OPTIMAL NETWORK BEAMFORMING FOR BI-DIRECTIONAL RELAY NETWORKS

Veria Havary-Nassab

Dept. Electrical and Computer Engineering
University of Toronto
10 King's College Road, Toronto, ON, M5S 3G4, Canada

Shahram Shahbazpanahi and *Ali Grami*

Faculty of Eng. and Applied Science
Univ. of Ontario Institute of Technology
2000 Simcoe St. N., Oshawa, ON, L1H 7K4, Canada

ABSTRACT

We consider a relay network which consists of two transceivers and r relay nodes. We study a half-duplex two-way relaying scheme. First, the two transceivers transmit their information symbols simultaneously and the relays receive a noisy mixture of the two transceiver signals. Then each relay adjusts the phase and the amplitude of its received signal by multiplying it with a complex beamforming coefficient and transmits the so-obtained signal. Aiming at optimally calculating the beamforming weight vector as well as the transceiver transmit powers, we minimize the *total* transmit power subject to two constraints on the receive signal-to-noise ratios (SNRs) at the two transceivers. We show that the optimal weight vector can be obtained through a simple iterative algorithm which enjoys a linear computational complexity per iteration.

Index Terms— Distributed beamforming, two-way relaying, distributed signal processing, bi-directional relaying, optimal power allocation, wireless relay networks.

1. INTRODUCTION

In cooperative wireless networks, different users act as relay nodes and collaborate with each other to establish a communication link between a transmitter and a receiver [1, 2]. The numerous relaying schemes presented in the literature study different kinds of processing on the signal received at the relay nodes. Examples of such schemes are amplify-and-forward approach, estimate-and-forward technique, decode-and-forward strategy, and compress-and-forward method. Typically, in a one-way relaying scheme, the communication is established in two steps. In the first step, the transmitter broadcasts its symbols to the relays. Then each relay processes its received signal based on the underlying relaying scheme and produces a new signal. In the second step, the so-obtained signal is transmitted by the relays to the receiver.

Recently, decentralized beamforming techniques have been presented for relaying schemes where the relays re-transmit an amplitude- and phase-adjusted version of their received signals [3–6]. Also, the problem of joint uplink-downlink beamforming has been studied for a single-relay network where the relay is equipped with multiple antennas [7]. Majority of the results published on decentralized beamforming consider a one-way relaying scheme where the relay nodes cooperate with each other to deliver the symbols transmitted by a source (or several sources) of information to a destination (or to several destinations). The literature considering *two-way* relaying strategies is scarce as compared to the volume of the published reports on one-way relaying schemes. In this paper, we study the problem of optimal decentralized beamforming for bi-directional (or two-way) wireless relay networks.

In a bi-directional relay network, the relays cooperate with each other to establish a reliable connection between two transceivers. Most of the published results on two-way relaying strategies consider the case of single relay strategies. Designing optimal bi-directional relaying schemes for a network with multiple relays is our focus in this paper. We study the problem of network beamforming for two-way relay communications. We consider a network consisting of two transceivers and r relay nodes. We assume that there is no direct link between the two transceivers, and therefore, they must communicate with each other through the relay network. Our relaying scheme consists of two phases. In the first phase, the two transceivers transmit their data to the relaying nodes. In the second phase, each relay node re-transmits an amplified and phase-steered version of its received signal. In fact, each relay multiplies its received signal by a complex coefficient and transmits the so-obtained signal in the second phase thereby collectively building a beam which covers both transceivers. Our goal is to optimally obtain the relay beamforming complex coefficients using a total transmit power minimization approach. More specifically, we aim to minimize the total transmit power, dissipated in the whole network, subject to two constraints on the quality of service (QoS) of the two transceivers. We show that this power minimization problem can be solved efficiently using an iterative steepest descent algorithm. We show how such a solution can be implemented in a distributed manner leading to a minimal communication among the transceivers and the relays. In fact, we show that the inter-node communication bandwidth required by our distributed two-way beamforming scheme will remain constant as the size of the network grows.

2. DATA MODEL

Let us consider a wireless network consisting of two transceivers and r relay nodes as shown in Fig. 1. We assume that there is no direct link between the two transceivers, and therefore, they must communicate with each other through the relay nodes. Each node of the network has a single antenna for both transmission and reception. Let f_{i1} and f_{i2} denote the complex coefficients representing the flat fading channels from the ith relay to the transceivers 1 and 2, respectively. We herein study a two-step two-way amplify-and-phase-adjust-and-forward relaying scheme. During the first step, both transceivers broadcast their data to the relays simultaneously. The relay received signals can be written, in vector form, as

$$\mathbf{x} = \sqrt{P_1}\mathbf{f}_1 s_1 + \sqrt{P_2}\mathbf{f}_2 s_2 + \boldsymbol{\nu} \tag{1}$$

where $\mathbf{x}$ is an $r \times 1$ complex vector whose ith entry is the signal received by the ith relay, P_k and s_k are, respectively, the transmit power of, and the information symbol transmitted by the kth transceiver for $k = 1, 2$, $\boldsymbol{\nu}$ is the $r \times 1$ complex vector of the relay

978-1-4244-2354-5/09/$25.00 ©2009 IEEE

noises, $\mathbf{f}_k \triangleq [f_{1k}\ f_{2k}\ \ldots\ f_{rk}]^T$ is the vector of channel coefficients between the relays and the kth transceiver, and $(\cdot)^T$ stands for the transpose operator. We assume that both transceivers know both channel vectors $\mathbf{f}_1$ and $\mathbf{f}_2$. The ith relay multiplies its received signal by a complex weight w_i^*, where $(\cdot)^*$ denotes complex conjugate. The relays then simultaneously transmit the so-obtained signals in the second step. The $r \times 1$ complex vector $\mathbf{t}$ of the relays transmit signals can then be expressed as $\mathbf{t} = \mathbf{W}\mathbf{x}$ where $\mathbf{W} \triangleq \mathrm{diag}([w_1^*\ w_2^*\ \ldots\ w_r^*]^T)$, and $\mathrm{diag}(\mathbf{a})$ denotes a diagonal matrix with the elements of the vector $\mathbf{a}$ as its diagonal entries. The signals received by transceivers 1 and 2 are given, respectively, by

$$y_1 = \mathbf{f}_1^T\mathbf{t} + n_1 = \mathbf{f}_1^T\mathbf{W}(\sqrt{P_1}\mathbf{f}_1 s_1 + \sqrt{P_2}\mathbf{f}_2 s_2 + \boldsymbol{\nu}) + n_1 \quad (2)$$

$$y_2 = \mathbf{f}_2^T\mathbf{t} + n_2 = \mathbf{f}_2^T\mathbf{W}(\sqrt{P_1}\mathbf{f}_1 s_1 + \sqrt{P_2}\mathbf{f}_2 s_2 + \boldsymbol{\nu}) + n_2 \quad (3)$$

where n_1 (n_2) is the receive noise at the first (second) transceiver. Using the fact that $\mathbf{a}^T\mathrm{diag}(\mathbf{b}) = \mathbf{b}^T\mathrm{diag}(\mathbf{a})$, we rewrite (2) and (3), respectively, as

$$y_1 = \sqrt{P_1}\mathbf{w}^H\mathbf{F}_1\mathbf{f}_1 s_1 + \sqrt{P_2}\mathbf{w}^H\mathbf{F}_1\mathbf{f}_2 s_2 + \mathbf{w}^H\mathbf{F}_1\boldsymbol{\nu} + n_1 \quad (4)$$

$$y_2 = \sqrt{P_1}\mathbf{w}^H\mathbf{F}_2\mathbf{f}_1 s_1 + \sqrt{P_2}\mathbf{w}^H\mathbf{F}_2\mathbf{f}_2 s_2 + \mathbf{w}^H\mathbf{F}_2\boldsymbol{\nu} + n_2\,. \quad (5)$$

where $\mathbf{F}_k \triangleq \mathrm{diag}(\mathbf{f}_k)$ for $k = 1, 2$, $\mathbf{w} \triangleq \mathrm{diag}(\mathbf{W}^H)$, and $(\cdot)^H$ stands for Hermitian (conjugate) transpose. Here, $\mathrm{diag}(\mathbf{A})$ is a vector which contains the diagonal entries of the square matrix $\mathbf{A}$. We assume that both transceivers calculate the weight vector $\mathbf{w}$. We will later show how each relay can calculate its own optimal beamforming weight based on its local channel information and based on a minimal amount of information that are broadcasted to all relays by the two transceivers. Note that the first term in (4) depends on the signal s_1 transmitted by transceiver 1 during the first time slot. As $\sqrt{P_1}\mathbf{F}_1\mathbf{f}_1$ is known at transceiver 1 and the weight vector $\mathbf{w}$ is going to be calculated at this transceiver, the first term in (4) is known at transceiver 1. Hence this term can be subtracted from y_1 and the residual signal can be processed at transceiver 1 to extract the information s_2. Similarly, the second term in (5) can be subtracted from y_2 and the residual signal can be processed at transceiver 2 to extract the information s_1. That is, the residual signals $\tilde{y}_1 \triangleq y_1 - \sqrt{P_1}\mathbf{w}^H\mathbf{F}_1\mathbf{f}_1 s_1$ and $\tilde{y}_2 \triangleq y_2 - \sqrt{P_2}\mathbf{w}^H\mathbf{F}_2\mathbf{f}_2 s_2$ expressed as

$$\tilde{y}_1 = \sqrt{P_2}\mathbf{w}^H\mathbf{F}_1\mathbf{f}_2 s_2 + \mathbf{w}^H\mathbf{F}_1\boldsymbol{\nu} + n_1 \quad (6)$$

$$\tilde{y}_2 = \sqrt{P_1}\mathbf{w}^H\mathbf{F}_2\mathbf{f}_1 s_1 + \mathbf{w}^H\mathbf{F}_2\boldsymbol{\nu} + n_2 \quad (7)$$

can be used at their corresponding transceivers to extract the desired information symbols. To optimally obtain the transmit powers P_1 and P_2 as well as the relay weight vector $\mathbf{w}$, the total transmit power consumed in the whole network can be minimized while guaranteeing that the receive SNRs at the two transceivers are kept above certain thresholds. In the following sections, we study this approach in further details.

3. POWER MINIMIZATION

Our goal is to obtain the beamforming weight vectors $\mathbf{w}$ and the transmit power P_1 and P_2 through the minimization of the total transmit power P_T, dissipated in the network, while maintaining the receive SNRs at both transceivers above certain levels, i.e., the receive SNRs are constrained to be larger than given pre-defined thresholds $\gamma_1 > 0$ and $\gamma_2 > 0$. Mathematically, we solve the following optimization problem:

$$\min_{P_1, P_2, \mathbf{w}} \ P_T \quad \text{subject to} \quad \mathrm{SNR}_1 \geq \gamma_1 \quad \text{and} \quad \mathrm{SNR}_2 \geq \gamma_2$$

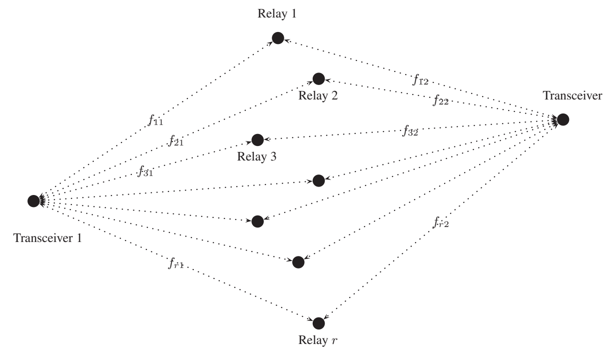

Fig. 1. A two-way relay network.

where SNR_k is the receive SNR at the kth transceiver for $k = 1, 2$. The total transmit power P_T can be expressed as

$$P_T = P_1 + P_2 + P_r \quad (8)$$

Here, P_r is the relay transmit power and is given by $P_r \triangleq E\{\mathbf{t}^H\mathbf{t}\} = \mathbf{w}^H\mathbf{D}\mathbf{w}$ where $\mathbf{D} \triangleq E\{\mathbf{X}^H\mathbf{X}\}$, $\mathbf{X} \triangleq \mathrm{diag}(\mathbf{x})$, and $E\{\cdot\}$ stands for statistical expectation. Using (1) and assuming that the relay noise vector $\boldsymbol{\nu}$ and the information symbols s_1, s_2 are all zero-mean mutually independent random variables, the matrix $\mathbf{D}$ can be written as $\mathbf{D} = P_1\mathbf{F}_1\mathbf{F}_1^H + P_2\mathbf{F}_2\mathbf{F}_2^H + \sigma^2\mathbf{I}$ where we have assumed that $E\{\boldsymbol{\nu}\boldsymbol{\nu}^H\} = \sigma^2\mathbf{I}$ and $\mathbf{I}$ stands for identity matrix. Without loss of generality we assume that $\sigma^2 = 1$ and $E\{|s_k|^2\} = E\{|n_k|^2\} = 1$, for $k = 1, 2$, where $|\cdot|$ stands for the amplitude of a complex number. Using (6) and (7), the receive SNRs can be written as

$$\mathrm{SNR}_1 = \frac{P_2\mathbf{w}^H\mathbf{h}\mathbf{h}^H\mathbf{w}}{1 + \mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w}}, \quad \mathrm{SNR}_2 = \frac{P_1\mathbf{w}^H\mathbf{h}\mathbf{h}^H\mathbf{w}}{1 + \mathbf{w}^H\mathbf{F}_2\mathbf{F}_2^H\mathbf{w}} \quad (9)$$

where $\mathbf{h} \triangleq \mathbf{F}_1\mathbf{f}_2 = \mathbf{F}_2\mathbf{f}_1 = \mathbf{f}_1 \odot \mathbf{f}_2$, and $\odot$ stands for element-wise Schur-Hadamard matrix product. Using (8) and (9), the optimization problem (8) can be rewritten as

$$\min_{P_1, P_2, \mathbf{w}} P_1(1 + \mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w}) + P_2(1 + \mathbf{w}^H\mathbf{F}_2\mathbf{F}_2^H\mathbf{w}) + \mathbf{w}^H\mathbf{w} \quad (10)$$

$$\text{subject to} \quad \frac{P_2\mathbf{w}^H\mathbf{h}\mathbf{h}^H\mathbf{w}}{1 + \mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w}} \geq \gamma_1 \text{ and } \frac{P_1\mathbf{w}^H\mathbf{h}\mathbf{h}^H\mathbf{w}}{1 + \mathbf{w}^H\mathbf{F}_2\mathbf{F}_2^H\mathbf{w}} \geq \gamma_2\,.$$

Note that the inequality constraints in (10) are satisfied with equality at the optimal point. This can be proved by contradiction. If, for example, the first constraint in (10) is not satisfied with equality at the optimum, then the optimal value of P_2 can be scaled down to satisfy this constraint with equality. However, reducing the optimal value of P_2 further reduces the objective function in (10) thereby contradicting the optimality. Based on this observation, the transceiver transmit powers can be obtained as

$$P_1 = \frac{\gamma_2(1 + \mathbf{w}^H\mathbf{F}_2\mathbf{F}_2^H\mathbf{w})}{\mathbf{w}^H\mathbf{h}\mathbf{h}^H\mathbf{w}} \text{ and } P_2 = \frac{\gamma_1(1 + \mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w})}{\mathbf{w}^H\mathbf{h}\mathbf{h}^H\mathbf{w}}\,. \quad (11)$$

Using (11), the optimization problem (10) can be turned into the following *unconstrained* optimization problem:

$$\min_{\mathbf{w}} \ \frac{(1 + \mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w})(1 + \mathbf{w}^H\mathbf{F}_2\mathbf{F}_2^H\mathbf{w})}{\mathbf{w}^H\mathbf{h}\mathbf{h}^H\mathbf{w}} + \beta\mathbf{w}^H\mathbf{w} \quad (12)$$

where $\beta \triangleq (\gamma_1+\gamma_2)^{-1}$. Let us define $\boldsymbol{\theta} \triangleq [\theta_1 \quad \theta_2 \quad \dots \quad \theta_r]^T$ and $\boldsymbol{\alpha} \triangleq [\alpha_1 \quad \alpha_2 \quad \dots \quad \alpha_r]^T$ where θ_i and α_i are the phase and the amplitude of w_i, respectively, (i.e., $w_i = \alpha_i e^{j\theta_i}$) for $i = 1, 2, \dots, r$. Since $\mathbf{D}_1 \triangleq \mathbf{F}_1\mathbf{F}_1^H$ and $\mathbf{D}_2 \triangleq \mathbf{F}_2\mathbf{F}_2^H$ are real-valued diagonal matrices, the numerator of the first term of the objective function in (12) can be written as $(1+\boldsymbol{\alpha}^T\mathbf{D}_1\boldsymbol{\alpha})(1+\boldsymbol{\alpha}^T\mathbf{D}_2\boldsymbol{\alpha})$, and therefore, it does not depend on $\boldsymbol{\theta}$. As a result, the minimization problem in (12) is equivalent to

$$\min_{\boldsymbol{\alpha}\succcurlyeq 0} \frac{(1+\boldsymbol{\alpha}^T\mathbf{D}_1\boldsymbol{\alpha})(1+\boldsymbol{\alpha}^T\mathbf{D}_2\boldsymbol{\alpha})}{p(\boldsymbol{\alpha})} + \beta\boldsymbol{\alpha}^T\boldsymbol{\alpha} \tag{13}$$

where $p(\boldsymbol{\alpha}) \triangleq \max_{\boldsymbol{\theta}} \mathbf{w}^H\mathbf{h}\mathbf{h}^H\mathbf{w}$. Note that

$$p(\boldsymbol{\alpha}) = \max_{\boldsymbol{\theta}} \left|\sum_{i=1}^{r} \alpha_i b_i e^{j(\phi_i-\theta_i)}\right|^2 = |\boldsymbol{\alpha}^T\mathbf{b}|^2 . \tag{14}$$

Here, b_i and ϕ_i denote the amplitude and the phase of the ith entry of $\mathbf{h}$, respectively, and $\mathbf{b} \triangleq [b_1 \ b_2 \ \dots \ b_r]^T$. The maximum in (14) is achieved when $\theta_i = \phi_i$ is chosen for any i. This means that the phase of w_i has to match to the phase of ith entry of $\mathbf{h}$ which is equal to the aggregated phase of the channel coefficients from the ith relay to the two transceivers. That is $\theta_i = \phi_i = \angle f_{i1} + \angle f_{i2}$ where $\angle z$ denotes the phase of the complex number z. Eventually, the optimization problem in (13) can be written as

$$\min_{\boldsymbol{\alpha}\succcurlyeq 0} \frac{(1+\boldsymbol{\alpha}^T\mathbf{D}_1\boldsymbol{\alpha})(1+\boldsymbol{\alpha}^T\mathbf{D}_2\boldsymbol{\alpha})}{\boldsymbol{\alpha}^T\mathbf{b}\mathbf{b}^T\boldsymbol{\alpha}} + \beta\boldsymbol{\alpha}^T\boldsymbol{\alpha} \tag{15}$$

The gradient of the objective function in (15) can be obtained as

$$\mathbf{g}(\boldsymbol{\alpha}) = \frac{(1+\zeta_1(\boldsymbol{\alpha}))(1+\zeta_2(\boldsymbol{\alpha}))}{\zeta_3(\boldsymbol{\alpha})} \left(\mathbf{Q}(\boldsymbol{\alpha})\boldsymbol{\alpha} - \frac{1}{\boldsymbol{\alpha}^T\mathbf{b}}\mathbf{b}\right) \tag{16}$$

where $\mathbf{Q}(\boldsymbol{\alpha})$ is an $r \times r$ *diagonal* matrix defined as

$$\mathbf{Q}(\boldsymbol{\alpha}) \triangleq \frac{1}{1+\zeta_1(\boldsymbol{\alpha})}\mathbf{D}_1 + \frac{1}{1+\zeta_2(\boldsymbol{\alpha})}\mathbf{D}_2 + \frac{\beta\zeta_3(\boldsymbol{\alpha})}{(1+\zeta_1(\boldsymbol{\alpha}))(1+\zeta_2(\boldsymbol{\alpha}))}\mathbf{I} . \tag{17}$$

Here, we have defined $\zeta_1(\boldsymbol{\alpha}) \triangleq \boldsymbol{\alpha}^T\mathbf{D}_1\boldsymbol{\alpha}$, $\zeta_2(\boldsymbol{\alpha}) \triangleq \boldsymbol{\alpha}^T\mathbf{D}_2\boldsymbol{\alpha}$, and $\zeta_3(\boldsymbol{\alpha}) \triangleq \boldsymbol{\alpha}^T\mathbf{b}\mathbf{b}^T\boldsymbol{\alpha}$. Equating $\mathbf{g}(\boldsymbol{\alpha})$ to zero yields

$$\mathbf{Q}(\boldsymbol{\alpha})\boldsymbol{\alpha} = \frac{1}{\boldsymbol{\alpha}^T\mathbf{b}}\mathbf{b} . \tag{18}$$

The optimal value of $\boldsymbol{\alpha}$ can be obtained by solving the r non-linear equations in (18).

Lemma 1: *The set of non-linear equations in (18) has only two solutions which are symmetric with respect to the origin. Moreover, one of the solutions has all positive entries.*

Proof: See [8].

It follows from Lemma 1 that the optimization problem (15) does not have any local solution and it has only one global solution. Therefore, the steepest descent algorithm can be used to obtain the global solution to (15). Let $\boldsymbol{\alpha}^{(k)}$ denote the value of $\boldsymbol{\alpha}$ at the kth iteration. Then the iterative steepest descent technique can be written as

$$\boldsymbol{\alpha}^{(k)} = \boldsymbol{\alpha}^{(k-1)} - \mu\mathbf{g}(\boldsymbol{\alpha}^{(k-1)}) \tag{19}$$

where μ is the parameter that controls the trade-off between the stability and the convergence speed of the algorithm. Note that if we choose the initial point as $\boldsymbol{\alpha}^{(0)} \succcurlyeq 0$, the iterative algorithm in (19) is guaranteed to converge to the optimal value $\boldsymbol{\alpha}_o \succcurlyeq 0$.

It can be observed from (16) and (17) that calculating the matrix $\mathbf{Q}(\boldsymbol{\alpha})$ and the gradient vector $\mathbf{g}(\boldsymbol{\alpha})$ requires the calculation of three quadratic terms, namely $\zeta_1(\boldsymbol{\alpha})$, $\zeta_2(\boldsymbol{\alpha})$, and $\zeta_3(\boldsymbol{\alpha})$. As the complexity of calculating each of these terms is $\mathcal{O}(r)$, the iterative steepest descent algorithm in (19) enjoys a linear computational complexity $\mathcal{O}(r)$ per iteration.

Lemma 2: *For symmetric QoS constraints where $\gamma_1 = \gamma_2 = \gamma$, at the optimal solution, half of the total transmit power will be allocated to the relaying nodes and the remaining half will be divided between the two transceivers.*

Proof: Let $\boldsymbol{\alpha}_o$ denote the optimal value of $\boldsymbol{\alpha}$. Then using (11), the optimal transmit powers of the two transceivers are given by

$$P_1^o = \frac{\gamma(1+\zeta_2(\boldsymbol{\alpha}_o))}{\zeta_3(\boldsymbol{\alpha}_o)} \text{ and } P_2^o = \frac{\gamma(1+\zeta_1(\boldsymbol{\alpha}_o))}{\zeta_3(\boldsymbol{\alpha}_o)} \tag{20}$$

Using (20), the total relay transmit power can be written, at the optimal solution, as

$$\begin{aligned}
P_r &= P_1^o\zeta_1(\boldsymbol{\alpha}_o) + P_2^o\zeta_2(\boldsymbol{\alpha}_o) + \boldsymbol{\alpha}_o^T\boldsymbol{\alpha}_o \\
&= \frac{\gamma(1+\zeta_2(\boldsymbol{\alpha}_o))}{\zeta_3(\boldsymbol{\alpha}_o)}\zeta_1(\boldsymbol{\alpha}_o) + \frac{\gamma(1+\zeta_1(\boldsymbol{\alpha}_o))}{\zeta_3(\boldsymbol{\alpha}_o)}\zeta_2(\boldsymbol{\alpha}_o) + \frac{1}{2}\boldsymbol{\alpha}_o^T\boldsymbol{\alpha}_o \\
&+ \frac{1}{2}\boldsymbol{\alpha}_o^T\boldsymbol{\alpha}_o \\
&= \frac{1}{2}\boldsymbol{\alpha}_o^T\boldsymbol{\alpha}_o + \frac{\gamma(1+\zeta_1(\boldsymbol{\alpha}_o))(1+\zeta_2(\boldsymbol{\alpha}_o))}{\zeta_3(\boldsymbol{\alpha}_o)}\times \\
&\underbrace{\left(\frac{\zeta_1(\boldsymbol{\alpha}_o)}{1+\zeta_1(\boldsymbol{\alpha}_o)} + \frac{\zeta_2(\boldsymbol{\alpha}_o)}{1+\zeta_2(\boldsymbol{\alpha}_o)} + \frac{(2\gamma)^{-1}\zeta_3(\boldsymbol{\alpha}_o)\boldsymbol{\alpha}_o^T\boldsymbol{\alpha}_o}{(1+\zeta_1(\boldsymbol{\alpha}_o))(1+\zeta_2(\boldsymbol{\alpha}_o))}\right)}_{\boldsymbol{\alpha}_o^T\mathbf{Q}(\boldsymbol{\alpha}_o)\boldsymbol{\alpha}_o=1} \\
&= \frac{\gamma(1+\boldsymbol{\alpha}_o^T\mathbf{D}_1\boldsymbol{\alpha}_o)(1+\boldsymbol{\alpha}_o^T\mathbf{D}_2\boldsymbol{\alpha}_o)}{\boldsymbol{\alpha}_o^T\mathbf{b}\mathbf{b}^T\boldsymbol{\alpha}_o} + \frac{1}{2}\boldsymbol{\alpha}_o^T\boldsymbol{\alpha}_o
\end{aligned} \tag{21}$$

To arrive at (21), we have used the fact that $\boldsymbol{\alpha}_o^T\mathbf{Q}(\boldsymbol{\alpha}_o)\boldsymbol{\alpha}_o = 1$ which in turn follows from multiplying (18) by $\boldsymbol{\alpha}^T$ from left and replacing $\boldsymbol{\alpha}$ with $\boldsymbol{\alpha}_o$. Note that the minimum total transmit power P_T^{min} for symmetric scenarios is given by

$$P_T^{min} = \frac{2\gamma(1+\boldsymbol{\alpha}_o^T\mathbf{D}_1\boldsymbol{\alpha}_o)(1+\boldsymbol{\alpha}_o^T\mathbf{D}_2\boldsymbol{\alpha}_o)}{\boldsymbol{\alpha}_o^T\mathbf{b}\mathbf{b}^T\boldsymbol{\alpha}_o} + \boldsymbol{\alpha}_o^T\boldsymbol{\alpha}_o . \tag{22}$$

Comparing (21) and (22), we conclude that at the optimal solution $P_r = \frac{1}{2}P_T^{min}$ holds true and the proof is complete. ∎

Note that we have assumed that the two transceivers use the iterative algorithm in (19) to calculate the optimal value $\boldsymbol{\alpha}_o$. Then one of the transceivers can send the ith entry of $\boldsymbol{\alpha}_o$ to the ith relay over a control channel. In such a scheme, the bandwidth of the control channel needs to be linearly increased as the number of relays is increased. Alternatively, each relay can use (18) to calculate the only entry of $\boldsymbol{\alpha}_o$ which it needs. To do so, the ith relay needs the ith entries of $\mathbf{D}_1$ and $\mathbf{D}_2$ matrices as well as three quadratic quantities: $\zeta_1(\boldsymbol{\alpha}_o)$, $\zeta_2(\boldsymbol{\alpha}_o)$, and $\zeta_3(\boldsymbol{\alpha}_o)$. In fact, the ith diagonal entry of $\mathbf{Q}(\boldsymbol{\alpha}_o)$ depends only on the ith entry of $\mathbf{D}_1$ and $\mathbf{D}_2$ and on the values of $\zeta_1(\boldsymbol{\alpha}_o)$, $\zeta_2(\boldsymbol{\alpha}_o)$, and $\zeta_3(\boldsymbol{\alpha}_o)$. Assuming that each relay knows its local channels and requiring the two transceivers to broadcast only $\zeta_1(\boldsymbol{\alpha}_o)$, $\zeta_2(\boldsymbol{\alpha}_o)$, and $\zeta_3(\boldsymbol{\alpha}_o)$ to all relays, then each relay can use the ith non-linear equation in (18) to calculate the only element of $\boldsymbol{\alpha}_o$ that it needs. Such a distributed scheme requires a minimal cooperation over the control channel.

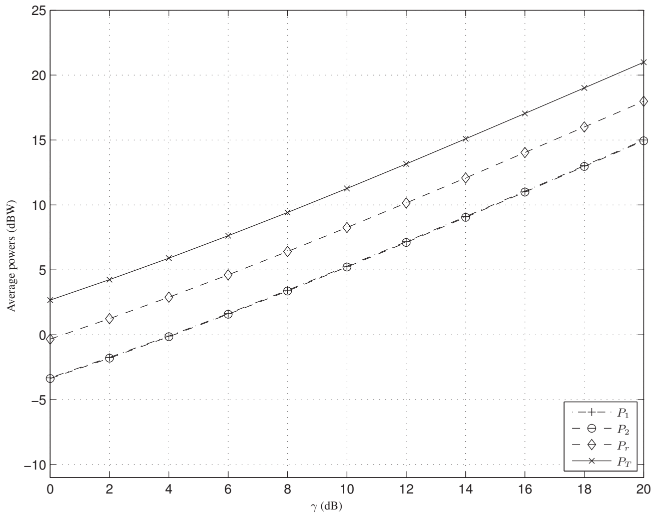

Fig. 2. The average minimum total transmit power, P_T, the corresponding average relay transmit power P_r, and the corresponding average transceiver powers P_1 and P_2 versus $\gamma_1 = \gamma_2 = \gamma$ for $\sigma^2_{f_1} = \sigma^2_{f_2} = 0$ dB.

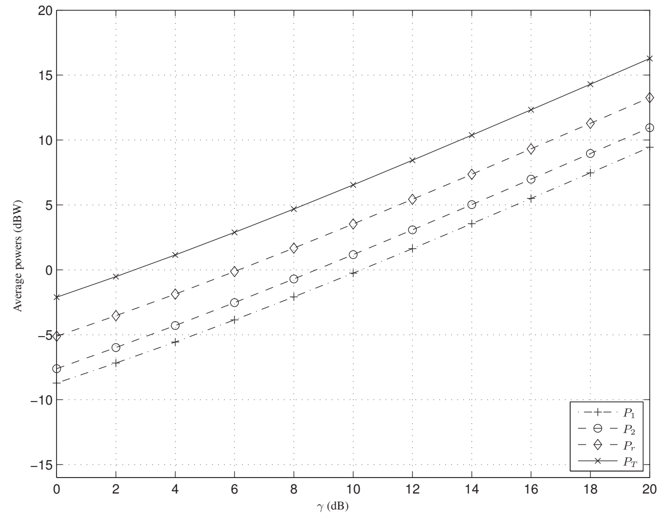

Fig. 3. The average minimum total transmit power, P_T, the corresponding average relay transmit power P_r, and the corresponding average transceiver powers P_1 and P_2 versus $\gamma_1 = \gamma_2 = \gamma$ for $\sigma^2_{f_1} = 7$ dB and $\sigma^2_{f_2} = 3$ dB.

4. SIMULATION RESULTS

Throughout our numerical experiments, we consider a network with 10 relay nodes where the channel vectors $\mathbf{f}_1$ and $\mathbf{f}_2$ are generated in each simulation run, as complex zero-mean Gaussian random vectors with variances $\sigma^2_{f_1}$ and $\sigma^2_{f_2}$, respectively. As these channel vectors are assumed to be known at the two transceivers, $\sigma^2_{f_1}$ and $\sigma^2_{f_2}$ are the measures of the quality of the corresponding channel vectors. The noise power σ^2 is assumed to be equal to 0 dBW. Fig. 2 shows the average total transmit power P_T, the average relay transmit power P_r, and the average transceiver powers P_1 and P_2 versus $\gamma = \gamma_1 = \gamma_2$ in dB for $\sigma^2_{f_1} = \sigma^2_{f_2} = 0$ dB. Fig. 3 illustrates the same quantities versus γ in dB for $\sigma^2_{f_1} = 7$ dB and $\sigma^2_{f_2} = 3$ dB. As can be seen from these two figures, the total relay power is 3 dB below the total transmit power for both scenarios. This observation is consistent with Lemma 2 as the QoS constraints are symmetric in both figures. It can also be observed that the transceiver powers P_1 and P_2 are equal for $\sigma^2_{f_1} = \sigma^2_{f_2} = 0$ dB while $P_2 > P_1$ for $\sigma^2_{f_2} < \sigma^2_{f_1}$. This means the transceiver with a better average channel quality requires less power in average.

5. CONCLUSIONS

We studied the problem of optimal distributed two-way beamforming for a relay network consisting of two transceivers and r relay nodes. Our approach was based on the minimization of the total transmit power while maintaining the receive SNRs at the two transceivers above certain given thresholds. We developed a simple iterative method which is guaranteed to converge to the optimal solution of our power minimization problem. We also proved that when SNR constraints are identical, half of the power is allocated to the two transceivers and the remaining half is allocated to the relay nodes.

6. REFERENCES

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity–part I: System description," *IEEE Trans. Commun.*, vol. 51, pp. 1927–1938, Nov. 2003.

[2] ——, "User cooperation diversity–part II: Implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, pp. 1939–1948, Nov. 2003.

[3] Y. Jing and H. Jafarkhani, "Network beamforming using relays with perfect channel information," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP'07)*, vol. 3, Honolulu, HI, Apr. 2007.

[4] V. Havary-Nassab, S. Shahbazpanahi, A. Grami, and Z. Q. Luo, "Distributed beamforming for relay networks based on second-order statistics of the channel state information," *IEEE Trans. Signal Process.*, vol. 56, pp. 4306–4316, Sep. 2008.

[5] V. Havary-Nassab, S. Shahbazpanahi, A. Grami, and Z.-Q. Luo, "Network beamforming based on second order statistics of the channel state information," in *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing, (ICASSP'08)*, Las Vegas, NV, USA, Mar./Apr. 2008, pp. 2605–2608.

[6] S. Fazeli-Dehkordy, S. Gazor, and S. Shahbazpanahi, "Distributed peer-to-peer multiplexing using ad hoc relay networks," in *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing, (ICASSP'08)*, Las Vegas, NV, USA, Mar. 31–Apr. 4, 2008, pp. 2373–2376.

[7] V. Havary-Nassab, S. Shahbazpanahi, A. Grami, and Z. Q. Luo, "Joint uplink-downlink beamforming for multi-antenna relaying schemes," *IEEE Trans. Signal Process.*, submitted, 2008.

[8] V. Havary-Nassab, S. Shahbazpanahi, and A. Grami, "Optimal distributed beamforming for two-way relay networks," *IEEE Trans. Signal Process.*, submitted, 2008.