APPROACHING USER CAPACITY IN A DSL SYSTEM VIA HARMONIC MEAN-RATE OPTIMIZATION

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ABSTRACT

In this paper we consider a Digital Subscriber Line (DSL) system with N orthogonal narrowband tones. Each user has a limited power budget, and our goal is to determine the power allocation of each user that enables the 'user capacity' of the system to be approached. In this paper, we use 'user capacity' to denote the maximum number of users that can be supported by the system, provided that each user is guaranteed to have a data rate that lies within a prescribed range. Finding a power allocation that enables this capacity to be approached directly can be quite cumbersome because it involves solving a (non-convex) integer-program. In order to circumvent this difficulty, in this paper we propose an alternate approach that is based on exploiting the fairness and per-tone convexity of the harmonic mean-rate objective. Using these features, we devise a computationally-efficient power allocation technique that enables the user capacity of the DSL system to be approached more closely than power allocation techniques that are more computationally demanding.

Index Terms— DSL systems, multi-tone communications, user-capacity, quality-of-service, dual algorithms

1. INTRODUCTION

Consider a Digital Subscriber Line (DSL) system in which K users share a common spectrum of N orthogonal tones [1]. Each user has a limited power budget and wishes to communicate as much data as possible. However, since a user is typically unable to decode the signals of other users, it treats the aggregate interference of these signals as additive Gaussian noise. The system is managed by a service provider that is assumed to know the crosstalk coefficients and the power budget of each user. The goal of the service provider is to allocate the users' power across tones so as to approach the user capacity of the system; that being the maximum number of users that can be supported by the system without breaching the quality-of-service (QoS) that the each user is entitled to have. (In this paper the QoS of a certain user is used to refer to the range of data rates within which this user can communicate reliably.)

The task of directly determining the power allocations that enable the DSL system to approach the user capacity is generally formidable because it involves solving an optimization problem with a (non-convex) integer-valued objective [2]. In fact, there are many instances in which the objective is continuous, but the problem of allocating power optimally across users and tones is known to be NP-hard [3]. For example, the problem of finding power-allocations that maximize the sum-rate was shown in [3] to be NP-hard. This problem can be solved approximately using the optimal spectrum balancing (OSB) algorithm developed in [4]. (The approximation in OSB follows from the discretization and exhaustive search that underlies this algorithm.) However, the computational cost of this algorithm is quite prohibitive, which makes it suitable only for systems with small number of users and tones. A less complex algorithm that can be used to provide approximate solutions for more practical systems is the so-called autonomous spectrum balancing (ASB) algorithm [5]. Unlike OSB, the ASB algorithm is decentralized in the sense that it does not require a central node to assign the power allocations to users. However, it requires auxiliary information that may not be always available in practice. Both the OSB and the ASB algorithms share common drawbacks. For instance, neither algorithm can be readily tailored to optimize alternative design objectives, nor to incorporate other design constraints. Moreover, neither algorithm takes QoS into direct consideration. In particular, both OSB and ASB tend to allocate power in such a way that favours stronger users to weaker ones. It is worth mentioning that, in addition to OSB and ASB, there are other power allocation techniques that can be used by the service provider to allocate the users' powers across tones. These techniques include the classic iterative water-filling algorithm (IWFA) [6] and the Successive Convex Approximation Low Complexity (SCALE) algorithm.

In contrast to other algorithms which focus on traditional sum-rate objectives, in this paper we consider the problem of maximizing the number of users that can be accommodated by the DSL system. The users belong to different categories depending on the QoS that they purchase from the system provider. This design objective is integer-valued and hence generally difficult to handle directly. As an alternative, we consider a design problem in which we maximize the (weighted) harmonic mean of the users' rates. This objective possesses two desirable features: first, from the users' perspective, this objective is known to be fairer than maximizing the sum-rate [3]; second, for single-tone systems, maximizing this objective can be cast as a convex optimization problem for which the global solution can be obtained efficiently. The first feature renders the harmonic mean a natural design objective for maximizing the number of users. This is because for one to be able to compare the number of users that can be supported by two systems, one ought to guarantee that the users obtain the same service in both systems. The second feature, on the other hand, enables us to design an efficient algorithm that exploits the per-tone convexity to provide power allocations that yield relatively high harmonic mean-rates. In particular, we begin by providing a convex lower bound on the harmonic mean-rate. Using

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the dual form, we decompose the problem of maximizing the convex lower bound into several convex optimization problems. These problems are not coupled across tones, and hence result in low design complexity. This feature renders this algorithm attractive for practical application in DSL systems with a large number of users and tones. Furthermore, in developing this algorithm we show how to incorporate different QoS levels. With the QoS guaranteed, we run an outer (quasi-bisection) algorithm for maximizing the number of users. In particular, for every number of users (with associated crosstalk coefficients), we solve a feasibility problem which serves as an indicator to whether this number of users can be supported by the system with the prescribed QoS levels. Finally, we provide numerical results that show that our approach can result in a DSL system that is fairer and supports more users than the more computationally-demanding approaches proposed in the literature.

2. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a DSL communication system in which N tones are shared by K users. (In practice, a 'user' may refer to a central office (CO) or a remote terminal (RT) that transmit data to a modem at the subscriber's end.) Let h_{jk}^n be the complex channel gain between the transmitter of User j and the receiver of User k on the n-th tone, where $n \in \mathcal{N} \triangleq \{1, \ldots, N\}$ and $j, k \in \mathcal{K} \triangleq \{1, \ldots, K\}$. In this notation h_{kk}^n denotes the channel gain between the transmitter of the k-th user and its intended receiver. Let $\alpha_{jk}^n \triangleq |h_{jk}^n|^2/|h_{kk}^n|^2$, and let s_k^n be the power allocated by User k to the n-th tone. Assuming that each user uses Gaussian signalling and that every user can only decode its intended messages, the maximum rate that User $k \in \mathcal{K}$ can achieve on the n-th tone is given by [7]

$$R_k^n(s_1^n, \ldots, s_K^n) = \log\Big(1 + \frac{s_k^n}{\Gamma\big(\sigma_k^n + \sum_{j \neq k} \alpha_{jk}^n s_j^n\big)}\Big), \quad (1)$$

where $\sigma_k^n \triangleq N_0/|h_{kk}^n|^2$ denotes the normalized noise variance observed by User k on the n-th tone, N_0 is the variance of the background Gaussian noise, and Γ is the so-called capacity gap, which is typically used to account for the non-Gaussianity of the signalling constellations used in practice [5]. Now, the total rate that can be reliably communicated by User k is given by $\sum_{n=1}^N R_k^n(s_1^n, \ldots, s_K^n)$.

Consider the situation in which the service provider wishes to maximize the number of users that the DSL system can support. For the service provider to do that, it may maximize the sum rate of all users, which is given by $\sum_{k=1}^K \sum_{n=1}^N R_k^n(s_1^n, \ldots, s_K^n)$. However, such an approach may result in power allocations that favour strong users to weaker ones. As an alternative, the service provider may consider a more balanced approach in which the objective is to maximize a weighted sum rate, $\sum_{k=1}^K w_k \sum_{n=1}^N R_k^n(s_1^n, \ldots, s_K^n)$ where the weights, $\{w_k\}_{k=1}^K$ are assigned in such a way that favours weak users to stronger ones. The drawback of this approach is that the way in which the weights ought to be assigned depends on the channel gains and the power budget in a non-linear fashion. An approach that might be more practical from a service provider's perspective is to maximize the harmonic mean-rate. This objective does not require weight assignment and is known to result in rates that are 'relatively' fair to all users [3]. The harmonic mean-rate can be written as

$$H(\mathbf{s}_1, \cdots, \mathbf{s}_K) = \Big(\sum_{k=1}^K \big(\sum_{n=1}^N R_k^n\big)^{-1}\Big)^{-1}. \quad (2)$$

Now, in order to find the power allocations that maximize this objective, we ought to solve the following optimization problem:

$$\begin{aligned} \min \quad & 1/H(\mathbf{s}_1, \cdots, \mathbf{s}_K), & (3a)\\ \text{subject to} \quad & \textstyle\sum_{n=1}^N s_k^n \leq P_k, \forall\, k, & (3b)\\ & 0 \leq s_k^n \leq S_{\max,k}^n, \forall\, k, & (3c) \end{aligned}$$

where in (3a), we have used the fact that maximizing $H(\mathbf{s}_1, \cdots, \mathbf{s}_K)$ is equivalent to minimizing $1/H(\mathbf{s}_1, \cdots, \mathbf{s}_K)$. We have also used $\mathbf{s}_k$ to denote the vector $[s_k^1, \cdots, s_k^N]^T$, P_k to denote the total power budget of User k, and $S_{\max,k}^n$ to denote the maximum signal power that User k can allocate to the n-th tone. In order for (3b) to be not redundant, we assume that $P_k \leq \sum_{n=1}^N S_{\max,k}^n$.

Although it is desirable from the service provider's perspective to be able to solve (3), for $N > 1$, this problem is known to be NP-hard [8], and hence difficult to solve in a computationally-efficient manner. As an alternative, in the next section we derive an upper bound on the objective in (3a). Unlike, the original problem in (3), this upper bound is convex and hence can be minimized using highly efficient numerical techniques.

3. AN UPPER BOUND AND QOS CONSTRAINTS

3.1. An upper bound on the harmonic mean-rate

In order to provide an approximate efficiently-computable solution for the NP-hard problem in (3), we begin by deriving an upper bound on $1/H(\mathbf{s}_0, \mathbf{s}_1, \cdots, \mathbf{s}_K)$. In particular, using the convexity of the function $f(x) = 1/x$, we invoke Jensen's inequality to write

$$\big(H(\mathbf{s}_1, \cdots, \mathbf{s}_K)\big)^{-1} = \sum_{k=1}^K \big(\sum_{n=1}^N R_k^n\big)^{-1} \leq \sum_{k=1}^K \sum_{n=1}^N (R_k^n)^{-1}, \quad (4)$$

where the inequality in (4) holds if and only if $N = 1$. Note that minimizing the right hand side (RHS) of (4) is equivalent to maximizing the harmonic mean of per-tone rates of users, whereas minimizing the left hand side of (4) is equivalent to maximizing the harmonic mean of the users' overall rates.

We will show that using the RHS of (4) as the design objective results in a convex optimization problem that possesses a special structure. In particular, by examining the Lagrange dual form, we manage to decompose the optimization problem into N decoupled convex optimization problems that can be solve with high efficiency. Consider the following optimization problem:

$$\begin{aligned} \min \quad & \textstyle\sum_{k=1}^K \sum_{n=1}^N (R_k^n)^{-1}, & (5a)\\ \text{subject to} \quad & \text{(3b) and (3c).} & (5b) \end{aligned}$$

Using the transformation in [8], we have

$$t_k^n = (R_k^n)^{-1}, \quad \text{and} \quad y_k^n = \log s_k^n. \quad (6)$$

Now, the optimization in (5) can be cast as

$$\begin{aligned} \min \quad & \textstyle\sum_{k=1}^K \sum_{n=1}^N t_k^n & (7a)\\ \text{subject to} \quad & y_k^n \leq \log(S_{\max,k}^n),\ t_k^n \geq 0, \quad \forall\, k, n, & (7b)\\ & \textstyle\sum_{n=1}^N 2^{y_k^n} \leq P_k, \qquad \forall\, k, n. & (7c)\\ & \log\Big(\big(\sigma_k^n 2^{-y_k^n} + \sum_{j \neq k} \alpha_{jk}^n 2^{(y_j^n - y_k^n)}\big)\big(2^{(t_k^n)^{-1}} - 1\big)\Big) \leq 0. & (7d) \end{aligned}$$

Using the observations in [8], it can be shown that this problem is convex, and hence, for small-to-moderate number of users and tones, can be solved using efficient interior point methods [2]. In Section 4 we will show how to exploit the structure of (7) to develop a highly efficient algorithm for solving problems with many users and tones.

3.2. QoS: Max Rate and Min Rate Constraints

In a DSL system the service provider determines the range of rates within which each user operates based on the QoS that this user has purchased. While the low endpoint of this range provides a guarantee on the minimum data rate that the user can operate at, the high endpoint may have an commercial value from the service provider's perspective. This is because the service provider is typically interested in limiting the maximum data rate that the user can communicate in order to encourage the user to purchase a higher QoS. If we denote the low and high endpoints by $R_{\min}$ and $R_{\max}$, respectively, the rate range over which User $k \in \mathcal{K}$ operates can be expressed as the set of rates that satisfy the following constraints:

$$\textstyle\sum_{n=1}^{N} R_k^n \leq R_{\max,k}, \quad \text{and} \quad \sum_{n=1}^{N} R_k^n \geq R_{\min,k}, \tag{8}$$

where R_k^n is defined in (1). Now, by applying the transformation in (6), it is easy to see that the first constraint in (8) can be readily incorporated in (7) without affecting the convexity of the optimization problem. Unfortunately, this transformation cannot be readily applied to cast the second constraint in (8) in a convex form. In order to circumvent this difficulty, for this constraint we consider a lower bound on $\sum_{n=1}^{N} R_k^n$. In particular, using (1), the concavity of the $\log(\cdot)$ function and applying Jensen's inequality one can show that

$$R_k^n \geq \log\big(1 + \sum_{j=1}^{K} \alpha_{jk}^n\big) + \frac{\log \sigma_k^n}{1 + \sum_{i=1}^{K} \alpha_{ik}^n} + \sum_{j=1}^{K} \frac{\alpha_{jk}^n \log s_j^n}{1 + \sum_{i=1}^{K} \alpha_{ik}^n} - \log\big(\sigma_k^n + \textstyle\sum_{j \neq k} \alpha_{jk}^n s_j^n\big) \triangleq R_{LB_k}^n \tag{9}$$

From (9) it can be seen that if $R_{LB_k}^n \geq R_{\min}$ then $R_k^n \geq R_{\min}$. Unlike the second constraint in (8), invoking the transformation in (6) the constraint that $R_{LB_k}^n \geq R_{\min}$ can be cast in a convex form and hence can be easily incorporated in (7). Prior to invoking the transformation, the resulting optimization problem can be cast as:

$$\begin{aligned} \min \quad & \textstyle\sum_{k=1}^{K} \sum_{n=1}^{N} (R_k^n)^{-1}, && \text{(10a)} \\ \text{subject to} \quad & \text{(3b) and (3c)}, && \text{(10b)} \\ & \textstyle\sum_{n=1}^{N} R_k^n \leq R_{\max,k}, \forall k \in \mathcal{K} && \text{(10c)} \\ & \textstyle\sum_{n=1}^{N} R_{LB_k}^n \geq R_{\min,k},\ \forall k \in \mathcal{K}, && \text{(10d)} \end{aligned}$$

where R_k^n and $R_{LB_k}^n$ are defined in (1) and (9), respectively.

4. A LAGRANGE DUAL-BASED SOLUTION

4.1. The Lagrange dual form

As pointed out in Section 3.1, even though (10) is convex, solving it directly can be quite computationally unwieldy for systems with large number of users and tones. In order to alleviate this drawback, we seek insight into the structure of (10) by considering the Lagrange dual function.

$$\begin{aligned} d(\lambda) = & \textstyle\sum_{k=1}^{K} \big(\varsigma_k R_{\min,k} - \lambda_k P_k - \mu_k R_{\max,k}\big) - \\ & \textstyle\sum_{k=1}^{K} \sum_{n=1}^{N} \Big(\log(1 + \sum_{j=1}^{K} \alpha_{jk}^n) + \dfrac{\log \sigma_k^n}{1 + \sum_{i=1}^{K} \alpha_{ik}^n}\Big) + \\ & \textstyle\sum_{n=1}^{N} \min\limits_{0 \leq s_k^n \leq S_{\max,k}^n} \Big\{ \sum_{k=1}^{K} \Big((R_k^n)^{-1} + \mu_k R_k^n + \lambda_k s_k^n \\ & + \varsigma_k \Big(\log\big(\sigma_k^n + \sum_{j \neq k} \alpha_{jk}^n s_j^n\big)\Big) - \sum_{j=1}^{K} \dfrac{\alpha_{jk}^n \log s_j^n}{1 + \sum_i \alpha_{ik}^n} \Big) \Big\}, \end{aligned} \tag{11}$$

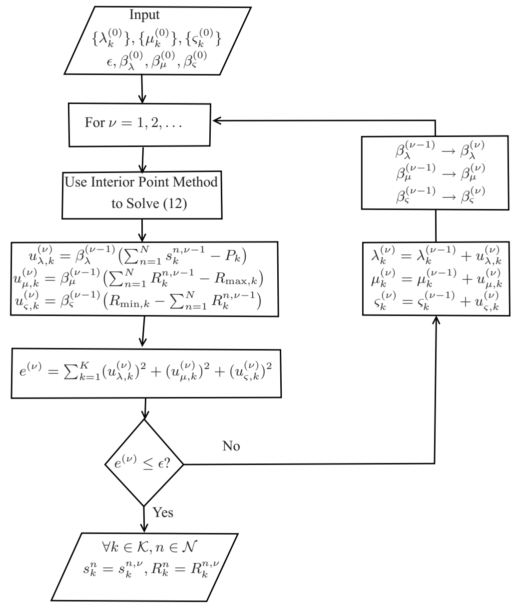

Fig. 1: Primal-dual updating algorithm for solving (10).

where $\lambda_k \geq 0$, $\varsigma_k \geq 0$, and $\mu_k \geq 0$ are the Lagrange multipliers that correspond to the constraints in (10).

In order to solve the per-tone minimization in (11), we use the transformation in (6) to cast this minimization as

$$\begin{aligned} \min \quad & \textstyle\sum_{k=1}^{K} \Big(t_k^n + \mu_k (t_k^n)^{-1} + \lambda_k 2^{y_k^n} + \varsigma_k \log\big(\sigma_k^n + \\ & \textstyle\sum_{j \neq k} \alpha_{jk}^n 2^{y_j^n}\big) - \varsigma_k \sum_{j=1}^{K} \dfrac{\alpha_{jk}^n y_j^n}{1 + \sum_i \alpha_{ik}^n} \Big), && \text{(12a)} \\ \text{subject to} \quad & y_k^n \leq \log(S_{\max,k}^n),\ t_k^n \geq 0, \qquad \forall\, k, && \text{(12b)} \\ & \log\big(\sigma_k^n 2^{-y_k^n} + \textstyle\sum_{j \neq k} \alpha_{jk}^n 2^{(y_j^n - y_k^n)}\big) \\ & \quad + \log(2^{(t_k^n)^{-1}} - 1) \leq 0. \qquad \forall\, k. && \text{(12c)} \end{aligned}$$

Using standard techniques [2], one can verify that, for a given set of dual variables $\{\lambda_k\}$, $\{\varsigma_k\}$, and $\{\mu_k\}$, the problem in (12) is convex and hence efficiently solvable using interior method techniques.

4.2. An efficient algorithm for solving (10)

In this section we use the observations made in the previous section to develop an efficient algorithm for solving (10). Our main strategy is to based on the standard primal-dual updating algorithm [2]. Our approach can be easily described using the flow chart in Figure 1. In this chart $s_k^{n,\nu}$ and $R_k^{n,\nu}$ are used to denote the power and rate of User k on the n-th tone at the ν-th iteration, respectively. The ν-th iterate of the dual variables are denoted by $\lambda_k^{(\nu)}$, $\mu_k^{(\nu)}$ and $\varsigma_k^{(\nu)}$. These variables are initiated with arbitrary positive values, and updated at each iteration ν. Using the iterates at the ν-th iteration, the problem in (12) is solved using an interior point method. Note that solving this problem is much less complex than solving (10) directly. This is because the problem in (12) is solved for each tone separately, whereas the one in (10) is coupled across the N tones and can be significantly more difficult to solve for large N. After solving (12), the dual variables are updated using a steepest-ascent algorithm with

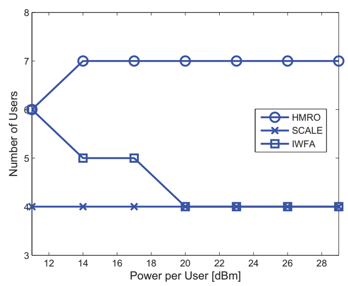

Fig. 2: Number of users supported by IWFA, SCALE and HMRO.

a diminishing stepsize. The algorithm stops after the sum of square difference between the old and the current dual variables falls below a small value $\epsilon \geq 0$. Note that because (10) is convex, this algorithm is guaranteed to converge to the global optimum, provided that the stepsize is chosen properly; see [2].

5. SIMULATION

In this section we compare the number of users that can be supported by the proposed harmonic mean-rate optimal (HMRO) algorithm with the number of users that can be supported by SCALE and IWFA in a DSL communication system with 256 tones. Assuming that there are seven users that we wish to accommodate in the DSL system, the crosstalk coefficients and the noise parameters of these users were generated using a practical DSL simulator.[1] The system model consists of 2 Central Office (CO) and 5 Remote Terminal (RT) lines, and all users are assumed to have identical power budgets. The lengths of the CO and RT lines are 5, 4, 3.5, 3.5, 3, 3 and 3 km, respectively, and the distances from the 5 RT's to the CO's are set to be 0.3, 0.5, 0.5, 3 and 3 km, respectively. The background noise variance is assumed to be $N_0 = -140$ dBm/Hz and the capacity gap is set to be 15 dB.The users are divided into basic and high-end service groups. The basic service group consists of Users 1 and 2 with $R_{\min,k} = 0.5$ Mbps and $R_{\max,k} = 2$ Mbps, $k = 1, 2$, and the high-end service group consists of Users 3–7 with $R_{\min,k} = 2$ Mbps and $R_{\max,k} = 12$ Mbps, $k = 3, \ldots, 7$.

Using these parameters, in Figure 2 we compare the number of users supported by SCALE and IWFA and the number of users supported by the proposed HMRO algorithm. From this figure it can be seen that for the considered range of power budgets, both SCALE and IWFA support fewer users than the proposed HMRO. For instance, SCALE can only support 4 users (Users 1 and 5–7) throughout the entire range of the considered power budgets. However, IWFA exhibits a more interesting behaviour. At an input power of 11 dBm, IWFA supports up to 6 users, but as the power increases, IWFA tends to favour stronger users (Users 6 and 7 in the current example) to weaker ones. This tendency eventually incurs a decrease in the number of users that the system can support. Finally, we consider the number of users that can be supported by the proposed HMRO. For an input power of 11 dBm, similar to IWFA, this algorithm supports 6 Users. However, by increasing the power budget, HMRO manages to accommodate all 7 users in the system. This performance advantage follows from the inherent fairness of the harmonic mean objective and the versatility with which the system designer can control the QoS of different classes of users.

[1] This simulator was provided by R. Cendrillon of Huawei Tech. Co. Ltd.

Finally, we compare the complexity of SCALE and IWFA with that of HMRO. Denoting the tolerance by ϵ, the complexity of IWFA and SCALE can be shown to be $\mathcal{O}\big(KN\log^2\epsilon\big)$ and $\mathcal{O}\big(KNL\log^2\epsilon\big)$, where L is the number of SCALE updates, respectively [9], whereas the complexity of HMRO is $\mathcal{O}(KN\log^2\epsilon)$. As a rough comparison, the average Matlab running time of the IWFA and SCALE for the scenario considered in this example is about 1 and 15 seconds, respectively, whereas that of HMRO is about 26 seconds. Hence, it can be seen that the computational complexity of HMRO is comparable to that of IWFA and SCALE, but it can support significantly more users than both algorithms.

6. CONCLUDING REMARKS

In this paper we have provided a technique for approaching the user capacity of a multitone DSL system; i.e., the maximum number of users that can be supported by the system under quality-of-service constraints. Since maximizing the number of users directly involves (non-convex) integer programming that is typically unwieldy to solve, we propose to use an indirect technique whereby we maximize a harmonic-mean rate objective. Unlike, other popular objectives, the harmonic-mean rate results in fair rate assignments that do not favour stronger users over weaker ones. In addition, the harmonic mean-rate possesses a per-tone convexity feature that enables us to devise a highly efficient algorithm for systems with a large number of tones. Numerical results show that the proposed technique enables the system to support significantly more users than systems designed to maximize sum or greedy individual rates.

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