DISTRIBUTED PEER-TO-PEER BEAMFORMING FOR MULTIUSER RELAY NETWORKS

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ABSTRACT

A computationally efficient distributed beamforming technique for multi-user relay networks is developed. The channel state information is assumed to be known at the relays or destinations, and the total relay transmitted power is minimized subject to the destination quality-of-service constraints. It is shown that this problem can be approximately converted to a convex second-order cone programming form. As a result, the proposed network beamforming technique offers a substantially reduced computational complexity than earlier state-of-the-art techniques that are based on semidefinite relaxation.

***Index Terms*—** Cooperative communications, distributed beamforming, peer-to-peer beamforming, relay networks

1. INTRODUCTION

Recently, user-cooperative techniques have attracted much attention in wireless communications [1]-[4]. In cooperative communication networks, users assist each other in transmitting their data through the network by means of signal relaying.

Several relaying strategies have been proposed. The most popular of them are amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF) relaying schemes [2]-[4]. Due to its simplicity, the AF scheme is of especial interest.

There has been a significant amount of recent work on AF relay beamforming [5]-[8]. Promising techniques have been developed in [6] and [7] that maximize the receiver signal-to-noise ratio (SNR) under the relay transmitted power constraints. However, the approaches of [6] and [7] are only applicable to the case of a single source-destination pair. In [8], an AF distributed peer-to-peer beamforming technique has been proposed that is applicable to multiple source-destination pairs and multiple relays. This technique uses the semidefinite relaxation (SDR) approach to approximate the original non-convex relay beamforming problem by a convex semidefinite programming (SDP) problem. However, the complexity of solving SDP problems is rather high.

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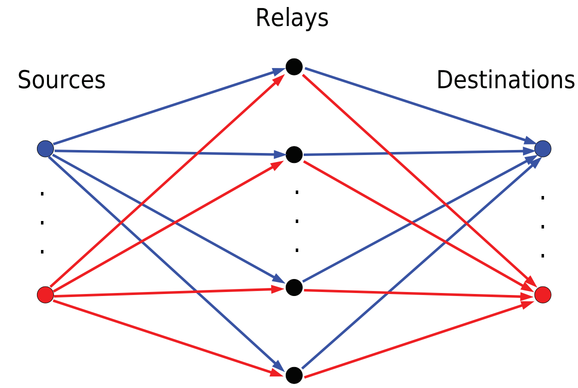

Fig. 1. Multi-user wireless relay network.

In this paper, we propose a computationally much simpler approach to distributed peer-to-peer beamforming in multiuser relay networks. In contrast to the approach of [8], we approximate the original non-convex relay beamforming problem by a convex second-order cone programming (SOCP) problem which can be solved much more efficiently than the SDP problem of [8]. Our simulation results show that the price for such drastic improvement in the computational complexity is only a moderate increase in the transmit power as compared to the SDP approach of [8].

2. SIGNAL MODEL

A half-duplex relay network with K source-destination pairs and R relays is considered, as depicted in Fig. 1. Without any loss of generality, the kth ($k = 1, \cdots, K$) source is assumed to transmit messages to the kth destination. It is also assumed that there are no direct links between the sources and the destinations. All nodes operate in a common frequency band. Each transmission from the source to the destination consists of two stages. In the first stage, the sources transmit signals to all the relays. In the second stage, the signals received at the relays are scaled by complex values and transmitted to the destinations. The following assumptions are used through this paper:

A1: The source transmitted symbols are statistically independent.

A2: The relay noise is spatially white.

A3: The relay noise, the destination noise, and the information symbols are mutually statistically independent.

The $R \times 1$ vector of signals received by the relays can be written as

$$\boldsymbol{r}(n) = \sum_{m=1}^{K} \boldsymbol{f}_m s_m(n) + \boldsymbol{\eta}(n) \tag{1}$$

where n is the time index, $\boldsymbol{f}_m$ is the $R \times 1$ vector of channel coefficients between the mth transmitting source and the relays, $s_m(n)$ is the signal transmitted by the mth source, and $\boldsymbol{\eta}(n)$ is the $R \times 1$ vector of the relay noise. Let $\boldsymbol{w} = [w_1, \cdots, w_R]^T$ be the relay weight vector, where w_i is the weight coefficient of the ith relay. Then, the $R \times 1$ vector of signals transmitted by the relays can be expressed as [8]

$$\boldsymbol{t}(n) = \boldsymbol{W}^H \boldsymbol{r}(n) \tag{2}$$

where $\boldsymbol{W} = \mathrm{diag}\{\boldsymbol{w}\}$ and $\mathrm{diag}\{\cdot\}$ is the operator that generates a diagonal matrix from a vector by placing its entries on the main diagonal. Let $\boldsymbol{g}_k$ denote the vector of channel coefficients between the relays and the kth destination. Using (1) and (2), the signal received by the kth destination can be written as [8]

$$\begin{aligned} y_k(n) &= \boldsymbol{g}_k^T \boldsymbol{t}(n) + \upsilon_k(n) \\ &= \boldsymbol{g}_k^T \boldsymbol{W}^H \sum_{m=1}^{K} \boldsymbol{f}_m s_m(n) + \boldsymbol{g}_k^T \boldsymbol{W}^H \boldsymbol{\eta}(n) + \upsilon_k(n) \\ &= \boldsymbol{g}_k^T \boldsymbol{W}^H \boldsymbol{f}_k s_k(n) + \boldsymbol{g}_k^T \boldsymbol{W}^H \sum_{m \neq k} \boldsymbol{f}_m s_m(n) \\ &\quad + \boldsymbol{g}_k^T \boldsymbol{W}^H \boldsymbol{\eta}(n) + \upsilon_k(n) \triangleq y_k^s(n) + y_k^i(n) + y_k^n(n) \end{aligned}$$

where $\upsilon_k(n)$ denotes the kth destination noise and

$$y_k^s(n) = \boldsymbol{g}_k^T \boldsymbol{W}^H \boldsymbol{f}_k s_k(n) \tag{3}$$

$$y_k^i(n) = \boldsymbol{g}_k^T \boldsymbol{W}^H \sum_{m \neq k} \boldsymbol{f}_m s_m(n) \tag{4}$$

$$y_k^n(n) = \boldsymbol{g}_k^T \boldsymbol{W}^H \boldsymbol{\eta}(n) + \upsilon_k(n) \tag{5}$$

are the desired signal, interference, and noise components at the kth destination, respectively.

3. THE PROPOSED TECHNIQUE

Throughout the paper, it will be assumed that the channel state information (CSI) (which is given by the vectors $\boldsymbol{f}_m$ and $\boldsymbol{g}_k$; $m, k = 1, \cdots, K$) is available to compute the relay weights. For example, this CSI may be available at destinations that need to compute the relay weights and feed them back to the relay nodes. It may also be possible that this CSI is available directly at the relays which then have to compute their own weight coefficients.

Let us consider the following network beamforming problem. Let us minimize the total relay transmit power subject to the destination quality-of-service (QoS) constraints. The signal-to-interference-plus-noise ratio (SINR) will be used as a measure of QoS. The latter problem can be written as

$$\min_{\boldsymbol{w}} \; P_t \qquad \text{s.t.} \qquad \mathrm{SINR}_k \geq \gamma_k, \quad k = 1, \cdots, K \tag{6}$$

where P_t is the total relay transmit power, SINR_k is the received SINR at the kth destination, and γ_k is the minimal required SINR at the kth destination.

The power P_t can be written as

$$\begin{aligned} P_t &= \mathrm{E}\{\boldsymbol{t}^H(n)\boldsymbol{t}(n)\} \\ &= \mathrm{E}\{\boldsymbol{r}^H(n)\boldsymbol{W}\boldsymbol{W}^H\boldsymbol{r}(n)\} \\ &= \mathrm{tr}\{\boldsymbol{W}^H \boldsymbol{R}_r \boldsymbol{W}\} \end{aligned} \tag{7}$$

where $\boldsymbol{R}_r = \mathrm{E}\{\boldsymbol{r}(n)\boldsymbol{r}^H(n)\}$ is the covariance matrix of the signals received at the relays, $\mathrm{E}\{\cdot\}$ is the statistical expectation, and $\mathrm{tr}\{\cdot\}$ denotes the trace. As the weight matrix $\boldsymbol{W}$ is diagonal, (7) can be rewritten as

$$P_t = \sum_{i=1}^{R} |w_i|^2 [\boldsymbol{R}_r]_{ii} = \boldsymbol{w}^H \boldsymbol{D} \boldsymbol{w} \tag{8}$$

where $\boldsymbol{D}$ is a diagonal matrix with $[\boldsymbol{D}]_{i,i} = [\boldsymbol{R}_r]_{i,i}$ and $[\cdot]_{il}$ denotes the (i,l)th element of a matrix. Using (1) and assumptions A1-A3, the covariance matrix $\boldsymbol{R}_r$ can be expressed as

$$\begin{aligned} \boldsymbol{R}_r &= \mathrm{E}\left\{\left(\sum_{m=1}^{K} \boldsymbol{f}_m s_m(n) + \boldsymbol{\eta}(n)\right)\left(\sum_{m=1}^{K} \boldsymbol{f}_m s_m(n) + \boldsymbol{\eta}(n)\right)^H\right\} \\ &= \sum_{m=1}^{K} p_m \boldsymbol{f}_m \boldsymbol{f}_m^H + \sigma_\eta^2 \boldsymbol{I} \end{aligned} \tag{9}$$

where $P_m = \mathrm{E}\{|s_m(n)|^2\}$ is the transmitted power of the mth source, σ_η^2 is the variance of the relay noise, and $\boldsymbol{I}$ is the identity matrix.

The received SINR at the kth destination is given by

$$\mathrm{SINR}_k = \frac{\mathrm{E}\{|y_k^s(n)|^2\}}{\mathrm{E}\{|y_k^i(n)|^2\} + \mathrm{E}\{|y_k^n(n)|^2\}}\,. \tag{10}$$

From (3), we obtain that the power of the signal component at the kth destination can be written as

$$\begin{aligned} \mathrm{E}\{|y_k^s(n)|^2\} &= \mathrm{E}\{|s_k(n)|^2\}\boldsymbol{g}_k^T \boldsymbol{W}^H \boldsymbol{f}_k \boldsymbol{f}_k^H \boldsymbol{W} \boldsymbol{g}_k^* \\ &= p_k \boldsymbol{w}^H \mathrm{diag}\{\boldsymbol{g}_k\} \boldsymbol{f}_k \boldsymbol{f}_k^H \mathrm{diag}\{\boldsymbol{g}_k^*\} \boldsymbol{w} \\ &= p_k \boldsymbol{w}^H (\boldsymbol{g}_k \odot \boldsymbol{f}_k)(\boldsymbol{g}_k \odot \boldsymbol{f}_k)^H \boldsymbol{w} \end{aligned} \tag{11}$$

where $\odot$ denotes the Schur-Hadamard matrix product and $(\cdot)^*$ denotes the complex conjugate. Using (4) and assumption A1, the interference power at the kth destination can be expressed as

$$\begin{aligned}
\mathrm{E}\{|y_k^i(n)|^2\} &= \boldsymbol{g}_k^T \boldsymbol{W}^H \left(\sum_{m\neq k} \mathrm{E}\{|s_m(n)|^2\} \boldsymbol{f}_m \boldsymbol{f}_m^H \right) \boldsymbol{W} \boldsymbol{g}_k^* \\
&= \boldsymbol{w}^H \mathrm{diag}\{\boldsymbol{g}_k\} \left(\sum_{m\neq k} p_m \boldsymbol{f}_m \boldsymbol{f}_m^H \right) \mathrm{diag}\{\boldsymbol{g}_k^*\} \boldsymbol{w} \\
&= \boldsymbol{w}^H \left(\sum_{m\neq k} p_m (\boldsymbol{g}_k \odot \boldsymbol{f}_m)(\boldsymbol{g}_k \odot \boldsymbol{f}_m)^H \right) \boldsymbol{w} \\
&= \boldsymbol{w}^H \boldsymbol{Q}_k \boldsymbol{w} \qquad (12)
\end{aligned}$$

where

$$\boldsymbol{Q}_k \triangleq \sum_{m\neq k} p_m (\boldsymbol{g}_k \odot \boldsymbol{f}_m)(\boldsymbol{g}_k \odot \boldsymbol{f}_m)^H.$$

Finally, using (5) and assumptions A2 and A3, we can write the power of the noise component $y_k^n(n)$ at the kth destination as

$$\begin{aligned}
\mathrm{E}\{|y_k^n(n)|^2\} &= \boldsymbol{g}_k^T \boldsymbol{W}^H \mathrm{E}\{\boldsymbol{\eta}(n)\boldsymbol{\eta}^H(n)\} \boldsymbol{W} \boldsymbol{g}_k^* + \sigma_\upsilon^2 \\
&= \sigma_\eta^2 \mathrm{tr}\{\boldsymbol{W}^H \boldsymbol{W} \boldsymbol{g}_k^* \boldsymbol{g}_k^T\} + \sigma_\upsilon^2 \\
&= \sigma_\eta^2 \boldsymbol{w}^H \boldsymbol{D}_k \boldsymbol{w} + \sigma_\upsilon^2 \qquad (13)
\end{aligned}$$

where the diagonal matrix $\boldsymbol{D}_k$ is defined by

$$[\boldsymbol{D}_k]_{i,i} = [\boldsymbol{g}_k^* \boldsymbol{g}_k^T]_{i,i}$$

and $\sigma_\upsilon^2 = \mathrm{E}\{|\upsilon_k(n)|^2\}$.

With (8) and (11)-(13), the original network beamforming problem (6) can be rewritten in the following form:

$$\begin{aligned}
&\min_{\boldsymbol{w}} \ \boldsymbol{w}^H \boldsymbol{D} \boldsymbol{w} \qquad (14) \\
&\text{s.t.} \ \frac{p_k \boldsymbol{w}^H (\boldsymbol{g}_k \odot \boldsymbol{f}_k)(\boldsymbol{g}_k \odot \boldsymbol{f}_k)^H \boldsymbol{w}}{\boldsymbol{w}^H \boldsymbol{Q}_k \boldsymbol{w} + \sigma_\eta^2 \boldsymbol{w}^H \boldsymbol{D}_k \boldsymbol{w} + \sigma_\upsilon^2} \geq \gamma_k, \ k = 1, \cdots, K.
\end{aligned}$$

The problem in (14) is generally non-convex, but can be approximately solved using the SDR approach [8]. However, the resulting relaxed SDP problem introduces extra variables, which augments the number of variables from R to R^2 and substantially increases the computational complexity of solving (14). To simplify the problem, let us rewrite the kth QoS constraint in (14) as

$$\frac{\sqrt{p_k} |\boldsymbol{w}^H (\boldsymbol{g}_k \odot \boldsymbol{f}_k)|}{\sqrt{\boldsymbol{w}^H \left(\boldsymbol{Q}_k + \sigma_\eta^2 \boldsymbol{D}_k\right) \boldsymbol{w} + \sigma_\upsilon^2}} \geq \sqrt{\gamma_k}. \qquad (15)$$

Introducing new notations

$$\begin{aligned}
\tilde{\boldsymbol{w}} &\triangleq [1, \boldsymbol{w}^T]^T \\
\tilde{\boldsymbol{h}}_k &\triangleq [0, (\boldsymbol{g}_k \odot \boldsymbol{f}_k)^T]^T \\
\boldsymbol{U}_k &\triangleq \begin{bmatrix} \sigma_\upsilon^2 & \boldsymbol{0}^T \\ \boldsymbol{0} & \boldsymbol{Q}_k + \sigma_\eta^2 \boldsymbol{D}_k \end{bmatrix}^{1/2}
\end{aligned}$$

we can rewrite (15) as

$$|\tilde{\boldsymbol{w}}^H \tilde{\boldsymbol{h}}_k| \geq \sqrt{\gamma_k / p_k}\, \|\boldsymbol{U}_k \tilde{\boldsymbol{w}}\| \qquad (16)$$

where $\|\cdot\|$ denotes the Euclidean norm of a vector. If there is only one source-destination pair ($K = 1$), then the phase of $\boldsymbol{w}$ can be always rotated (without affecting the value of P_t) so that (16) is equivalent to

$$\tilde{\boldsymbol{w}}^H \tilde{\boldsymbol{h}}_1 \geq \sqrt{\gamma_1 / P_1}\, \|\boldsymbol{U}_1 \tilde{\boldsymbol{w}}\| \qquad (17)$$

where it is guaranteed by the right-hand side of inequality (17) that $\tilde{\boldsymbol{w}}^H \tilde{\boldsymbol{h}}_1$ is real and non-negative.

Although in the $K > 1$ we cannot make all $\tilde{\boldsymbol{w}}^H \tilde{\boldsymbol{h}}_i$, $i = 1, \ldots, K$ real-valued, we can use a somewhat related idea to obtain a simple approximation of the QoS constraints in the case of multiple source-destination pairs ($K > 1$). In the latter case, we have

$$|\tilde{\boldsymbol{w}}^H \tilde{\boldsymbol{h}}_k| \geq \mathrm{Re}\{\tilde{\boldsymbol{w}}^H \tilde{\boldsymbol{h}}_k\} \qquad (18)$$

and, therefore, for each $k = 1, \ldots, K$, we can strengthen the constraint (16) as

$$\mathrm{Re}\{\tilde{\boldsymbol{w}}^H \tilde{\boldsymbol{h}}_k\} \geq \sqrt{\gamma_k / p_k}\, \|\boldsymbol{U}_k \tilde{\boldsymbol{w}}\| \qquad (19)$$

where $\mathrm{Re}\{\cdot\}$ denotes the real part.

Introducing

$$\boldsymbol{V} \triangleq \begin{bmatrix} 0 & \boldsymbol{0}^T \\ \boldsymbol{0} & \boldsymbol{D} \end{bmatrix}^{1/2}$$

and using (16)-(18), the problem (14) can be expressed as

$$\begin{aligned}
&\min_{\tilde{\boldsymbol{w}}} \ \|\boldsymbol{V} \tilde{\boldsymbol{w}}\| \\
&\text{s.t.} \ \mathrm{Re}\{\tilde{\boldsymbol{w}}^H \tilde{\boldsymbol{h}}_k\} \geq \sqrt{\gamma_k / p_k} \|\boldsymbol{U}_k \tilde{\boldsymbol{w}}\|, \ k = 1, \cdots, K. \qquad (20)
\end{aligned}$$

The problem in (20) is convex and belongs to the class of SOCP problems. It can be solved with the worst-case complexity of $\mathcal{O}(R^3 K^{1.5})$ using interior point methods [9]. Note here that if the SDR approach of [8] is used to approximately solve (14), then the worst-case complexity will be $\mathcal{O}(R^4 (R + K)^{2.5})$ [10]. Therefore, the gain in computations of our approach with respect of the SDR technique of [8] is at least $\mathcal{O}(RK)$.

In some applications, it is useful to add individual relay power constraints to the problem in (6). For the ith relay, the individual power constraint can be written as $|w_i|^2 [\boldsymbol{D}]_{i,i} \leq p_i^t$ where p_i^t is the maximal transmitted power of the ith relay. Such individual relay power constraints can be straightforwardly added to (20) which in the latter case still can be expressed in the SOCP form.

4. SIMULATION RESULTS

Throughout our simulations, we consider a network with $R = 20$ relays and assume Rayleigh flat-fading channels whose coefficients have unit variance. The relay and destination noise

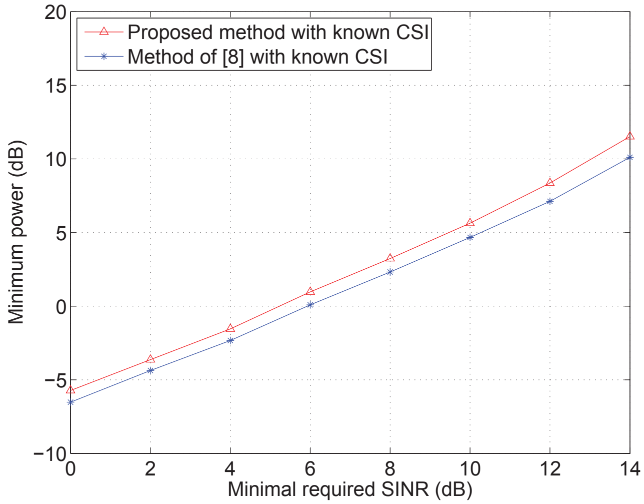

Fig. 2. Total relay transmitted power versus minimal required SINR. $K = 2$.

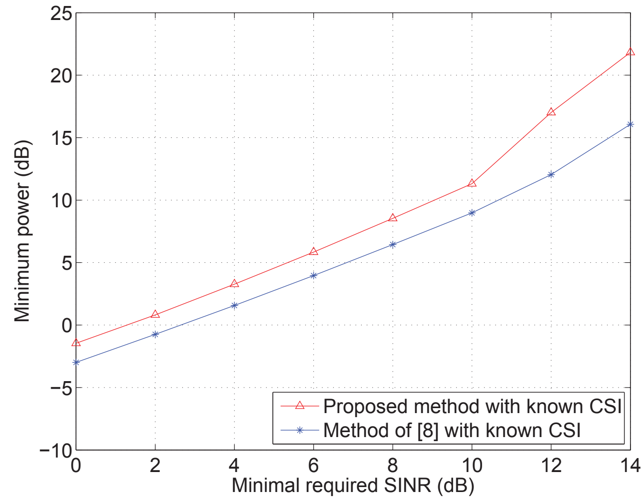

Fig. 3. Total relay transmitted power versus minimal required SINR. $K = 3$.

powers are assumed to be equal to each other. The transmitted power of each source is equal to 10 dB with respect to these noise powers. The binary phase shift keying (BPSK) modulation has been used. The proposed SOCP approach is compared with the SDR-based technique of [8] (that uses the instantaneous rather than the second-order statistics-based CSI in this particular case).

As in the case of single source-destination pair ($K = 1$) the transition between the problems (14) and (20) is exact, in our simulations we only address the case $K > 1$. Fig. 2 and 3 display the total relay transmit power versus the minimal required SINR at the destination for $K = 2$ and $K = 3$, respectively. From these figures, we can see that the proposed method has only a moderate reduction in performance as compared to the method of [8]. Therefore, the proposed technique represents a computationally attractive alternative to the method of [8].

5. REFERENCES

[1] J. N. Laneman and G. W. Wornell, "Energy efficient antenna sharing and relaying for wireless networks," *IEEE WCNC'00*, vol. 1, pp. 7-12, Chicago, Oct. 2000.

[2] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, pp. 3062-3080, Dec. 2004.

[3] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inform. Theory*, vol. 51, pp. 3037-3063, Sept. 2005.

[4] T. M. Cover and A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inform. Theory*, vol. 25, pp. 572-584, Sept. 1979.

[5] P. Larsson, "Large-scale cooperative relaying network with optimal combining under aggregate relay power constraint," *Proc. Future Telecomm. Conf.*, pp. 166-170, Beijing, China, Dec. 2003.

[6] Y. Jing and H. Jafarkhani, "Network beamforming using relays with perfect channel information," *Proc. IEEE ICASSP'07*, vol. 3, pp. 473-476, Honolulu, HI, Apr. 2007.

[7] V. Havary-Nassab, S. Shahbazpanahi, A. Grami, and Z.-Q. Luo, "Network beamforming based on second order statistics of the channel state information," *Proc. IEEE ICASSP'08*, vol. 2, pp. 2605-2608, Las Vegas, NV, Apr. 2008.

[8] S. Fazeli-Dehkordy, S. Gazor, and S. Shahbazpanahi, "Distributed peer-to-peer multiplexing using ad hoc relay networks," *Proc. IEEE ICASSP'08*, vol. 2, pp. 2373-2376, Las Vegas, NV, Apr. 2008.

[9] M. S. Lobo, L. Vandenberghe, S. Boyd, and H. Lebret, "Applications of second-order cone programming," *Linear Algebra and Its Applications*, vol. 284, pp. 193-228, 1998.

[10] L. Vandenberghe and S. Boyd, "Semidefinite programming," *SIAM Review*, vol. 38, pp. 49-95, 1996.