GAME THEORY FOR PRECODING IN A MULTI-USER SYSTEM: BARGAINING FOR OVERALL BENEFITS

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ABSTRACT

A precoding strategy for multi-user spectrum sharing over an interference channel is proposed and analyzed from a game-theoretic perspective. The proposed strategy is based on finding the Nash bargaining solution for precoding matrices in a cooperative scenario over frequency selective channels under a spectrum mask constraint. An in-time update of the precoding matrices is enabled by using time slots to guarantee the effectiveness of the bargaining solution when the number of users varies. A dual decomposition approach is exploited to construct a distributed structure for solving the bargaining problem. The proposed distributed algorithm realizes the physical process of bargaining, which is not present in the Nash bargaining theory.

Index Terms— Linear precoding, interference channel, Nash bargaining, cooperative game, duality, dual decomposition.

1. INTRODUCTION

As the demand for spectrum resources keeps increasing, improving spectrum efficiency is necessary for alleviating the spectrum scarcity. One approach to improve spectrum efficiency is through the user cooperation on the same spectrum band, i.e., spectrum sharing [1], [2]. In most cases, wireless users in a system interfere with each other if they are active simultaneously, and the corresponding communication channel is called an *interference channel*. There is a significant amount of work studying the capacity on an interference channel from the information-theoretic perspective, such as [3]-[6]. A recent research topic is applications of game-theoretic approaches for investigation of interference channels.

Equilibria and bargaining theories of game theory can be explored to analyze the actions of the game players for non-cooperative and cooperative cases, respectively. There are some existing game theoretic studies of an interference channel for both cases. A twouser cooperative game over a flat fading interference channel is studied in [7], where the users agree to cooperate by sharing the spectrum in a frequency division multiplexing (FDM) manner. In [8], this game is extended to the case of multiple players communicating over a frequency selective channel, and joint time division multiplexing (TDM) and FDM is adopted. A two-user vector game over a multiple-input single-output (MISO) interference channel is investigated in and the non-cooperative and cooperative beamforming vectors are derived [9]. A matrix-valued multi-user non-cooperative precoding game over a frequency selective interference channel is analyzed in [10]. The cooperative Nash bargaining (NB) based solution for the precoding matrices for a two-user game is given in our previous paper [11].

In this paper, we first extend the result of [11] to the case of precoding matrices design for a multi-user game. Then an algorithm is developed to realize the bargaining process among users, and solve the bargaining problem in a distributed manner.

2. SYSTEM MODEL

Consider block transmissions in an M-user wireless communication system, for example, an orthogonal frequency-division multiplexing (OFDM) system. The sampled signal vector received by user i can be written as

$$\mathbf{y}_{i} = \mathbf{H}_{ii}\mathbf{F}_{i}\mathbf{s}_{i} + \sum_{j=1, j\neq i}^{j=M} \mathbf{H}_{ji}\mathbf{F}_{j}\mathbf{s}_{j} + \mathbf{n}_{i}, \quad i \in \Omega = \{1, 2, ...M\}$$
(1)

where \mathbf{H}_{ji} is the $N \times N$ matrix of sampled channel responses between transmitter j and receiver i (the channel is assumed to be wideband frequency selective), \mathbf{s}_i is the $N \times 1$ information symbol block of user i, \mathbf{F}_i is the $N \times N$ precoding matrix of user i, and \mathbf{n}_i is the $N \times 1$ additive Gaussian noise vector with $E\{\mathbf{n}_i\mathbf{n}_i^H\} = \sigma_i^2 \mathbf{I}$, \mathbf{I} denotes an identity matrix, and $(\cdot)^H$ stands for the Hermitian transpose. The information symbols are assumed to be uncorrelated and $E\{\mathbf{s}_i\mathbf{s}_i^H\} = \mathbf{I}$.

Consider the wireless users as players, their choices of precoding matrices as strategies, and the corresponding transmission rates as their payoffs. The precoding design problem can be viewed as a game in which benefit of each player depends on the precoding strategies of all other players. The utility space of this game is an *M*-dimensional rate region of the players. The information rate that a single user *i* can achieve under the strategy set of all users $\{\mathbf{F}_i | i \in \{1, 2, ..., M\}\}$ is [10]

$$R_{i} = \log(|\mathbf{I} + \mathbf{F}_{i}^{H}(\mathbf{H}_{ii}^{T})^{H}R_{-i}^{-1}\mathbf{H}_{ii}^{T}\mathbf{F}_{i}|), \quad \forall i$$
(2)

where $R_{-i} = \sigma_i^2 + \sum_{j=1, j \neq i}^{j=M} \mathbf{H}_{ji}^T \mathbf{F}_j \mathbf{F}_j^H (\mathbf{H}_{ji}^T)^H$ is the noise plus in-

terference for user *i*, and $|\cdot|$ and $(\cdot)^T$ stand for the determinant and transpose, respectively.

A spectral mask constraint is adopted to limit the maximal power that each user can allocate on a specific frequency bin. Denote the maximal power that user *i* can allocate on the frequency bin *k* as $p_i^{\max}(k)$. With proper cyclic prefix incorporation in transmitted symbols, the channel matrix \mathbf{H}_{ji} can be diagonalized as $\mathbf{H}_{ji} = \mathbf{W} \Omega_{ji} \mathbf{W}^H$, with **W** being the $N \times N$ IFFT matrix and Ω_{ji} being the following diagonal matrix $\Omega_{ji} = diag(H_{ji}(1), H_{ji}(2), ..., H_{ji}(N))$, where $H_{ji}(k)$ is the channel frequency-response of the *k*th frequency bin from transmitter *j* to receiver *i*. Then the spectral mask constraint for user *i* on frequency bin *k* can be expressed as [10]

$$E\{|[\mathbf{W}^{H}\mathbf{F}_{i}\mathbf{s}_{i}]_{k}|^{2}\} = [\mathbf{W}^{H}\mathbf{F}_{i}\mathbf{F}_{i}^{H}\mathbf{W}]_{kk} \le p_{i}^{\max}(k), \forall i, \forall k.$$
(3)

It is assumed that the receivers know the channel information perfectly and feed it back to the transmitters without errors.

3. PRECODING STRATEGY OVER FREQUENCY SELECTIVE CHANNEL: M-USER COOPERATIVE GAME

A Nash equilibrium (NE) solution for the *M*-user game is a set of precoding strategies $\{\mathbf{F}_i^{\rm NE}\}$ satisfying the spectrum mask constraint (3) and

$$R_i(\mathbf{F}_i^{\text{NE}})|_{\mathbf{F}_j = \mathbf{F}_j^{\text{NE}}, \ \forall j \neq i} \ge R_i(\mathbf{F}_i)|_{\mathbf{F}_j = \mathbf{F}_j^{\text{NE}}, \ \forall j \neq i}$$
(4)

which means that no precoding matrix \mathbf{F}_i can generate better payoff for user *i* than \mathbf{F}_j^{NE} given that all other users also apply their strategies in the NE. The NE based strategies are given as [10]

$$\mathbf{F}_{i}^{\text{NE}} = \mathbf{W} \sqrt{diag(\mathbf{p}_{i}^{\max})}, \quad \forall i$$
(5)

where $\mathbf{p}_i^{\max} = [p_i^{\max}(1), p_i^{\max}(2), ..., p_i^{\max}(N)]$ is the vector representing the spectral mask for user *i*. It is easy to verify that $\{\mathbf{F}_i^{\text{NE}}\}$ here also constitutes a dominant strategy equilibrium, in which each user chooses the best strategy independent on the choices of other users.

However, NE may result in an inefficient solution for all users. This is due to the lack of coordination among users. If the users are willing to cooperate with each other, they can expect better benefits. One possible approach to cooperate is through the time sharing on the frequency bins. By using certain frequency bins exclusively during certain portions of time, i.e., TDM/FDM, users are able to avoid interference from other users. Although TDM/FDM rate region (the utility space of the TDM/FDM cooperative game) is not the capacity region of the interference channel, it is an effective method for the users to cooperate. Moreover, the time sharing among users guarantees a convex utility space, and thus guarantees the existence of the NB solution.

As a part of cooperative game theory, the NB proposes a unique solution in a convex set that satisfies four axioms in [12], and maximizes the following Nash function

$$F = \prod_{i} (P_i - P_i^c) \tag{6}$$

where P_i is user *i*'s payoff (information rate) in cooperative case and P_i^c is user *i*'s payoff in non-cooperative case (NE). The NB solution, if exists, provides supplementary benefits for all users as compared to the non-cooperative solution. Moreover, the benefits among users are distributed based on the so called proportional fairness [13].

To perform the TDM/FDM and find the NB solution of precoding strategies in a cooperative game, we first partition the time into time slots, each with length T. Then it is easier for the users to perform time sharing. Each user is allocated some portion of time on certain frequency bins in every slot such that no user is kept waiting for a long time on any frequency bin. Moreover, considering the case when the number of users may change slowly over time, the partitioning of time slots enables an in-time update of bargaining solution if time slots are small enough, and previous solution can be terminated with least loss of correctness.

The cooperative solution can be obtained through the following steps:

- Step 1. Initialization: users are in non-cooperative state and the NE solution is obtained.
- *Step 2.* Computation: the cooperative NB solution for the precoding matrices is calculated.

Step 3. Implementation: Implement the NB solution for one time slot. If the number of players changes during this time slot, go back to step 1; otherwise, repeat step 3.

The following proposition is in order.

Proposition 1: Precoding matrices corresponding to the NB solution of the M-player TDM/FDM cooperative game over a frequency selective channel are in the form

$$\mathbf{F}_i = \mathbf{W} \mathbf{\Lambda}_i, \quad \forall i \in \{1, 2, ..., M\}$$
(7)

where $\Lambda_i = \Gamma_i(t) \sqrt{diag(\mathbf{p}_i^{\max})}$, $\Gamma_i(t)$ is a diagonal matrix with its *k*th element

$$\boldsymbol{\Gamma}_{i}^{k}(t) = \begin{cases} 1, & \text{if } t \in [b_{i}^{k}, e_{i}^{k}] \\ 0, & \text{if } t \notin [b_{i}^{k}, e_{i}^{k}] \end{cases}, \quad \forall i$$
(8)

 b_i^k and e_i^k represent, respectively, the starting and ending moments between which frequency bin k is allocated to user i in a slot [0, T]. The following conditions are satisfied

$$\sum_{i} \mathbf{\Gamma}_{i}(t) = \mathbf{I}, \quad \forall t \in [0, T]$$

$$\mathbf{\Gamma}_{i}(t) \odot \mathbf{\Gamma}_{j}(t) = \mathbf{0}, \quad \forall i \neq j, \; \forall t \in [0, T]$$
(9)

where $t \in [0, T]$ is the time instant in a current slot and \odot denotes the Hadamard product.

Equation (7) states that the diagonal structure remains the same in the cooperative game but the elements change. The reason that this diagonal structure remains unchanged follows from the property of the dominant strategy equilibrium, and the proof is omitted here due to space limitation. The first equation in (9) states that no frequency bin should be vacant at any time, while the second equation requests that no frequency bin be used by more than one user at any time.

It is the length of $[b_i^k, e_i^k]$ denoted as α_i^k , rather than the specific values of b_i^k and e_i^k , that affects the payoffs of the players. Once the time proportions α_i^k are fixed, the order of using frequency bins actually does not matter to the users. Thus, the key problem is to calculate the fraction of time α_i^k that user *i* obtains on a frequency bin *k*, for all users and all frequency bins. Mathematically, it can be formulated as the following optimization problem

$$\begin{array}{ll}
\max_{\{\alpha_i^k\}} & \prod_i (R_i - R_i^{\text{NE}}) \\
\text{s.t.} & 0 \le \alpha_i^k \le 1, \ \forall i, \ \forall k, \\
& \sum_i \alpha_i^k \le 1, \ \forall i, \ R_i > R_i^{\text{NE}}, \ \forall i.
\end{array}$$
(10)

According to the bargaining theory, the NB solution exists if and only if problem (10) has at least one feasible point. Also note that the problem is convex if $R_i > R_i^{\text{NE}}$ for all users. We next develop a distributed solution to this problem.

4. REALIZATION OF NB VIA A DISTRIBUTED STRUCTURE

Although the NB theory gives a solution of a cooperative game, it does not provide a constructive way of reaching this solution. Thus, the bargaining process of solving a specific game needs to be physically realized. From practical point of view, it is preferable to decompose the original problem (10) to distributively solvable subproblems. First note that the problem (10) can be rewritten as

$$\begin{array}{ll}
\max_{\{\alpha_i^k\}} & \sum_i \log(R_i - R_i^{\text{NE}}) \\
\text{s.t.} & 0 \le \alpha_i^k \le 1, \ \forall i, \ \forall k, \\
& \sum_i \alpha_i^k \le 1, \ \forall i, \ R_i > R_i^{\text{NE}}, \ \forall i.
\end{array}$$
(11)

This is a convex optimization problem with a coupling constraint, which can be solved through dual decomposition. The Lagrange dual problem of this problem is given as

$$\max_{\{\alpha_i^k\}} \sum_{i} \log(R_i - R_i^{\text{NE}}) - \sum_{k} \lambda^k (\sum_{i} \alpha_i^k - 1)$$
s.t. $0 \le \alpha_i^k \le 1, \forall i, \forall k, R_i > R_i^{\text{NE}}, \forall i.$
(12)

which can be further converted to a two-level optimization problem with the lower level subproblem given as ¹

$$\begin{array}{l} \max_{\{\alpha_i^k\}} & \log(R_i - R_i^{\text{NE}}) - \sum_k \lambda^k \alpha_i^k \\ \text{s.t.} & 0 \le \alpha_i^k \le 1, \ \forall k, \ R_i > R_i^{\text{NE}}, \ \forall i \end{array}$$
(13)

for each user *i*, and the higher level master problem given as

$$\begin{array}{ll} \min_{\{\lambda^k\}} & \sum_i U_i(\lambda) + \sum_k \lambda^k \\ \text{s.t.} & \lambda^k \ge 0, \ \forall k \end{array}$$
(14)

where $U_i(\lambda)$ is the maximum value of the objective function in (13) given $\lambda = [\lambda^1, \lambda^2, ..., \lambda^N]$. It is the dual problem, rather than the original one, that can be solved using the distributed structure with a coordinator. However, since the original problem is convex, strong duality holds and the solutions of the dual and the original problems are the same if slater condition [15] is satisfied. For this specific problem, we have the following proposition.

Proposition 2: The slater condition is guaranteed to be satisfied as long as the NB solution exists, i.e., there exists $0 \le \alpha_i^k \le 1$ such that

$$R_i > R_i^{\rm NE}, \ \forall i. \tag{15}$$

The proof is omitted here due to space limitations.

Note that proposition 1 in the previous section may be used to further simplify the dual problem. Substituting (7) into the objective function of the sub-problem (13) and considering the cooperation in a unit of time (T=1), the final form of the lower level subproblem can be written as

$$\begin{aligned} \max_{\alpha_i^k} & \log(\sum_{k=1}^N \alpha_i^k R_i^k - R_i^{\text{NE}}) - \sum_k \lambda^k \alpha_i^k \\ \text{s.t.} & 0 \le \alpha_i^k \le 1, \ \forall k, \ \sum_{k=1}^N \alpha_i^k R_i^k > R_i^{\text{NE}}, \forall i. \end{aligned}$$
(16)

where $R_i^k = \log(1 + |H_{ii}(k)|^2 p_i^{\max}(k) / \sigma^2)$ is the rate on the frequency bin k for user i.

The lower level subproblems are solved distributively by the corresponding users. Note that each Lagrange problem (16) is guaranteed to be convex and thus a unique solution exists. More importantly, the information required to solve the *i*th subproblem, i.e. R_i^k and R_i^{NE} , is local to user *i*.

Table 1. Dual decomposition algorithm for NB.

1. The coordinator initializes $\lambda = \lambda^0$ and broadcasts it to all
users.
2. Each user solves (16) according to the present value of λ
and transmits its solutions for $\{\alpha_i^k\}$ to the coordinator.

3. The coordinator updates λ according to the gradient of the master problem (14): $\hat{\lambda}^k = [\lambda^k - \delta(1 - \sum \alpha_i^k)]_+, \forall k$.

4. If $\forall k, |\hat{\lambda^k} - \lambda^k| \le \xi$, stop; otherwise broadcast $\hat{\lambda}$ and go to step 2.



Fig. 1. Instantaneous information rate of users and the corresponding values of Nash function versus number of iterations.

A coordinator is required to solve the higher level master problem. Since the overhead of information exchange and computation is not significant, one user may be selected as a coordinator. All users may also work as the coordinators in a round-robin manner.

The complete process of solving the dual problem can be summarized using the implementation algorithm shown in Table 1, where δ and ξ are the step length and the stopping threshold respectively, and $(\cdot)_+$ denotes the projection onto non-negative subspaces.

Note that the coefficients $\{\lambda_k\}_{k=1}^N$ have specific physical meanings. Indeed the coefficient λ_k represents the risk that cooperation among users breaks up due to a conflict on sharing frequency bin k. Thus, in the lower level subproblems, the objective for each user consists of two parts. Taking user i as an example, it can be described as follows. On one hand, a larger α_i^k is preferred to increase the total information rate. On the other hand, if α_i^k becomes too large, the cooperation may break up and the payoff of user i returns to the inferior competitive solution.

5. SIMULATION RESULTS

Consider a four-user case with six frequency bins. The channel is assumed to be Rayleigh fading with noise power equaling to 0.01 for each user.

For a given step length δ and stopping threshold ξ , the iterations of bargaining are shown in Fig. 1. The four curves on the upper side of the figure are the instantaneous information rates that the users can achieve, and the curve at the bottom shows the corresponding value of the Nash function, i.e., the objective function of the original optimization problem (10). The final NB and NE solutions and

¹This technique is similar to the one used in [14].

the comparison between them are shown in Table 2. It can be seen that all users gain supplementary benefit from cooperation. The corresponding final allocation of time portions on each frequency bin for each user is shown in Fig. 2. It can be seen that the frequency bins 1, 2, 3, and 4 are occupied exclusively by the users 3, 4, 1, and 2, respectively. The frequency bins 5 and 6 are shared between the users 1 and 4, and 2 and 3, respectively. Fig. 3 shows the effect



Fig. 2. Allocations of time portions on frequency bins $\{\alpha_i^k\}$.

of the step length on the convergence speed of the algorithm. With the values of $\delta \in \{0.3, 0.2, 0.1\}$, the values of the Nash functions are shown in the corresponding sub-figures. It can be seen that the algorithm is quite time-efficient with a good choice of step length.



Fig. 3. Nash function versus number of iterations under different step length, $\delta \in \{0.3, 0.2, 0.1\}$.

6. CONCLUSIONS

The precoding strategies for the multi-user cooperative game over frequency selective fading interference channels are developed. Using TDM/FDM in cooperation, the problem of finding the optimal precoding matrices in a cooperative case turns to be equivalent to the problem of determining time portions allocated to each user. The latter problem is convex and can be solved in a distributed manner using a dual decomposition method, which physically realizes the process of bargaining among users. The physical meaning of the Lagrange multipliers in the dual problem are shown to be the risks that

Table 2 . Value of NE and NB solutions corresponding to Fig	. 1	
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User	NE Solution	NB solution	Increased by
1	1.1296	2.2707	101.02%
2	1.4014	2.4906	77.72%
3	1.2952	2.3992	85.24%
4	1.6957	2.4175	42.56%

cooperation may break up. The simulation results demonstrate the advantages of the cooperative strategy over the non-cooperative one.

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