MULTI-ANTENNA COGNITIVE RADIO SYSTEMS: ENVIRONMENTAL LEARNING AND CHANNEL TRAINING

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ABSTRACT

This paper presents a multi-antenna cognitive radio (CR) system that is capable of operating concurrently with the primary radio (PR) link. The operation of the CR system consists of three stages: environmental learning, CR channel training and CR data transmission. In environmental learning stage, partial channel information between PR and CR are obtained blindly, based on which the transmit beamforming and the receive beamforming strategies are designed at CR to remove/reduce the interference to and from PR, respectively. We characterize all the interference values analytically and study the problem of *learning/training tradeoff* associated with the proposed scheme. The optimal balancing between learning and training is examined via the minimum mean square error (MSE) of the channel estimation. It is shown that for a given total learning/training time, there indeed exists a optimal learning time that minimizes the MSE of the channel estimation, yet the interference power to the PR is regulated.

Index Terms— Cognitive Radio, transmit beamforming, receive beamforming, learning, channel training.

1. INTRODUCTION

The original idea behind cognitive radio (CR) expects the CR user to detect the frequency bands that are not currently occupied by the primary radio (PR) and to start the opportunistic transmission on these empty bands, while the CR user must release the bands once PR user becomes active. As a consequence, spectrum sensing is recognized as the key technique and has attracted a lot of attentions [1]- [3].

With the introducing of multiple antennas at the CR transmitter (CR-Tx) [4], CR is allowed to transmit even if the PR link is active, provided that the resultant interference power or the so-called *interference temperature* at each PR terminal is kept below certain predefined threshold. Intuitively, with the aid of multiple antennas, CR-Tx could set a null along the direction from CR-Tx to PR, whereas a strong beam can be built along the direction from CR-Tx to CR receiver (CR-Rx).

In this work, we consider a more practical CR scenario, where both CR-Tx and CR-Rx are equipped with multiple antennas. This system is called multi-antenna CR system shown

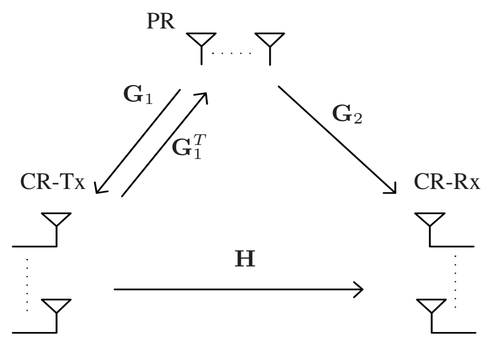

Fig. 1. System model for the multi-antenna CR system.

in Fig. 1, where there are M_1 antennas at CR-Tx and M_2 antennas at CR-Rx. Assume that both CR terminals stay within the boundary of M_p antennas of PR that operate in time division duplex (TDD) mode. For simplicity, we assume that these M_p antennas purely belong to one PR terminal,[1] whose function switches between the transmitting, that occupies a factor α of the overall time, and the receiving that occupies a factor $(1-\alpha)$ of the overall time. However, we do not expect CRs to know in which period PR is devoted to transmitting or receiving.

Let us represent the channels from PR to CR-Tx and CR-Rx by the $M_1 \times M_p$ matrix $\mathbf{G}_1$, and the $M_2 \times M_p$ matrix $\mathbf{G}_2$, respectively. The channel from CR-Tx to CR-Rx is denoted by the $M_2 \times M_1$ matrix $\mathbf{H}$. Since CRs operate in the same frequency at PR, the reverse channels from CR-Tx to PR is denoted $\mathbf{G}_1^T$. Although more general scenario when CR-Tx and CR-Rx operate under the TDD mode can be considered, here, we will only present the main idea from the one way transmission. Furthermore, we require more antennas at CRs, i.e., $M_1 > M_p$ and $M_2 > M_p$ which is a reasonable cost for CRs to achieve the concurrent transmission with PR. It is assumed that PR are oblivious to the existence of the CR link.

The operation of the multi-antenna CR system consists of three stages: environmental learning, CR channel training and CR data transmission. In environmental learning stage, partial channel information between PR and CR are obtained blindly, based on which the transmit beamforming and the receive

[1]Discussions considering PR transceiver pairs can be found in [5].

beamforming strategies are designed at CR to remove/reduce the interference to and from PR, respectively, for the period of CR channel training and data transmission.

2. INITIALIZING THE SYSTEM

Suppose CR is going to spend N symbol periods during the initialization, which is divided into N_l symbol periods for learning the channels from PR to CRs and $N_t = N - N_l$ symbol periods for training the channel from CR-Tx to CR-Rx.

2.1. Environmental Learning

Considering that PR switches between transmitting and receiving, the signals sent from PR can be expressed as

$$\mathbf{s}_p(n) = \begin{cases} \tilde{\mathbf{s}}_p(\tilde{n}) & \text{if PR transmits} \\ \mathbf{0} & \text{otherwise,} \end{cases} \qquad n = 1\ldots, N_l, \quad (1)$$

where $\tilde{n}$ is another set of index and $\tilde{\mathbf{s}}_p(\tilde{n})$ are the independent and identically distributed (i.i.d.) random signals with covariance matrix $\sigma_s^2\mathbf{I}$. Assuming that the learning period is sufficiently long, which is reasonable in order to achieve the reliable learning, there is $\mathbf{R}_p = \mathrm{E}[\mathbf{s}_p(n)\mathbf{s}_p^H(n)] = \alpha\sigma_s^2\mathbf{I}$.

The signals received at CR-Tx and CR-Rx are

$$\mathbf{y}_j(n) = \mathbf{G}_j\mathbf{s}_p(n) + \mathbf{z}_j(n), \quad n = 1, \ldots, N_l, \qquad (2)$$

for $j = 1, 2$, where $\mathbf{z}_j(n)$ represents the complex i.i.d. Gaussian noise, each entry having the variance σ_{nj}^2.

The covariance matrices of the received signals are

$$\mathbf{R}_j = \mathrm{E}[\mathbf{y}_j(n)\mathbf{y}_j^H(n)] = \underbrace{\alpha\sigma_s^2\mathbf{G}_j\mathbf{G}_j^H}_{\mathbf{Q}_j} + \sigma_{nj}^2\mathbf{I}. \qquad (3)$$

The eigen-decomposition (EVD) of $\mathbf{R}_j$ can be expressed as

$$\mathbf{R}_j = \mathbf{V}_j\mathbf{\Lambda}_j\mathbf{V}_j^H + \sigma_{nj}^2\mathbf{U}_j\mathbf{U}_j^H, \qquad (4)$$

where $\mathbf{V}_j$ is the $M_j \times M_p$ matrix that spans the same space as $\mathbf{G}_j$, while $\mathbf{U}_j$ is the $M_j \times (M_j - M_p)$ matrix that spans the orthogonal space of $\mathbf{G}_j$. Correspondingly, $\mathbf{\Lambda}_j$ is the diagonal matrix that contains largest M_p eigenvalues of $\mathbf{R}_j$.

If no additional training symbols are sent from PR to CRs, one can only obtain the subspace information of $\mathbf{G}_j$. Nonetheless, knowing $\mathbf{V}_j$ and $\mathbf{U}_j$ is sufficient to help design the CR systems. If we restrict CR-Tx to transmit only through the space spanned by $\mathbf{U}_1^*$ and CR-Rx to receive only through the space spanned by $\mathbf{U}_2$, then the interference to and from PR can be completely removed during CR transmission since $\mathbf{U}_j^H\mathbf{G}_j = \mathbf{0}$. This scheme is called *cognitive beamforming*.

Practically, however, CRs can only obtain a limited samples of the received signals. Then, the sample covariance matrix is constructed as

$$\hat{\mathbf{R}}_j = \sum_{n=1}^{N_l}\mathbf{y}_j(n)\mathbf{y}_j^H(n). \qquad (5)$$

By applying EVD to $\hat{\mathbf{R}}_j$, we obtain the noisy version of the matrices $\mathbf{U}_j$ as $\hat{\mathbf{U}}_j$. From [6], the first order perturbation in the estimated $\mathbf{U}_j$ is approximated by

$$\Delta\mathbf{U}_j = \hat{\mathbf{U}}_j - \mathbf{U}_j \approx -(\mathbf{Q}_j)^\dagger\Delta\mathbf{R}\mathbf{U}_j, \qquad (6)$$

where $\dagger$ denotes the pseudo-inverse and $\Delta\mathbf{R}_j = \hat{\mathbf{R}}_j - \mathbf{R}_j$.

2.2. CR Data Transmission

Before proceeding to the CR channel training, we need first recognize the channels that are needed at CR-Rx. To protect PR, the information symbols $\mathbf{d}(n)$ sent from CR-Tx will be precoded by the matrix $\hat{\mathbf{U}}_1^*$, named as *cognitive transmit beamforming*. The received signal at CR-Rx is then

$$\mathbf{y}(n) = \mathbf{H}\hat{\mathbf{U}}_1^*\mathbf{d}(n) + \mathbf{G}_2\mathbf{s}_p(n) + \mathbf{z}_2(n). \qquad (7)$$

The second term on the right hand side (RHS) denotes the interference from PR to CR, which was not handled in the existing literatures [4]. In this sense, we propose the concept of *cognitive receive beamforming*, i.e., CR-Rx will process the received signals by pre-multiplication with $\hat{\mathbf{U}}_2^H$. The received signal then becomes

$$\tilde{\mathbf{y}}(n) = \mathbf{F}\mathbf{d}(n) + \Delta\mathbf{U}_2^H\mathbf{G}_2\mathbf{s}_p(n) + \tilde{\mathbf{z}}_2(n), \qquad (8)$$

where $\mathbf{F} = \hat{\mathbf{U}}_2^H\mathbf{H}\hat{\mathbf{U}}_1^*$ and $\tilde{\mathbf{z}}_2(n) = \hat{\mathbf{U}}_2^H\mathbf{z}_2(n)$. The residue interference $\Delta\mathbf{U}_2^H\mathbf{G}_2\mathbf{s}_p(n)$ goes to zero if the estimated $\hat{\mathbf{U}}_2$ becomes perfect. Another advantage of applying both transmit and receive beamforming appears when CRs operate under TDD mode, where one can easily verify that the reverse channel from CR-Rx to CR-Tx becomes $\mathbf{F}^T$, which maintains the reciprocity of the TDD transmission and will lessen the burden of feeding back channels from both direction.

Therefore, the task of channel estimation should focus on estimating $\mathbf{F}$ considering both the residue interference and the equivalent noise.

2.3. CR Channel Estimation

Suppose the training sequence from CR-Tx contains $\mathbf{t}(n)$, $n = N_l + 1, \ldots, N_l + N_t$, which also must be precoded by $\hat{\mathbf{U}}_1$ in order to reduce the interference to PR.

Denote

$$\begin{aligned}
\tilde{\mathbf{Y}} &= [\tilde{\mathbf{y}}(N_l+1), \tilde{\mathbf{y}}(N_l+2), \ldots, \tilde{\mathbf{y}}(N_l+N_t)] \\
\mathbf{T} &= [\mathbf{t}(N_l+1), \mathbf{t}(N_l+2), \ldots, \mathbf{y}(N_l+N_t)] \\
\mathbf{S}_p &= [\mathbf{s}_p(N_l+1), \mathbf{s}_p(N_l+2), \ldots, \mathbf{s}_p(N_l+N_t)] \\
\tilde{\mathbf{Z}}_2 &= [\tilde{\mathbf{z}}_2(N_l+1), \tilde{\mathbf{z}}_2(N_l+2), \ldots, \tilde{\mathbf{z}}_2(N_l+N_t)].
\end{aligned}$$

In this work we do not assume any channel statistics at CR before the system initialization,[2] so the least square (LS) channel estimation of $\mathbf{F}$ is

$$\hat{\mathbf{F}} = \tilde{\mathbf{Y}}\mathbf{T}^\dagger = \mathbf{F} + (\Delta\mathbf{U}_2^H\mathbf{G}_2\mathbf{S}_p + \tilde{\mathbf{Z}}_2)\mathbf{T}^\dagger. \qquad (9)$$

[2] If channel statistics are known, we can use the linear minimum mean square error (LMMSE) channel estimator.

It can be calculated that

$$\begin{aligned}
&\mathrm{E}[\mathbf{S}_p^H\mathbf{G}_2^H\Delta\mathbf{U}_2\Delta\mathbf{U}_2^H\mathbf{G}_2\mathbf{S}_p] \\
&\overset{(a)}{=}\frac{\sigma_{n2}^2(M_2-M_p)}{N_l}\mathrm{E}[\mathbf{S}_p^H\mathbf{G}_2^H\mathbf{Q}_2^\dagger\mathbf{R}_2\mathbf{Q}_2^\dagger\mathbf{G}_2\mathbf{S}_p] \\
&=\frac{\sigma_{n2}^2(M_2-M_p)\alpha\sigma_s^2}{N_l} \\
&\qquad\times\Big(tr(\mathbf{G}_2^H\mathbf{Q}_2^\dagger\mathbf{G}_2)+\sigma_{n2}^2 tr(\mathbf{G}_2^H\mathbf{Q}_2^\dagger\mathbf{Q}_2^\dagger\mathbf{G}_2)\Big) \\
&=\frac{(M_2-M_p)\sigma_{n2}^2\beta_2}{N_l}\mathbf{I}, \qquad (10)
\end{aligned}$$

where "(a)" is derived by using the property [6]

$$\mathrm{E}[\Delta\mathbf{R}\mathbf{A}\Delta\mathbf{R}]=\frac{1}{N_l}tr(\mathbf{R}\mathbf{A})\mathbf{R}$$

for any matrix $\mathbf{A}$, and β_2 is defined as $M_p+\sigma_{n2}^2 tr(\mathbf{Q}_2^\dagger)$. Although the exact value of $\mathbf{Q}_2$ is not known to CR-Rx, which brings some trouble when identifying the system parameters, we may replace $\mathbf{Q}_2$ by its maximum likelihood (ML) estimate $\hat{\mathbf{Q}}_2$ that can be obtained according to the algorithms in [5].

Following the standard approach [7], the channel estimation targets to minimize the mean square error (MSE):

$$\begin{aligned}
J &\triangleq \mathrm{E}[tr((\hat{\mathbf{F}}-\mathbf{F})^H(\hat{\mathbf{F}}-\mathbf{F}))] \\
&=(M_2-M_p)\sigma_{n2}^2(\frac{\beta_2}{N_l}+1)tr((\mathbf{T}\mathbf{T}^H)^{-1}). \qquad (11)
\end{aligned}$$

Therefore, β_2 is the only scalar that needs to be informed to CR-Tx, which can be achieved via a very lower rate feedback channel.

Due to the non-perfect environmental learning, the residue interference $\mathbf{G}_1^T\Delta\mathbf{U}_1^*\mathbf{t}(n)$ is non-zero at PR. The interference at PR is normally characterized by the *interference temperature* defined as:

$$I(n)=\mathrm{E}[\|\mathbf{G}_1^T\hat{\mathbf{U}}_1^*\mathbf{t}(n)\|^2]=\frac{\|\mathbf{t}(n)\|^2\sigma_{n1}^2\beta_1}{\alpha\sigma_s^2N_l}, \qquad (12)$$

where similar derivation steps as in (10) are adopted and β_1 is defined as $M_p+\sigma_{n1}^2 tr(\mathbf{Q}_1^\dagger)$.

In fact, there is no way to restrict the instant interference $I(n)$ at each time slot n since $\hat{\mathbf{U}}_1$ itself contains the randomness. Therefore, we only need to deal with the average interference during the training, defined as

$$I=\frac{1}{N_t}\sum_{n=N_l+1}^{N_l+N_t}I(n)=\frac{\sigma_{n1}^2\beta_1 tr(\mathbf{T}\mathbf{T}^H)}{\alpha\sigma_s^2N_lN_t}. \qquad (13)$$

Suppose the average interference temperature that can be tolerated at PR is ζ. Then, the following constraint should be satisfied during the training:

$$tr(\mathbf{T}\mathbf{T}^H)\le\frac{\zeta\alpha\sigma_s^2N_lN_t}{\sigma_{n1}^2\beta_1}. \qquad (14)$$

Note that, the single scalar $\zeta\alpha\sigma_s^2$ should be a standard parameter that can be obtained from PR.

3. LEARNING/TRAINING TRADEOFF

At a first glance, larger value of N_l is desirable from the viewpoint of environmental learning at CRs. On the other side, larger value of N_t is preferable for channel estimation by assuming the average power of CR-Tx is P_a. Even if the total power constraint P_t is applied, one cannot expect N_t to be small; otherwise the average interference I in (13) will exceed the threshold. Note that a similar tradeoff has also been studied in [3] between spectrum sensing and data transmission for single antenna CR system. However here, we propose the tradeoff between the environmental learning and the channel training in a multi-antenna CR system.

3.1. Average Power Constraint

In this case, the maximum power that CR-Tx can spend during training is N_tP_a. The optimization is derived as

$$\begin{aligned}
&\min_{T,N_l,N_t}\ (\frac{\beta_2}{N_l}+1)tr((\mathbf{T}\mathbf{T}^H)^{-1}) \qquad (15)\\
&\text{s.t. } tr(\mathbf{T}\mathbf{T}^H)\le\min\left\{\frac{\zeta\alpha\sigma_s^2N_lN_t}{\sigma_{n1}^2\beta_1},N_tP_a\right\}, \\
&N_l+N_t=N,\quad N_t\ge(M_1-M_p),
\end{aligned}$$

where the last constraint is required to successfully execute the channel estimation. It can be easily known that the optimal $\mathbf{T}\mathbf{T}^H$ is a scale of identity matrix regardless of the parameters N_l, N_t. The optimization is decoupled into the following two cases:

1): When $P_a\ge\frac{\zeta\alpha\sigma_s^2N_l}{\sigma_{n1}^2\beta_1}$: $\mathbf{T}\mathbf{T}^H=\frac{\zeta\alpha\sigma_s^2N_lN_t}{\sigma_{n1}^2\beta_1(M_1-M_p)}\mathbf{I}$ and N_l should be found from

$$\begin{aligned}
&\min_{N_l}\ f_1(N_l)=\frac{1}{N_l(N-N_l)}(\frac{\beta_2}{N_l}+1) \qquad (16)\\
&\text{s.t. } N_l\le N-(M_1-M_p).
\end{aligned}$$

The solution can be directly found by checking all the roots of $\dot{f}(N_l)$ and the boundary point $N_l=N-(M_1-M_p)$, whose explicit expression is omitted due to the lack of space.

2): When $P_a<\frac{\zeta\alpha\sigma_s^2N_l}{\sigma_{n1}^2\beta_1}$: $\mathbf{T}\mathbf{T}^H=\frac{N_tP_a}{(M_1-M_p)}\mathbf{I}$ and N_l should be found from

$$\begin{aligned}
&\min_{N_l}\ f_2(N_l)=\frac{1}{(N-N_l)}(\frac{\beta_2}{N_l}+1) \qquad (17)\\
&\text{s.t. } N_l\le N-(M_1-M_p).
\end{aligned}$$

The optimal solution can be found similarly as in (16).

3.2. Total Power Constraint

In this case, we assume that the total power reserved for training is P_t. The optimization problem is the same as (15) except that the first constraint in (15) is replaced by $tr(\mathbf{T}\mathbf{T}^H)\le\min\left\{\frac{\zeta\alpha\sigma_s^2N_lN_t}{\sigma_{n1}^2\beta_1},P_t\right\}$. The related discussion is similar to that in Section 3.1 and is omitted for brevity.

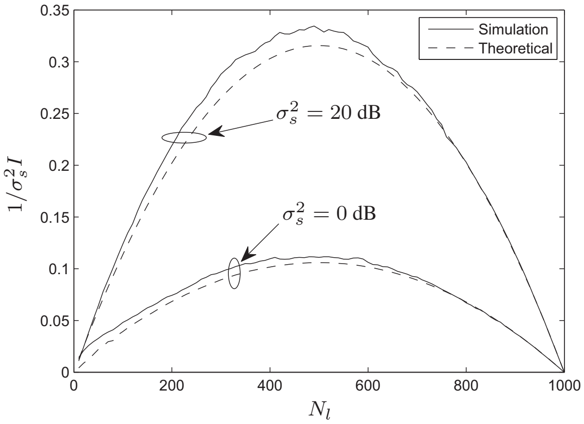

Fig. 2. Inverse of normalized interference temperature versus environmental learning time.

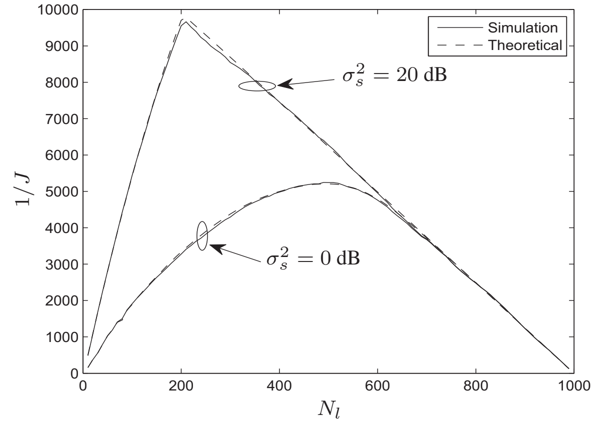

Fig. 3. Inverse of the channel estimation error versus environmental learning time.

4. SIMULATIONS

We consider a PR terminal with $M_p = 2$ antennas transmitting with probability $\alpha = 0.5$, and a CR system with $M_1 = M_2 = 4$ antennas. The total initialization time assigned for CR is $N = 1000$ and we fix the average transmit power of CR-Tx as 20 dB.

In the first example, we numerically examine the theoretical expression of the interference temperature (13) for $\sigma_s^2 = 0$ dB and $\sigma_s^2 = 20$ dB, respectively. The ML estimate $\hat{\mathbf{Q}}_j$, $j = 1, 2$ are used to derive β_j. The figure of merit is the inverse of the normalized interference temperature (INIT) $1/(\sigma_s^2 I)$. As shown in Fig. 2, the numerical and theoretical results match each other quite well. The higher σ_s^2 yields higher INIT because it has a smaller β_2.

In Fig. 3, we provide the numerical results of the inverse channel estimation MSE $1/J$ versus N_l for $\sigma_s^2 = 0$ dB and $\sigma_s^2 = 20$ dB, respectively. For simplicity, the threshold ζ is normalized according to $\zeta\alpha\sigma_s^2 = 1$ for different σ_s^2. The analytical performance curve is also displayed in the same figure. Clearly, analytical results match the numerical ones, which means that the solution N_l to (15) can well guide the optimality of the practical initialization.

5. CONCLUSIONS

In this work, we present a new CR scheme when both CR terminals are equipped with multiple antennas. With the help of the environmental learning, we design the transmit beamforming and receive beamforming that could reduce the interference to and from PR during the CR training and transmission. We found that there is a tradeoff between environmental learning and channel training if the overall initialization time is fixed. Finally, numerical examples are provided to corroborate the proposed studies.

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