TRANSMIT BEAMFORMING FOR WIRELESS MULTICASTING USING CHANNEL ORTHOGONALIZATION AND LOCAL REFINEMENT

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ABSTRACT

The problem of transmit beamforming for single-group multicasting is considered, where the objective is to transmit *common* information to a (large) number of users. The transmitter is assumed to have accurate downlink channel state information (CSI) for all users, and the objective is to design the beamformer weights to minimize the total transmitted power subject to meeting the quality-of-service (QoS) constraints of all users. This is an NP-hard problem that has recently drawn considerable interest (e.g., in the context of UMTS-LTE / E-MBMS). Several channel orthogonalization-based methods are proposed to solve this problem in an approximate way. Our techniques are shown to offer an improved performance-to-complexity tradeoff as compared to the original semidefinite relaxation (SDR) based multicasting technique.

Index Terms— Channel orthogonalization, multicasting, transmit beamforming

1. INTRODUCTION

Let us consider the single-group multicasting problem where a multiple-antenna transmitter has to broadcast information to multiple single-antenna receivers within a certain service area. The transmitter is assumed to have complete knowledge of the downlink user channel state information (CSI). Such a single-group multicasting problem has been considered in [1], where the following particular formulation of this problem has been addressed: Find the transmit beamformer weight vector that minimizes the total transmitted power subject to multiple quality-of-service (QoS) constraints, one for each receiver. The authors of [1] have proved that this problem is generally non-convex and NP-hard. They have proposed an elegant approach to approximately solve this problem using the semidefinite relaxation (SDR) technique [2], [3]. However, because of the SDR step, the approach of [1] is computationally quite demanding. Therefore, the design of simpler multicasting methods is of significant interest.

In this paper, we develop a new approach to approximately solve the aforementioned multiple-antenna multicasting problem using channel orthogonalization. The key idea of our approach is to orthogonalize the user downlink channel vectors (e.g., using the QR-decomposition technique or any of its variants such as the Gram-Schmidt algorithm) to satisfy the QoS constraints in a simple way. Using channel orthogonalization in this context was previously proposed in [4], but the approach of [4] demonstrates good results only when the number of users is less than the number of transmit antennas. In particular, the simulation results in [4] show that even for a relatively small number of users, the approach of [4] can reach the performance of the SDR method only when rather low numbers of randomizations are chosen in the latter technique.

In practice, the number of users is typically greater than the number of transmit antennas, and this is the case that we consider in this paper (also note that the NP-hardness proof in [1] applies to the case when the number of users is greater than or at least equal to the number of transmit antennas). Moreover, we propose a local search algorithm that can be combined with our approach to further improve its performance.

Several alternative techniques based on the ideas of channel orthogonalization and local search are formulated below. They are demonstrated to provide a substantially improved performance-to-complexity tradeoff as compared to the SDR technique of [1]. In particular, our techniques can be tuned to achieve nearly the same performance (in terms of the total transmitted power) as the SDR method of [1] at a substantially lower computational cost, or, alternatively, achieve an improved performance as compared to the SDR technique at nearly the same computational complexity.

2. PROBLEM FORMULATION

Let us consider a single-group wireless multicasting scenario where an N-antenna transmitter broadcasts data to M single-antenna users with frequency-flat quasi-static channels [1]. The signal-to-noise ratio (SNR) of the ith user is given by

$$\gamma_i = |\mathbf{w}^H \mathbf{h}_i|^2 / \sigma_i^2$$

where the $N \times 1$ vectors $\mathbf{w}$ and $\mathbf{h}_i$ are the transmit beamformer weight vector and the downlink channel vector of the ith user, respectively, σ_i^2 is the additive white Gaussian noise (AWGN) variance of the ith user, and $(\cdot)^H$ denotes the Hermitian transpose.

The problem of finding the weight vector that minimizes the total transmitted power subject to the user QoS constraints can be expressed as [1]:

$$\min_{\mathbf{w}} \|\mathbf{w}\|^2 \quad \text{s.t.} \quad |\mathbf{w}^H \tilde{\mathbf{h}}_i|^2 \geq 1 \ \text{ for all } \ i = 1, \ldots, M \qquad (1)$$

where

$$\tilde{\mathbf{h}}_i = \mathbf{h}_i / \sqrt{\gamma_{\min,i} {\sigma_i}^2}$$

is the ith user's normalized downlink channel vector and $\gamma_{\min,i}$ is the prescribed minimum SNR for the ith user.

3. SDR TECHNIQUE

The problem in (1) is a quadratically constrained quadratic program (QCQP) with non-convex constraints [3]. It has been shown in [1]

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that this problem is NP-hard. Defining the matrices

$$\mathbf{Q}_i \triangleq \tilde{\mathbf{h}}_i \tilde{\mathbf{h}}_i^H, \quad \mathbf{X} \triangleq \mathbf{w}\mathbf{w}^H$$

and using the fact that

$$|\mathbf{w}^H \tilde{\mathbf{h}}_i|^2 = \text{trace}\{\mathbf{w}\mathbf{w}^H \tilde{\mathbf{h}}_i \tilde{\mathbf{h}}_i^H\}$$

the problem in (1) can be rewritten as [1]:

$$\min_{\mathbf{X}} \text{trace}\{\mathbf{X}\} \quad \text{s.t.} \quad \text{trace}\{\mathbf{X}\mathbf{Q}_i\} \geq 1, \ \ i = 1, \ldots, M, \qquad (2)$$
$$\mathbf{X} \succeq 0, \ \ \text{rank}\{\mathbf{X}\} = 1.$$

Following the SDR approach, the authors of [1] proposed to relax the problem in (2) by dropping the rank constraint $\text{rank}\{\mathbf{X}\} = 1$. This results in a convex semidefinite programming (SDP) problem which can be solved in polynomial time using available convex optimization tools [5].

It has been found numerically in [1] that the optimal solution of the latter SDP problem, $\mathbf{X}_{\text{opt}}$, generally is not rank-one. In the latter case, to obtain the optimal weight vector from $\mathbf{X}_{\text{opt}}$, randomization techniques have to be used [1]. The main idea of these techniques is to use $\mathbf{X}_{\text{opt}}$ for generating multiple candidate weight vectors and to select the candidate vector that requires the smallest scaling to satisfy (2). The dominant computational cost of the SDR approach is given by $\mathcal{O}\left((M + N^2)^{3.5}\right)$ [1].

4. THE PROPOSED APPROACH

Since generally we have more users than transmit antennas, let us choose a subset of N channel vectors from $\{\tilde{\mathbf{h}}_i\}_{i=1}^M$ to generate a set of orthonormal vectors $\{\mathbf{q}_i\}_{i=1}^N$. As the vectors $\mathbf{q}_i$ $(i = 1, \ldots, N)$ span the whole N-dimensional space, the desired weight vector $\mathbf{w}$ can be represented as

$$\mathbf{w} = \sum_{i=1}^{N} c_i \mathbf{q}_i \qquad (3)$$

where $\mathbf{c} = [c_1, \ldots, c_N]^T$ is the vector of complex coefficients and $(\cdot)^T$ denotes the transpose. From (3), it follows that

$$\|\mathbf{w}\|^2 = \|\mathbf{c}\|^2. \qquad (4)$$

The key idea of our approach is to choose each component of $\mathbf{w}$ in (3) to satisfy the QoS constraints corresponding to the chosen subset of channel vectors with equality. The remaining $M - N$ QoS constraints can be then satisfied by scaling the so-obtained vector $\mathbf{w}$ so that the most violated constraint is satisfied with equality.

In Subsection 4.1, we introduce a new multicasting technique that is based on the QR-decomposition of the channel vectors. In Subsection 4.2, we consider a particular case of using the Gram-Schmidt algorithm to compute the QR-decomposition. As the way of selecting the subset of N channel vectors out of the available M vectors may be critical for performance, in both our techniques we use multiple choices of channel vector subsets involved in the orthogonalization process.

In Subsection 4.3, a simple local search technique is proposed that can be used in conjunction with the developed channel orthogonalization based techniques to further improve their performance.

4.1. The QR-Decomposition Based Technique

For the sake of clarity, let us hereafter assume that $M \geq N$ (note that the extension to the reverse case $M < N$ is trivial). Let us consider the $N \times M$ matrix $\mathbf{G} \triangleq [\tilde{\mathbf{h}}_1, \ldots, \tilde{\mathbf{h}}_M]$ whose columns are the vectors $\tilde{\mathbf{h}}_i$ $(i = 1, \ldots, M)$. Let the $N \times N$ matrix $\mathbf{H}$ be obtained by dropping any $M - N$ columns of $\mathbf{G}$ and possibly reordering the remaining N columns. Applying QR-decomposition to $\mathbf{H}$, we have

$$\mathbf{H} = [\mathbf{q}_1, \ldots, \mathbf{q}_N] \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1N} \\ 0 & \gamma_{22} & \cdots & \gamma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \gamma_{NN} \end{bmatrix} \triangleq \mathbf{Q}\mathbf{R} \qquad (5)$$

where $\gamma_{ii} > 0$ for all $i = 1, \ldots, N$.

Using (3)-(5) and keeping in (1) only the N QoS constraints that correspond to the columns of $\mathbf{H}$, the latter problem can be rewritten as

$$\min_{\mathbf{c}} \|\mathbf{c}\|^2 \quad \text{s.t.} \quad |\mathbf{c}^H \mathbf{r}_i| \geq 1 \ \ \text{for all} \ \ i = 1, \ldots, N \qquad (6)$$

where $\mathbf{r}_i$ is the ith column of $\mathbf{R}$.

Although the problem (6) has the same mathematical form as (1), an important difference between these two problems is that the vectors $\mathbf{r}_i$ inherit the upper-triangular structure of the matrix $\mathbf{R}$. Moreover, the number of constraints in (6) is less than in (1). These two facts make it possible to satisfy the constraints in (6) with equalities by successive computation of the coefficients c_i, $i = 1, \ldots, N$. In particular, from the first constraint $|\mathbf{c}^H \mathbf{r}_1| = 1$, we have

$$|c_1 \gamma_{11}| = 1$$

and, therefore, $|c_1| = 1/\gamma_{11}$. Note that the phase of c_1 can be chosen arbitrarily. Indeed, due to the successive nature of computing the coefficients c_i $(i = 1, \ldots, N)$, any change of $\arg\{c_1\}$ will only cause a rotation of the computed weight vector and, clearly, such a rotation will not affect the cost function. Therefore, without any loss of generality, we set $\arg\{c_1\} = 0$, that is, the first coefficient is computed as

$$c_1 = 1/\gamma_{11}. \qquad (7)$$

From the kth constraint $|\mathbf{c}^H \mathbf{r}_k| = 1$ $(k = 2, \ldots, N)$, we have:

$$\left| \sum_{i=1}^{k} c_i^* \gamma_{ik} \right| = 1 \qquad (8)$$

where $(\cdot)^*$ denotes the complex conjugate. Defining

$$\beta_k \triangleq \sum_{i=1}^{k-1} c_i^* \gamma_{ik}$$

for $k = 2, \ldots, K$, we can rewrite (8) as

$$|c_k^* \gamma_{kk} + \beta_k| = 1. \qquad (9)$$

Equation (9) illustrates the kth step of our successive algorithm to compute the vector $\mathbf{c}$. In this step, all c_i for $i = 1, \ldots, k-1$ have already been computed (that is, β_k is given), and c_k should be computed from (9) so that the increase of the cost function $\|\mathbf{c}\|^2$ caused by c_k is minimal. Clearly, this is equivalent to choosing c_k that satisfies (9) and has the smallest absolute value.

From (9) and nonnegativity of γ_{kk}, it readily follows that such an optimal value of c_k should satisfy the following property:

$$\arg\{c_k\} = \begin{cases} -\arg\{\beta_k\}, & |\beta_k| < 1, \\ -\arg\{\beta_k\} + \pi, & |\beta_k| > 1. \end{cases} \qquad (10)$$

Note that in the case $|\beta_k| = 1$, $c_k = 0$ and, therefore, the value of $\arg\{c_k\}$ is immaterial.

From (10), we have that

$$|c_k^* \gamma_{kk} + \beta_k| = \begin{cases} |c_k|\gamma_{kk} + |\beta_k| = 1, & |\beta_k| < 1, \\ |\beta_k| - |c_k|\gamma_{kk} = 1, & |\beta_k| > 1. \end{cases} \quad (11)$$

From (10) and (11), it readily follows that in the kth step ($k = 2, \ldots, N$) of our sequential algorithm and for any $|\beta_k|$, the coefficient c_k can be obtained as

$$c_k = \frac{1 - |\beta_k|}{\gamma_{kk}} e^{-j \arg\{\beta_k\}}. \quad (12)$$

Equations (7) and (12) define the proposed technique to successively compute the coefficients c_k, from $k = 1$ to $k = N$. After computing the whole coefficient vector $\mathbf{c}$ in this way, the associated weight vector can be obtained from (3). The remaining $M - N$ QoS constraints to be satisfied are those corresponding to the $M - N$ dropped columns of $\mathbf{G}$. To satisfy these constraints, we check all of them and then rescale the resulting weight vector so that the most violated remaining constraint is satisfied with equality.

Since the initial choice of the columns of $\mathbf{H}$ (and their particular order in $\mathbf{H}$) can greatly affect the resulting performance, we compute multiple candidate values of $\mathbf{w}$ that correspond to different choices of dropped columns in $\mathbf{G}$ and different orders of the remaining columns in $\mathbf{H}$. From these candidate weight vectors, the vector with the smallest norm (i.e., with the lowest total transmitted power) is finally chosen.

The process of finding the best (in terms of performance) ordered subset of N vectors out of the set of M channel vectors $\{\tilde{\mathbf{h}}_i\}_{i=1}^M$ requires checking $M!/(M-N)!$ possibilities. Clearly, for large M and N this can be computationally prohibitive. Therefore, we propose to check $n_{\mathrm{QR}} \ll M!/(M-N)!$ random permutations where n_{QR} is a design parameter that can be used to trade off between computational complexity and performance. The resulting dominant complexity of our algorithm is given by $\mathcal{O}(n_{\mathrm{QR}}(N^3 + MN))$. Therefore, for a reasonably low choice of n_{QR}, the proposed technique represents a computationally attractive alternative to the SDR technique.

4.2. The Gram-Schmidt Orthogonalization Based Technique

As the computational complexity of the QR-decomposition technique can be still rather high, in this subsection we consider a computationally more efficient *ad hoc* approach to properly choose the columns of $\mathbf{H}$ and their order in the case when the Gram-Schmidt procedure is used to orthogonalize the channel vectors.

We start by choosing an arbitrary initial channel vector $\mathbf{f}_1$ from the set $\{\tilde{\mathbf{h}}_i\}_{i=1}^M$. Hereafter, we denote the N vectors chosen from this set at the N steps of the Gram-Schmidt procedure as $\mathbf{f}_i$, $i = 1, \ldots, N$ (the way of selecting these vectors will be discussed in the sequel). The Gram-Schmidt orthogonalization process can be written as

$$\mathbf{b}_k = \mathbf{f}_k - \sum_{i=1}^{k-1} \mathbf{q}_i^H \mathbf{f}_k \, \mathbf{q}_i, \quad \mathbf{q}_k = \mathbf{b}_k / \|\mathbf{b}_k\| \quad (13)$$

for $k = 2, \ldots, N$ where $\mathbf{q}_1 = \mathbf{f}_1 / \|\mathbf{f}_1\|$. In each kth step, the intermediate weight vector can be computed as:

$$\mathbf{w}_k = \sum_{i=1}^{k-1} c_i \mathbf{q}_i \quad (14)$$

where the principle of computing the coefficients c_i is the same as in the QR-decomposition based technique discussed in the previous subsection. Our key idea of selecting the channel vectors $\mathbf{f}_i$ from $\{\tilde{\mathbf{h}}_i\}_{i=1}^M$ can be described as follows. In each kth step of the Gram-Schmidt procedure, we select the vector for which it is most difficult to satisfy the corresponding QoS constraint. That is, the vector having the smallest magnitude of its inner product with $\mathbf{w}_k$ is selected. It can be shown that this criterion for selecting the channel vectors at any kth step amounts to choosing the vector that corresponds to the smallest $|\beta_k|$. As the component that is added to the weight vector in any step is orthogonal to the channel vectors selected in the previous steps, it will not affect any of the previously satisfied constraints.

Finally, (3) is used to compute the resulting $\mathbf{w}$. This vector is then rescaled to satisfy the most violated of the remaining $M - N$ constraints with equality.

The whole orthogonalization process has to be repeated M times, where each time a new channel vector is chosen as the initial vector $\mathbf{f}_1$ for the Gram-Schmidt procedure. As a result, we end up with M candidate weight vectors and the one having the smallest norm is chosen as the final weight vector. The complexity of this technique is $\mathcal{O}(MN^3 + M^2N)$.

4.3. Refinements by local search

We have found that a simple approach can be used to improve the multicasting techniques developed in Subsections 4.1 and 4.2. The idea is to perform an unconstrained local search for any candidate weight vector $\mathbf{w}_{\mathrm{cand},i}$ used in these techniques. The algorithm takes this vector as a starting point and searches for another vector $\tilde{\mathbf{w}}_i$ in its neighborhood that maximizes the worst user SNR. This is achieved by finding a local maximum of the function

$$f(\tilde{\mathbf{w}}_i) = \min_k |\tilde{\mathbf{w}}_i^H \tilde{\mathbf{h}}_k|$$

for all values of i. The resulting vectors are then treated as the candidate weight vectors. Note that global maximization of the worst user SNR under a power constraint is also non-convex, NP-hard, and closely related to our original problem [1]; but what we advocate here is only local refinement, which can be easily accomplished using a variety of standard methods such as the Nedler-Mead simplex algorithm which is used in the MATLAB function `fminsearch`.

5. SIMULATION RESULTS

Throughout our simulations, we assume a Rayleigh fading channel with i.i.d. circularly symmetric unit-variance channel coefficients.

In all our examples, we compare the two proposed techniques (both combined with the local search algorithm) and the SDR technique of [1]. The acronyms QRLS and GSLS stand for the QR-decomposition based technique with local search and the Gram-Schmidt technique with local search, respectively. The `fminsearch` MATLAB function has been used for the local search. All our results are averaged over 1000 Monte Carlo runs. To implement the SDR approach, we have followed the guidelines of [1] where three different randomization procedures have been used in parallel, with 1000 randomizations for each. Throughout the simulation examples, it is assumed that $\sigma_i^2 = 1$ for all $i = 1, \ldots, M$. The parameters of the QRLS and GSLS have been chosen in two different regimes:

Fast regime to make the total complexity of the QRLS and GSLS techniques substantially lower than that of the SDR technique. In this regime, $N_{\mathrm{QR}} = 50$ has been taken, and the following parameters of the `fminsearch` function have been chosen: `MaxFunEvals` $= 500$ and `MaxFunEvals` $= 125$ for GSLS and

QRLS, respectively; and MaxIter $= 200$ and MaxIter $= 125$ for GSLS and QRLS, respectively. In this regime, according to a simple MATLAB time count comparison, the GSLS and QRLS computation times are on average 21% and 42% of that of the SDR technique, respectively.

Slow regime to make the total complexity of the QRLS and GSLS techniques similar to that of the SDR technique. In this case, in the GSLS method the tolerance parameter TolX of fminsearch has been set equal to 10^{-3} and the parameters MaxFunEvals and MaxFunEvals were not limited. In the QRLS case, the following parameters have been selected: MaxFunEvals $= 1000$ and MaxFunEvals $= 400$. In this regime, the GSLS and QRLS MATLAB times are on average 93% and 116% of that of the SDR technique, respectively.

In the first example, we assume that the prescribed minimum SNR $\gamma_{\min,i} = 1$ and use the so-called *boost ratio* [1]

$$\eta = \|\mathbf{w}_{\text{fin}}\|^2/\text{trace}\{\mathbf{X}_{\text{opt}}\}$$

to characterize the performance, where $\mathbf{w}_{\text{fin}}$ is the final beamformer weight vector of each technique tested and $\mathbf{X}_{opt}$ is the solution of the relaxed problem obtained from (2) by dropping the rank constraint. It should be stressed that $\text{trace}\{\mathbf{X}_{\text{opt}}\}$ is a lower bound on the transmitted power corresponding to the original problem (1); see [1] for more details.

The mean and the standard deviation (std) values of the boost ratio are summarized in Table 1 for different values of N and M. As can be observed from this table, the QRLS and GSLS techniques in the slow regime have generally improved values of the boost ratio as compared to the SDR technique. In the fast regime, the boost ratios of the QRLS are GSLS methods are generally comparable to that of the SDR method.

Table 1. Comparison of the boost ratios of the QRLS, GSLS, and SDR techniques.

N/M	4/8		4/16		8/16	
	Mean	Std	Mean	Std	Mean	Std
SDR	1.13	0.16	1.49	0.31	1.82	0.38
QRLS fast	1.27	0.14	1.52	0.22	1.98	0.30
GSLS fast	1.27	0.16	1.44	0.24	1.78	0.30
QRLS slow	1.10	0.09	1.38	0.19	1.84	0.25
GSLS slow	1.07	0.11	1.27	0.19	1.45	0.19

In our second example, we compare the total transmitted power of the SDR, QRLS, and GSLS techniques versus the prescribed minimum SNR. In this example, we assume that $N = 4$ and $M = 16$, and all the other parameters are the same as in the previous example.

From this figure, we observe that the "slow" GSLS and QRLS techniques have a significantly reduced total transmit power as compared to the SDR technique. For example, the power improvements of "slow" GSLS with respect to the SDR technique are close to 1 dB for all the SNR values tested. The "fast" GSLS and QRLS result in nearly the same transmit power values as the SDR method.

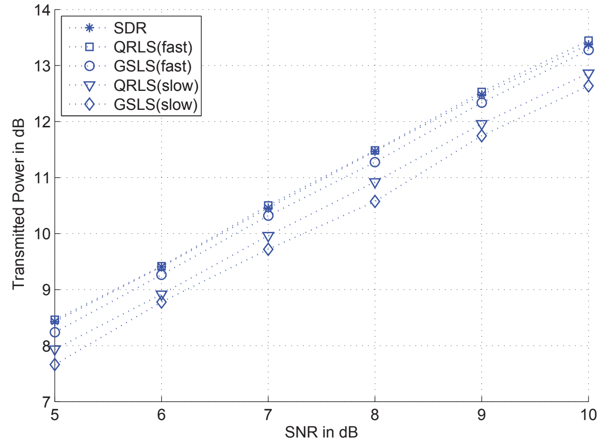

Fig. 1. Total transmitted power versus prescribed minimum SNR.

Therefore, the proposed techniques offer improved (and more flexible) performance-to-complexity tradeoffs than the SDR approach.

6. CONCLUSIONS

Several methods have been proposed to approximately solve the single-group multicasting problem using a combination of channel orthogonalization and local refinement. The proposed techniques have been shown to offer improved and more flexible performance-to-complexity tradeoffs as compared to the popular SDR multicasting technique. In particular, our techniques can be tuned to achieve nearly the same performance as the SDR method at a significantly lower computational cost, or, alternatively, achieve a substantially improved performance relative to the SDR technique at nearly the same complexity.

7. REFERENCES

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