STRUCTURED SPECTRUM BALANCING IN DSL MULTIUSER COMMUNICATIONS

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ABSTRACT

Finding the power allocations that maximize the sum-rate of a K-user N-tone Digital Subscriber Line (DSL) system is known to be NP-hard. In this paper we devise a polynomial-time algorithm to approximate the maximum sum rate of the system. The development of this algorithm is guided by the fact that, to approach the sum-rate maximum, the users should operate in an FDMA-mode over frequency tones where the crosstalk coefficients exceed a certain threshold, and should share the tones for which the crosstalk coefficients are sufficiently small. Drawing on this insight, the algorithm partitions the N tones into three sections and imposes an appropriate signalling structure on each section. The first section contains those tones for which the crosstalk coefficients are small and uses an iterative water-filling technique to determine the power allocations. The second section contains the tones with intermediate crosstalk coefficients and uses a primal-dual algorithm, and the third section contains the tones with large crosstalk coefficients and uses a dual FDMA algorithm. To decouple the overall optimization of power allocation across the three sections, we use tools from Lagrangian duality and sensitivity analysis to devise an iterative scheme that can optimally allocate each user's power budget to the three sections. Our numerical simulations, show that the sum-rate of the proposed algorithm is very close to that of the 'optimal' spectrum balancing algorithm, but requires considerably less computational effort.

Index Terms— DSL systems, multi-tone communications, water-filling, spectrum balancing, dual algorithms

1. INTRODUCTION

In a typical Digital Subscriber Line (DSL) system several users share a common spectrum of N orthogonal narrowband tones. The users do not collaborate and cannot decode each other's transmissions [1]. Hence each user treats the interference from the transmissions of other users as additive Gaussian noise. Due to crosstalk interference, the performance of each user depends not only on its own transmission power spectra, but also those of others sharing the same spectrum. The goal of the system designer is to determine the power that each user ought to allocate to each tone in order for the system to achieve the maximum sum-rate [2].

The problem of finding the power allocations that maximize the sum-rate is known to be NP-hard [3], and several algorithms have been proposed in order to provide approximate solutions for it. For example, the so-called optimal spectrum balancing (OSB) algorithm [4] uses the structure of the primal and Lagrange dual problems to perform exhaustive search for finding optimal power allocations. Although the OSB was shown in [4] to yield high sum-rates, its computational cost becomes prohibitive even for relatively small number of users. Another technique for optimizing the power allocations in a DSL system is the autonomous spectrum balancing (ASB) algorithm [5]. This algorithm does not require a central node and is based on the assumption that the users observe a common 'reference line' that enables them to adjust their power allocations. However this 'reference line' is evaluated using empirical parameters which may not be readily available to the users. In addition to OSB and ASB, a low-complexity power allocation algorithm that uses successive convex approximation (SCALE) of the sum-rate objective was developed in [6]. Similar to ASB, this algorithm does not require the existence of a central node. Instead, it requires the users to exchange their parameters using a message passing algorithm. In the numerical simulations section, we will compare the performance of the algorithm proposed herein with that of OSB and SCALE.

The algorithm proposed herein is guided by the fact that [2] if the crosstalk coefficients between users exceed a certain threshold on some tones, these tones ought to be operated in a frequency division multiple access (FDMA) mode in order to approach the maximum sum-rate of the DSL system. That is, none of these tones ought to be occupied by more than one user. In addition, we note that if the crosstalk coefficients between users are close to zero on some tones, then it is easy to see that, the sum-rate of the system is maximized if all the users allocate their powers on these tones using a water-filling approach. These observations suggest that, in a general DSL system, one can use the crosstalk coefficients to partition the tones into sections and utilize a different optimization technique on each section. In particular, we propose to partition the tones into three sections, where the membership of a tone in one of the sections depends on whether the crosstalk coefficients on this tone lie below, in between or above two thresholds. To determine these thresholds, we use a quasi-bisection optimization technique for relaxing the (somewhat stringent) thresholds provided in [2].

With the tones partitioned into sections, we deploy an sum-rate maximization algorithm that is tailored to the signalling structure of each section. In particular, for the section in which the users operate in an FDMA mode, we use the FDMA sum-rate maximization algorithm developed in [7]. The computational efficiency of this algorithm stems from decoupling the Lagrange multiplies that govern the power allocated on each tone. Now, for the section in which the crosstalk coefficients are close to zero, we use the classical iterative water-filling algorithm (IWFA) [8]. In each iteration of this algorithm, each user updates its power allocation so as to water-fill [9] on the noise-plus-interference levels observed in the previous iteration. Finally, we consider the section of tones in which the crosstalk coefficients assume intermediate values that are neither large enough to operate the tones in an FDMA mode, nor small enough to operate them in an IWFA mode. In this section we propose using the primal-dual updates algorithm described in [10]. In this algorithm the primal and the dual variables are updated iteratively using a standard gradient ascent algorithm. In order to determine the power that

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each user allocates to each section of tones, we develop an iterative technique that uses a sensitivity analysis to determine the section of tones that yields the highest sum-rate gain for a given power increment. This sensitivity analysis exploits the Lagrange dual variables generated by the algorithms deployed in the three sections. That is, the FDMA sum-rate maximization algorithm, the IWFA algorithm and the primal-dual updates algorithm.

Finally, we provide numerical examples that show that the proposed algorithm enables the DSL system to achieve sum-rates that are very close to those achieved by the OSB algorithm. However, because of the structure that underlies our algorithm, we manage to avoid the exhaustive search required by the OSB algorithm and hence to significantly reduce the, otherwise prohibitive, computational burden.

2. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a DSL communication system in which N tones are shared by K users. Let h_{jk}^n be the complex channel gain between the transmitter of User j and the receiver of User k on the n-th tone, where $n \in \mathcal{N} \triangleq \{1, \ldots, N\}$ and $j, k \in \mathcal{K} \triangleq \{1, \ldots, K\}$. In this notation h_{kk}^n denotes the channel gain between the transmitter of the k-th user and its intended receiver. Let the crosstalk coefficient between Users j and k on the n-th tone be denoted by $\alpha_{jk}^n \triangleq |h_{jk}^n|^2/|h_{kk}^n|^2$, and let s_k^n be the power allocated by User k to the n-th tone. Assuming that each user uses Gaussian signalling and that every user can only decode its intended messages, the maximum rate that User $k \in \mathcal{K}$ can achieve on the n-th tone is given by [9]

$$R_k^n(s_1^n, \ldots, s_K^n) = \log\Big(1 + \frac{s_k^n}{\Gamma\big(\sigma_k^n + \sum_{j \neq k} \alpha_{jk}^n s_j^n\big)}\Big), \quad (1)$$

where $\sigma_k^n \triangleq N_0/|h_{kk}^n|^2$ denotes the normalized noise variance observed by User k on the n-th tone, N_0 is the variance of the background Gaussian noise, and Γ is the so-called capacity gap, which is typically used to account for the non-Gaussianity of the signalling constellations used in practice [5]. Now, the system designer's goal is to find the power allocation that maximizes the over all sum-rate $\sum_{k=1}^{K} \sum_{n=1}^{N} R_k^n(s_1^n, \ldots, s_K^n)$, provided that the total and per-tone powers utilized by each user do not exceed certain thresholds. In addition, the system designer may wish to enforce a bit-cap in order to ensure that the rates communicated on each tone can be supported by commercial modulators [4]. Using these constraints, it can be shown that the power allocation problem can be formulated as

$$\begin{aligned} \max \quad & \textstyle\sum_{k=1}^{K} \sum_{n=1}^{N} R_k^n(s_1^n, \ldots, s_K^n), & \text{(2a)} \\ \text{subject to} \quad & \textstyle\sum_{n=1}^{N} s_k^n \leq P_k, \ \forall\, k, & \text{(2b)} \\ & 0 \leq s_k^n \leq S_{\max,k}^n, \ \forall\, k, n, & \text{(2c)} \\ & \big(2^{B_k^n} - 1\big)^{-1} s_k^n - \textstyle\sum_{j \neq k} \alpha_{jk}^n s_j^n \leq \sigma_k^n, \ \forall\, k, n, & \text{(2d)} \end{aligned}$$

where P_k, $S_{\max,k}^n$ and B_k^n are the total power budget, the spectral mask and the bit-cap of User k on the n-th tone, respectively.

3. PROBLEM DECOMPOSITION

Solving (2) directly is known to be NP-hard [11], which makes the task of finding a global optimal solution rather formidable even for relatively small systems. As an alternative, we propose to use inherent features of the optimal solution [2] in order to decompose (2) into three subproblems that are relatively easy to solve. In the next sections we will describe our partitioning methodology.

3.1. FDMA-operated tones

Let $\mathcal{F} \subseteq \mathcal{N}$ be the set of tones for which

$$\alpha_{jk}^n \alpha_{kj}^n \geq \tfrac{1}{4} - \delta_1, \quad \forall\, j \neq k \in \mathcal{K}, \quad (3)$$

where $\delta_1 \in [0, \frac{1}{4}]$ is a designed parameter to be determined. Let $P_{\mathcal{F},k}$ be the power allocated by User $k \in \mathcal{K}$ to the tones in $\mathcal{F}$. In order to understand the role of δ_1, we note that one of the key results in [2], implies that if $\delta_1 = 0$, then all the tones in $\mathcal{F}$ must be operated in an FDMA mode in order for the maximum sum-rate to be approached. However, since this condition is only sufficient, in some scenarios it may be too stringent and higher sum-rates can be obtained if more tones are operated in an FDMA mode. Hence, by operating the tones in $\mathcal{F}$ in an FDMA mode, δ_1 can be regarded as a parameter for relaxing the condition in [2].[1] For a given $P_{\mathcal{F},k}$, the algorithm in [7] can be used to find power allocations that maximize the sum-rate achieved on the tones in $\mathcal{F}$. This algorithm exploits the FDMA structure to decouple the power allocated across tones by assigning each tone to the user with the, so-called, highest 'shadow rate'. For a target precision of ϵ_1, the complexity of this algorithm can be shown to be $\mathcal{O}(K^2 \log^2 \epsilon_1)$ [2].

3.2. IWFA-operated tones

For this section, let $\mathcal{W} \subseteq \mathcal{N} - \mathcal{F}$ be the set of tones for which

$$\alpha_{jk}^n \alpha_{kj}^n \leq \delta_2, \quad \forall\, j \neq k \in \mathcal{K}, \quad (4)$$

where $\delta_2 < \frac{1}{4} - \delta_1$ is a parameter that plays a role similar to the one played by δ_1 in (3). Let $P_{\mathcal{W},k}$ be the power that User $k \in \mathcal{K}$ assigns the tones in $\mathcal{W}$. Now, if for the tones in $\mathcal{W}$, $\alpha_{jk}^n = \alpha_{kj}^n = 0$, then δ_2 can be set equal to zero. Since in this case the users are completely decoupled, the maximum sum-rate can be achieved by classic water-filling [9]. However, in practice it is rarely the case that the crosstalk coefficients are exactly equal to zero, and $\mathcal{W}$ will contain those tones with small, but finite, crosstalk coefficients that satisfy (4) with sufficiently small δ_2. In this case, the maximum sum-rate on $\mathcal{W}$ can be approached by using the iterative, instead of the classic, water-filling algorithm (IWFA) [8]. The complexity of this algorithm is $\mathcal{O}(KN \log^2 \epsilon_1)$. Similar to δ_1, the value of δ_2 ought to be adjusted in order to maximize the sum-rate of the DSL system.

3.3. Tones with unstructured signalling

Finally, let $\mathcal{M} \subseteq \mathcal{N} - \mathcal{F} - \mathcal{W}$ be the set of tones on which the crosstalk coefficients do not satisfy either (3) or (4), and let $P_{\mathcal{M},k}$ be the power allocated by User $k \in \mathcal{K}$ to the tones in $\mathcal{M}$. Since the crosstalk coefficients on $\mathcal{M}$ assume intermediate values, the optimal signalling structure on $\mathcal{M}$ does not necessarily resemble any of the standard signalling patterns. Hence, we settle for power allocations that are locally sum-rate optimal. Such allocations can be found using the standard the primal-dual updates algorithm described in [10]. Similar to IWFA, the complexity of this algorithm is $\mathcal{O}(KN \log^2 \epsilon_1)$.

4. POWER BUDGET PARTITIONING

In the previous section a framework for partitioning the N tones into $\mathcal{F}, \mathcal{W}$, and $\mathcal{M}$ sections was presented. The powers allocated

[1] Note that while (3) gives a sufficient FDMA optimality condition for any $K \geq 2$, a tighter condition can be used for $K = 2$; see [2].

by any User $k \in \mathcal{K}$ to these sections are $P_{\mathcal{F},k}$, $P_{\mathcal{W},k}$, and $P_{\mathcal{M},k}$, respectively, where for (2b) to be satisfied, we must have[2]

$$P_{\mathcal{F},k} + P_{\mathcal{W},k} + P_{\mathcal{M},k} = P_k. \tag{5}$$

Our goal now is to find locally optimal $P_{\mathcal{F},k}$, $P_{\mathcal{W},k}$, and $P_{\mathcal{M},k}$ that enable the maximum sum-rate of the DSL system to be approached. In order to do that, we begin by introducing the following definitions. Let the k-th entry of $\mathbf{\Delta}_i \triangleq [\Delta_{i,1} \cdots \Delta_{i,K}]$ be an additional power by which User k increments $P_{\mathcal{F},k}$ and $P_{\mathcal{W},k}$ for $i = 1$ and 2, respectively. With $\Delta_{i,k}$ defined as such, satisfying the power constraint in (5), implies that the decrement of $P_{\mathcal{M},k}$ is $\Delta_{3,k} = -\sum_{i=1}^{2} \Delta_{i,k}$. For given $P_{\mathcal{F},k}$, $P_{\mathcal{W},k}$, and $P_{\mathcal{M},k}$, let $g_1(\mathbf{\Delta}_1)$ be the (locally) optimal sum-rate on the $\mathcal{F}$ tones for a power increment $\mathbf{\Delta}_1$. That is, for every (sufficiently small) $\mathbf{\Delta}_1$, $g_1(\mathbf{\Delta}_1)$ is the solution of

$$\begin{aligned} \max \quad & \textstyle\sum_{n\in\mathcal{F}}\sum_{k\in\mathcal{K}} R_k^n, & \text{(6a)}\\ \text{subject to} \quad & \text{(2c) and (2d)}, \quad \forall\, n \in \mathcal{F}, & \text{(6b)}\\ & \textstyle\sum_{n\in\mathcal{F}} s_k^n - P_{\mathcal{F},k} = \Delta_{1,k}, \quad \forall\, k \in \mathcal{K}. & \text{(6c)} \end{aligned}$$

Similarly, one can define $g_2(\mathbf{\Delta}_2)$ and $g_3(\mathbf{\Delta}_3)$ for the sum-rates on the $\mathcal{W}$ and $\mathcal{M}$ tones, respectively. We note that, apart from the constraint in (5), the optimization problems that correspond to the functions $\{g_i(\cdot)\}_{i=1}^3$ are decoupled. Moreover, each of these problems can be solved efficiently using the techniques outlined in Section 3.

For brevity, we will focus on $g_1(\mathbf{\Delta}_1)$, and the analysis for the other two functions follows similar paths. Let us consider the Lagrange dual form of (6). For this dual let k-th entry of $\boldsymbol{\lambda}_1(\mathbf{\Delta}_1) \in \mathbb{R}^K$ be the Lagrange dual variable that corresponds to the k-th constraint in (6c). Now, using the sensitivity theorem in [10, Proposition 3.2.2], we have

$$\nabla_{\mathbf{\Delta}_1} g_1(\mathbf{\Delta}_1) = -\boldsymbol{\lambda}_1(\mathbf{\Delta}_1). \tag{7}$$

This implies that the k-th entry in $\boldsymbol{\lambda}_1(\mathbf{\Delta}_1)$ can be used to quantify the increase in the sum-rate of User k on $\mathcal{F}$ that corresponds to a power increment of $\Delta_{1,k}$. Using a similar observation, the Lagrange dual vectors $\boldsymbol{\lambda}_i(\mathbf{\Delta}_i),\ i = 2, 3$ can be used to quantify the additional sum-rate that each user can obtain by increasing its power budget by a small $\Delta_{i,k},\ i = 2, 3$, on the $\mathcal{W}$ and $\mathcal{M}$ tones, respectively.

Now, for any initial power partition for which (5) is satisfied and power increment vectors $\{\mathbf{\Delta}_i\}_{i=1}^3$, a (local) maximum of total sum-rate that can be achieved on the $\mathcal{F}$, $\mathcal{W}$ and $\mathcal{M}$ tones is given by $\sum_{i=1}^3 g_i(\mathbf{\Delta}_i)$, and the the sum-rate increase that corresponds to increment vectors $\mathbf{\Delta}_1$ and $\mathbf{\Delta}_2$ is

$$\nabla_{\mathbf{\Delta}_1}\Big(\textstyle\sum_{i=1}^3 g_i(\mathbf{\Delta}_1)\Big) = \boldsymbol{\lambda}_3 - \boldsymbol{\lambda}_1, \quad \text{and} \tag{8a}$$

$$\nabla_{\mathbf{\Delta}_2}\Big(\textstyle\sum_{i=1}^3 g_i(\mathbf{\Delta}_2)\Big) = \boldsymbol{\lambda}_3 - \boldsymbol{\lambda}_2, \tag{8b}$$

respectively. Notice that in writing (8) we have used the fact that for the power partitions perturbed by $\mathbf{\Delta}_i,\ i = 1, 2, 3$, to satisfy (5), $\sum_{i=1}^3 \mathbf{\Delta}_i = 0$.

Using (8), we can now use a standard gradient ascent algorithm to find power partitions that yields (locally) maximum total sum-rate. In order to do that, let the k-th entry of $\mathbf{P}_{\mathcal{F}}^{(\nu)}$, $\mathbf{P}_{\mathcal{W}}^{(\nu)}$ and $\mathbf{P}_{\mathcal{M}}^{(\nu)} \in \mathbb{R}_+^K$, be the ν-th iterates of the power partitioning of the k-th user on the $\mathcal{F}, \mathcal{W}$ and $\mathcal{M}$ tones, respectively, and let $\boldsymbol{\lambda}_i^{(\nu)},\ i = 1, 2, 3$ be the Lagrange dual vectors generated by the algorithms outlined in Section 3; viz, FDMA power allocation algorithm described in [7], the IWFA algorithm [8], and the primal-dual updates algorithm [10], respectively. The steepest ascent algorithm for updating the power partitions can now be expressed as

$$\mathbf{P}_{\mathcal{F}}^{(\nu+1)} = \mathbf{P}_{\mathcal{F}}^{(\nu)} + \mu_1(\boldsymbol{\lambda}_3 - \boldsymbol{\lambda}_1), \tag{9a}$$

$$\mathbf{P}_{\mathcal{W}}^{(\nu+1)} = \mathbf{P}_{\mathcal{W}}^{(\nu)} + \mu_2(\boldsymbol{\lambda}_3 - \boldsymbol{\lambda}_2), \tag{9b}$$

$$\mathbf{P}_{\mathcal{M}}^{(\nu+1)} = \mathbf{P} - \mathbf{P}_{\mathcal{F}}^{(\nu+1)} - \mathbf{P}_{\mathcal{W}}^{(\nu+1)}, \tag{9c}$$

where $\mathbf{P} \in \mathbb{R}_+^K$ is the vector of the power budgets of the K users in $\mathcal{K}$, and $\mu_1, \mu_2 > 0$ are two (diminishing) stepsizes. Our Structured Spectrum Balancing (SSB) can be summarized using the flow chart in Figure 1. As shown in this chart, the tone allocation parameters δ_1 and δ_2 in (3) and (4) are determined using a two-dimensional bisection search with convergence accuracy ϵ_2. For the steepest ascent algorithm, we used a convergence accuracy ϵ_1. Using the complexity orders given in Section 3, and the exponential convergence of the bisection method, one can show that the complexity of the SSB algorithm is $\mathcal{O}\big(N_P(2KN + K^2)\log^2\epsilon_2 \log^2\epsilon_1\big)$, where N_P is the maximum number of iterations of the gradient ascent algorithm.

It is worth mentioning that since the power partitioning problem is not convex, the performance of the algorithm in (9) depends, in general, on the initial power partitions, $\mathbf{P}_{\mathcal{F}}^{(0)}$, $\mathbf{P}_{\mathcal{W}}^{(0)}$ and $\mathbf{P}_{\mathcal{M}}^{(0)}$. In order to generate 'good' initial partitions, we begin by assuming that α_{jk}^n is zero for all $n \in \mathcal{N}$ and $j \neq k \in \mathcal{K}$. In this case the optimum power allocation is given by the classic water-filling technique. Denoting the power allocated by User k to the n-th tone by $s_k^{n,0}$, we choose the k-th entries of the initial power partitions to be $P_{\mathcal{F},k}^{(0)} = \sum_{n\in\mathcal{F}} s_k^{n,0}$, $P_{\mathcal{W},k}^{(0)} = \sum_{n\in\mathcal{W}} s_k^{n,0}$, and $P_{\mathcal{M},k}^{(0)} = P_k - P_{\mathcal{F},k}^{(0)} - P_{\mathcal{W},k}^{(0)}$. Our extensive numerical experiments have shown that this initialization procedure typically results in sum-rates that are close to the optimal ones achieved by the significantly more complex OSB algorithm.

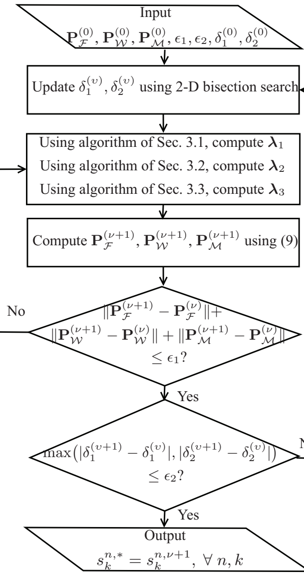
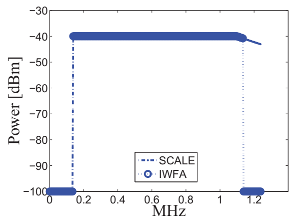


Fig. 1: A flow chart of the SSB algorithm

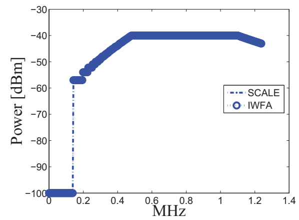
5. NUMERICAL RESULTS

In this section we compare the sum-rate and the power spectral density (PSD) obtained by OSB, IWFA and SCALE with the sum-rate

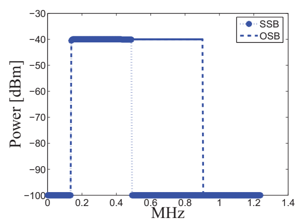
[2] It can be seen that if each user occupies at least one tone in $\mathcal{F}$ the optimal solution of (2) must satisfy (2b) with equality.



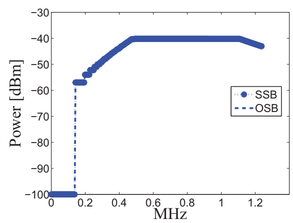
(a) IWFA and SCALE PSD of CO



(b) IWFA and SCALE PSD of RT



(c) OSB and SSB PSD of CO



(d) OSB and SSB PSD of RT

Fig. 2: A PSD Comparison for IWFA, SCALE, OSB and SSB

and PSD obtained by the proposed SSB. Due to the prohibitive computational complexity of OSB, we restrict our attention in this example to a 2-user scenario and a 256-tone DSL system. The crosstalk coefficients and spectral masks of this system were generated using a practical DSL simulator.[3] In particular, we simulated a scenario with one 5 km Central Office (CO) line and one 5 km Remote Terminal (RT) line, where the distance between the CO and the RT was taken to be 2.5 km . The overall power budget of both users was set at 20 dBm, the capacity gap, Γ, at 15, the background noise variance at -140 dBm/Hz, and the bit-cap at 15 bits per tone.

For this scenario, IWFA and SCALE achieve relatively low sum-rates of about 5.82 and 5.84 Mbps, respectively, whereas OSB achieves an 'optimal' rate of about 7.62 Mbps. On the other hand, the proposed SSB algorithm achieves a sum-rate of about 7.60 Mbps, which is only slightly less than the sum-rate achieved by OSB. Figure 2 shows the powers allocated by the four algorithms, and as can be seen from this figure, the power allocations of SSB resemble, to a large extent, those of OSB. However, these allocations vary quite significantly from the power allocations of both IWFA and SCALE.

A key advantage of SSB is that it exploits the structure of optimal power allocations to avoid the exhaustive search and the discretization that underlie the OSB algorithm. In order to provide a rough comparison between the computational complexity of OSB and SSB, we measured the Matlab running time of both algorithms for the current 2-user example. For OSB this time was about 530 seconds, whereas for SSB this time was 22 seconds only.[4] This running time difference becomes more dramatic for systems with more users because the proposed SSB relies on polynomial-time algorithms that are significantly more efficient than the exhaustive search of OSB.

As another example, we compare the sum-rate of our proposed SSB algorithm with that of IWFA and SCALE in a 6-user scenario.[5] For this example we use similar parameters to those used in the previous example, and we generate the crosstalk coefficients using the same DSL simulator, but for 2 co-located CO's and 4 Rt's. The lengths of the CO lines were chosen to be 5 and 4 km and those of the RT lines were chosen to be 5, 5, 4, and 4 km, respectively. The distance between the CO's and the RT's was chosen to be 0.2, 0.2, 3 and 3 km, respectively. For this scenario, IWFA and SCALE could achieve sum-rates of only 13.0 Mbps and 14.1 Mbps, respectively, whereas SSB could achieve a sum-rate of 16.7 Mbps.

6. CONCLUSIONS

In this paper we have provided an efficient algorithm for approaching the maximum sum-rate of DSL communication systems. Unlike previously proposed algorithms, this algorithm exploits the inherent structure that underlies optimal power allocations to partition the tones into three sections. For each section, it imposes a signalling structure and maximizes the sum-rate that can be achieved by this structure. We have shown via numerical simulations that the proposed algorithm achieves sum-rates that are very close to those achieved by the significantly more computationally demanding 'optimal' spectrum balancing algorithm.

7. REFERENCES

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[3] This simulator was provided by R. Cendrillon of Huawei Tech. Co. Ltd.

[4] The corresponding running times of IWFA and SCALE are 1 and 15 seconds, respectively.

[5] The computational complexity of OSB has made it rather difficult for us to provide a sum-rate comparison for this example.