{\nu+1}, \mathbf{s}_k, \mathbf{s}_{k+1}^\nu, \cdots, \mathbf{s}_K^\nu)\Big|_{\mathbf{s}_k = \mathbf{s}_k^{\nu+1}}\Big]_{\mathcal{P}_k}, \quad (8) \end{aligned}$$

whereas the jammer solves

$$\mathbf{s}_0^{\nu+1} = \Big[\mathbf{s}_0^{\nu+1} + \nabla_{\mathbf{s}_0} U_0(\mathbf{s}_0, \mathbf{s}_1^{\nu+1}, \cdots, \mathbf{s}_K^{\nu+1})\Big|_{\mathbf{s}_0 = \mathbf{s}_0^{\nu+1}}\Big]_{\mathcal{P}_0}, \quad (9)$$

where we use $[\cdot]_{\mathcal{P}_k}$ to denote the projection operator onto the polyhedron defined in (6). That is, for any vector $x \in \mathbb{R}^N$

$$[x]_{\mathcal{P}_k} = \arg\min_{y \in \mathcal{P}_k} \|y - x\|. \quad (10)$$

[2] In [4] we extend our results to include the case in which the users and the jammer may update their power allocations at arbitrary time instants using, possibly, outdated noise-plus-interference measurements.

Using (3) and (4), we can compute the gradients $\nabla_{\mathbf{s}_k} U_k$ explicitly. In particular, the n-th entry of $\nabla_{\mathbf{s}_k} U_k$ for $k \in \{0\} \cup \mathcal{K}$, $[\nabla_{\mathbf{s}_k} U_k]_n$, can be expressed as

$$\left[\nabla_{\mathbf{s}_k} U_k(\mathbf{s}_0^\nu, \mathbf{s}_1^{\nu+1}, \cdots, \mathbf{s}_{k-1}^{\nu+1}, \mathbf{s}_k, \mathbf{s}_{k+1}^\nu, \cdots, \mathbf{s}_K^\nu)\Big|_{\mathbf{s}_k=\mathbf{s}_k^{\nu+1}}\right]_n = \frac{1}{\sigma_k^n + \sum_{j=1}^{k} \alpha_{jk}^n s_j^{n,\nu+1} + \sum_{j=k+1}^{K} \alpha_{jk}^n s_j^{n,\nu} + \alpha_{0k}^n s_0^{n,\nu}}, \ \forall k \in \mathcal{K}, \tag{11}$$

$$\left[\nabla_{\mathbf{s}_0} U_0(\mathbf{s}_0, \mathbf{s}_1^{\nu+1}, \cdots, \mathbf{s}_K^{\nu+1})\Big|_{\mathbf{s}_0=\mathbf{s}_0^{\nu+1}}\right]_n = \sum_{k=1}^{K}\left(\frac{\alpha_{0k}^n s_k^{n,\nu+1}}{\sum_{j=1,\, j\neq k}^{K} \alpha_{jk}^n s_k^{n,\nu+1} + \sigma_k^n + \alpha_{0k}^n s_0^{n,\nu}} \times \frac{1}{\sum_{j=1}^{K} \alpha_{jk}^n s_k^{n,\nu+1} + \sigma_k^n + \alpha_{0k}^n s_0^{n,\nu}}\right), \tag{12}$$

where, in (11) and (12), we have used that $\alpha_{kk}^n = 1$ for all $k \in \mathcal{K}$.

From (11) and (12) we observe that for User $k \in \mathcal{K}$ to update its power allocation, it is sufficient to measure its own noise-plus-interference level on each tone, whereas for the jammer to update its power allocation, it needs, not only to know the power transmitted by each user, but also to know the noise-plus-interference level experienced by each user on every tone.

3.2. Convergence Analysis

We now present sufficient conditions under which this algorithm converges to the unique Nash equilibrium of the game. Applying [7, Proposition 11.13] it can be seen that a tuple of power strategies $\{\mathbf{s}_k^*\}_{k=0}^K$ achieves equilibrium if and only if

$$\mathbf{s}_k^* = \left[\mathbf{s}_k^* + \theta \nabla_{\mathbf{s}_k} U_k(\mathbf{s}_0^*, \mathbf{s}_1^*, \cdots, \mathbf{s}_{k-1}^*, \mathbf{s}_k, \mathbf{s}_{k+1}^*, \cdots, \mathbf{s}_K^*)\Big|_{\mathbf{s}_k=\mathbf{s}_k^*}\right]_{\mathcal{P}_k}, \quad k \in \mathcal{K} \tag{13a}$$

$$\mathbf{s}_0^* = \left[\mathbf{s}_0^* + \theta \nabla_{\mathbf{s}_0} U_0(\mathbf{s}_0, \mathbf{s}_1^*, \cdots, \mathbf{s}_K^*)\Big|_{\mathbf{s}_0=\mathbf{s}_0^*}\right]_{\mathcal{P}_0}, \tag{13b}$$

for some $\theta > 0$. Since our generalized iterative water-filling algorithm (8)–(9) corresponds to setting $\theta = 1$ in (13), then if this algorithm converges to a power strategy $\{\mathbf{s}_k^*\}_{k=0}^K$, then it must be a Nash equilibrium of the game (7). We now present sufficient conditions under which the generalized IWFA converges to a unique Nash equilibrium point. In particular, let

$$A = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ -\alpha_{12} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\alpha_{1K} & -\alpha_{2K} & \cdots & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & \alpha_{21} & \alpha_{31} & \cdots & \alpha_{K1} \\ 0 & 0 & \alpha_{32} & \cdots & \alpha_{K2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \alpha_{K,K-1} \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix},$$
$$\text{and} \quad \beta = \begin{bmatrix} \alpha_{01} & \cdots & \alpha_{0K} \end{bmatrix}^T, \tag{14}$$

where we define $\alpha_{jk} \triangleq \left\|[\alpha_{jk}^1, \cdots, \alpha_{jk}^N]\right\|_2$ for all $j \in \{0\} \cup \mathcal{K}$, $k \in \mathcal{K}, j \neq k$. Furthermore, for every $k \in \mathcal{K}$, let F_k be an $N \times NK$ block-diagonal matrix whose n-th $1 \times K$ diagonal block is f_k^n; i.e.,

$$F_k \triangleq \begin{bmatrix} f_k^1 & 0 & \cdots & 0 \\ 0 & f_k^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & f_k^N \end{bmatrix}, \tag{15}$$

where the i-th entry of f_k^n, $[f_k^n]_i$, $i = 1, \ldots, K$, is given by

$$[f_k^n]_k = \frac{(S_{\max,0}^n)^2}{(d_{\min,k}^n)^2 (c_{\min,k}^n + S_{\max,0}^n)^2} + \frac{\sum_{j=1,\, j\neq k}^{K} \alpha_{jk}^n S_{\max,j}^n}{(\sum_{j=1,\, j\neq k}^{K} \alpha_{jk}^n S_{\max,j}^n + \eta_k^n) c_{\min,k}^n d_{\min,k}^n} + \frac{S_{\max,0}^n}{d_{\min,k}^n (c_{\min,k}^n + S_{\max,0}^n)} \times \left(\frac{1}{d_{\min,k}^n} + \frac{1}{c_{\min,k}^n} + \frac{S_{\max,k}^n}{c_{\min,k}^n (\alpha_{0k}^n c_{\min,k}^n + S_{\max,k}^n)}\right), \tag{16}$$

$$[f_k^n]_i = \frac{2 S_{\max,0}^n S_{\max,k}^n}{c_{\min,k}^n (\alpha_{0k}^n c_{\min,k}^n + S_{\max,k}^n) d_{\min,k}^n (c_{\min,k}^n + S_{\max,0}^n)} \alpha_{ik}^n + \frac{(S_{\max,k}^n)^2 d_{\min,k}^n + 2 S_{\max,k}^n c_{\min,k}^n (\alpha_{0k}^n c_{\min,k}^n + S_{\max,k}^n)}{d_{\min,k}^n (c_{\min,k}^n)^2 (\alpha_{0k}^n c_{\min,k}^n + S_{\max,k}^n)^2} \alpha_{ik}^n, \quad i \neq k,\ i \in \mathcal{K} \tag{17}$$

where

$$c_{\min,k}^n = \frac{1}{\alpha_{0k}^n}\left(\sum_{j=1,\, j\neq k}^{K} \alpha_{jk}^n \eta_j^n + \sigma_k^n\right), \tag{18}$$

$$d_{\min,k}^n = c_{\min,k}^n + \frac{1}{\alpha_{0k}^n} \eta_k^n, \tag{19}$$

with η_k^n being a lower bound on $s_k^{n,\nu}$. That is, for every iteration ν, $\eta_k^n \leq s_k^{n,\nu}$, $\forall k \in \mathcal{K}, n \in \mathcal{N}$. In [4, Appendix B] we show that η_k^n is given by

$$\eta_k^n = \left[\tfrac{1}{N}\Big(P_k + \sum_{i=1}^{m_k} \sigma_k^{\pi_k(i)}\Big) + \Big(\tfrac{1}{N} - 1\Big) \sum_{j=0,\, j\neq k}^{K} \alpha_{jk}^n S_{\max,j}^n - \sigma_k^{\pi_k(n)}\right]^+,$$

where m_k is the largest integer for which

$$(m_k - 1)\Big(\sigma_k^{\pi_k(j)} + \sum_{i=0,\, i\neq k}^{K} \alpha_{ik}^{\pi_k(j)} S_{\max,i}^{\pi_k(j)}\Big) \leq P_k + \sum_{i=1}^{m_k-1} \sigma_k^{(i)},$$

is satisfied for all $j \leq m_k$. For each User $k \in \mathcal{K}$, $\sigma_k^{(i)}$ denotes the noise variance that satisfies $\sigma_k^{(i)} \leq \sigma_k^{(i+1)}$, for all $i = 1, \ldots, N-1$, and $\pi_k(\cdot)$ denotes the tone permutation that satisfies

$$X_k^{\pi_k(1)} \leq \cdots \leq X_k^{\pi_k(N)}, \text{ where } X_k^n = \sigma_k^n + \sum_{j=0,\, j\neq k}^{K} \alpha_{jk}^n S_{\max,j}^n.$$

Theorem 1 (Convergence of GIWFA) *Suppose there exists a scalar $\tau \in (0, 1)$ such that the following conditions are satisfied*

$$\left(1 + (1-\tau)^{-2} \Big\|\sum_{k=1}^{K} F_k\Big\|_2^2\right)\left(\|A^{-1}B\|_2^2 + \|A^{-1}\beta\|_2^2\right) < 1, \tag{20}$$

$$\max_n \sum_{k=1}^{K} \frac{S_{\max,k}^n \left(2 c_{\min,k}^n + \frac{S_{\max,k}^n}{\alpha_{0k}^n}\right)}{(c_{\min,k}^n)^2 \left(c_{\min,k}^n + \frac{S_{\max,k}^n}{\alpha_{0k}^n}\right)^2} \leq \tau + 1, \tag{21}$$

$$\min_n \sum_{k=1}^{K}\left(\frac{(\alpha_{0k}^n)^3 \eta_k^n}{\left(\sum_{j=1,\, j\neq k}^{K} \alpha_{jk}^n S_{\max,j}^n + \alpha_{0k}^n S_{\max,0}^n + \sigma_k^n\right)^2} \times \frac{1}{\sum_{j=1,\, j\neq k}^{K} \alpha_{jk}^n S_{\max,j}^n + \eta_k^n + \alpha_{0k}^n S_{\max,0}^n + \sigma_k^n} + \frac{(\alpha_{0k}^n)^3 \eta_k^n}{\sum_{j=1,\, j\neq k}^{K} \alpha_{jk}^n S_{\max,j}^n + \alpha_{0k}^n S_{\max,0}^n + \sigma_k^n} \times \frac{1}{\left(\sum_{j=1,\, j\neq k}^{K} \alpha_{jk}^n S_{\max,j}^n + \eta_k^n + \alpha_{0k}^n S_{\max,0}^n + \sigma_k^n\right)^2}\right) \geq 1 - \tau. \tag{22}$$

Then the noncooperative game (7) *has a unique Nash equilibrium, and the iterates generated by the GIWFA algorithm converges to this unique equilibrium linearly.*

Proof: See Appendix A in [4]. □

Notice that the conditions (20)–(22) only depend on the power budget of each user, its maximum allowable power on each tone and the cross-talk coefficients. In many practical scenarios, these parameters, or a reasonably accurate estimate thereof, may be known *a priori* to the system designer. Hence, these conditions allow the system designer to study the impact that a potential jammer may have on the users' utilities as well as the utility of the whole system. In Section 4 we will present numerical results that show that for scenarios in which the conditions of Theorem 1 are met, both the users and the jammer converge, and in [4] we provide instances to show that the violation of these conditions may cause the algorithm to oscillate.

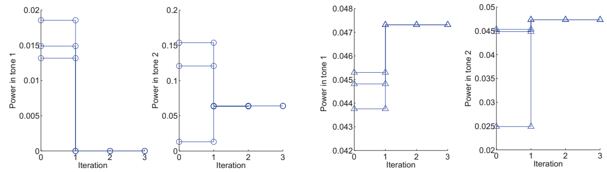
Observe that for any τ, the condition in (20) implies the standard IWFA convergence conditions. In particular, for any such τ for which (20) holds, we have $\|A^{-1}B\|_2 < 1$. However, in contrast with the convergence conditions of standard IWFA, the convergence of the GIWFA in the presence of the jammer depends, not only on the crosstalk coefficients, but also on the power budgets of both the users and the jammer.

Condition (22) implies that $\min_n \sum_{k=1}^{N} s_k^{n,\nu} \geq \min_n \sum_{k=1}^{N} \eta_k^{n,\nu} > 0$. Thus if $s_k^{n,*} \equiv \lim_{\nu\to\infty} s_k^{n,\nu}$, then $\min_n \sum_{k=1}^{N} s_k^{n,*} > 0$. In words, this says that the Nash equilibrium computed by the GIWFA has the property that, under the assumption of Theorem 1, every tone n is used by at least one user k. In fact, it can be verified that if one of the tones is abandoned by all users at any iteration, the jammer's strategy to reduce the system utility may cause the GIWFA to oscillate. Since in the jammer-free case the users compete but, otherwise have no interest in reducing the system utility, the oscillation phenomenon is less likely to occur in the jammer-free case. Another insight offered by Theorem 1 is that if the jammer's maximum signal power $S^n_{\max,0}$ on tone n is sufficiently large so that $\eta_k^n = 0$ for all k, then (22) cannot be satisfied and the convergence of the GIWFA is in jeopardy.

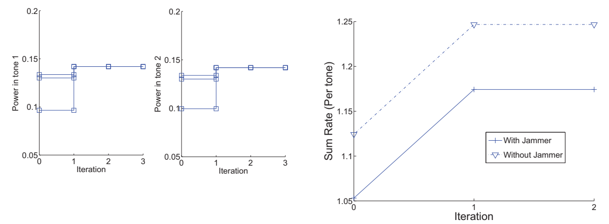
4. NUMERICAL RESULTS

In this section we provide a numerical example that illustrates the sufficiency of the conditions given in Theorem 1 for the convergence of the decentralized GIWFA algorithm. For this example, we choose the number of users, $K = 4$, and the number of tones $N = 10$. The maximum allowable power per tone is set to be constant across tones for each user as well as for the jammer; i.e., we set $S^n_{\max,k} = S_{\max,k}$, $n = 1, \ldots, 10$ for $k = 0, \ldots, 4$.

Example 1 *In this example, the system parameters (i.e,* $\alpha^n_{jk}, \sigma^n_k, P_k, S_{\max}, k, \forall\ j \neq k,\ k = 0, \ldots, 4$*) are selected at random so as to satisfy the conditions in Theorem 1. The users and the jammer update their power allocations using the GIWFA algorithm described in Section 3.1. For this scenario, in Figures 1(a) and 1(b) we plot the power allocations of Users 1 and 2 versus the iteration number for all the tones. For the same scenario, in Figure 1(c) we plot the power allocations of the jammer versus the iteration number. In each of the plots, three randomly chosen allocations were used to initialize the fixed-point algorithm. Since the system parameters were chosen to meet the conditions of Theorem 1, the algorithm converges to a unique Nash equilibrium, irrespective of the initial power allocations. In order to quantify the jammer's impact on the overall system performance, the sum rate of all the users over the ten tones is plotted versus the iteration number in Figure 1(d).*



(a) Power allocations of User 1 (b) Power allocations of User 2



(c) Power allocations of jammer (d) Sum rate with/without jammer

Fig. 1. The GIWFA iterates converge to a unique Nash equilibrium irrespective of the initial power allocation.

5. CONCLUSION

In this paper we considered a communication scenario in which K users and a jammer share N orthogonal tones. We modelled this scenario as a non-cooperative game, and developed a (decentralized) extension of the IWFA algorithm for the users and the jammer to update their power allocations. For this decentralized GIWFA algorithm, we derived sufficient conditions under which the iterates of the algorithm converge to the unique Nash equilibrium of the game.

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