EFFICIENT SEMIDEFINITE RELAXATION FOR ENERGY-BASED SOURCE LOCALIZATION IN SENSOR NETWORKS

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ABSTRACT

Recently, energy-based localization using acoustic energy measurements has received much attention in wireless sensor networks. Since the objective function of the energy-based maximum likelihood (ML) localization is non-convex, the global solutions are hardly obtained without good initial estimates. In this paper, we relax this non-convex problem as a convex semidefinite programming (SDP), based on which a good estimate can be obtained and be improved by a procedure called randomization. Simulation results show that the proposed method is effective and outperforms the existing methods.

Index Terms— Acoustic energy, Source localization, Maximum-likelihood, Sensor networks, Semidefinite relaxation (SDR)

1. INTRODUCTION

Recently, energy-based localization has received much attention in wireless sensor networks [1]-[6]. In comparison with the localization methods based on the information of time of arrivals (TOA) or time differences of arrivals (TDOA), *etc*., energy-based localization is often an appropriate choice that requires less resources to localize a source in practical applications [2].

Recently, energy-based source localization is approached by a number of researchers. Li and Hu proposed an energy decay model [1] and verified the model in the real field experiment, according to which the quadratic elimination (QE) method is proposed. However, simulation results [4, 5] show that the performance of the QE method is not so satisfactory. Sheng and Hu gave an approach to maximum likelihood multiple-source localization [2] with the use of local search methods, which, however, is quite time-consuming and may trap into the local minima. Blatt and Hero presented a localization method in [3] with distributed implementation via projection-onto-convex-sets (POCS), which converges to the global solution. However, it suffers from the convex hull problem: the localization method will fail when the source lies out of the convex hull of the corresponding sensor locations. Therefore, the POCS method is not suitable for localization in *ad hoc* networks where the sensor structure is not fixed. Most recently, different two-stage closed-form weighted least square (WLS) methods [4, 5] were proposed with less computation for energy-based localization. Since the second-order noise terms are ignored in the methods, the performance of the WLS methods will be highly degraded when the noise becomes large.

In this paper, we propose a new method for energy-based localization via semidefinite relaxation (SDR). As shown in [3] and [6], the cost function in the ML formulation is non-convex. By using SDR, this non-convex problem can be relaxed as a convex semidefinite programming (SDP). And a standard randomization procedure is often used to refine the solution by the SDP.

2. SIGNAL MODEL

Here we adopt the energy decay model that is used in [2]-[4]. Consider a network composed of N sensors where each sensor receives M noisy measurements within a time interval. The intensity of the source attenuates inversely proportionally to the distance from the source to the sensor, i.e., the received signal at the ith sensor can be written as

$$z_i(t) = \frac{\sqrt{g_i}a(t-\tau_i)}{\|\boldsymbol{x}-\boldsymbol{s}_i\|^{\beta/2}} + w_i(t), \quad i = 1, \ldots, N, \tag{1}$$

where $a(t)$ is the intensity of the source signal measured 1 m from the source and $w_i(t)$ is the zero-mean Gaussian noise with variance $\sigma^2_{w_i}$, which is assumed to be independent to $a(t)$. For simplicity, we assume $\sigma^2_{w_i}$ is known and is identical for all sensors, i.e., $\sigma^2_{w_i} = \sigma^2_w$ for $i = 1, \ldots, N$. $\sqrt{g_i}$ is the gain of sensor i, τ_i is the time delay from the source to the sensor, and $\boldsymbol{x}$ and $\boldsymbol{s}_i$ represent the location of the source and the ith sensor, respectively. β is the decay factor, and is equal to 2 in free space, or slightly greater than 2 in the presence of reflections and reverberations [1]. In this paper, we formulate the localization problem with the decay factor of $\beta \geq 2$.

The received energy at the ith sensor is averaged over M signal measurements

$$y_i = \frac{1}{M}\sum_{m=0}^{M-1} z_i^2\left(t_s + \frac{m}{f_s}\right), \tag{2}$$

where t_s is the starting time and f_s is the sampling frequency.

Substituting (1) into (2), expanding (2) yields

$$\begin{aligned} y_i =& \frac{g_i}{M\|\boldsymbol{x}-\boldsymbol{s}_i\|^\beta}\sum_{m=0}^{M-1} a^2\left(t_s + \frac{m}{f_s} - \tau_i\right) \\ &+ \frac{2\sqrt{g_i}}{M\|\boldsymbol{x}-\boldsymbol{s}_i\|^{\beta/2}}\sum_{m=0}^{M-1} a\left(t_s + \frac{m}{f_s} - \tau_i\right) w_i\left(t_s + \frac{m}{f_s}\right) \\ &+ \frac{1}{M}\sum_{m=0}^{M-1} w_i^2\left(t_s + \frac{m}{f_s}\right) \end{aligned} \tag{3}$$

The cross term can be neglected when M is large due to the zero-mean and independence assumption of $a(t)$ and $w_i(t)$. Neglecting the time delay and letting $P = \frac{1}{M}\sum_{m=0}^{M-1} a^2\left(t_s + \frac{m}{f_s}\right)$ and $v_i = \frac{1}{M}\sum_{m=0}^{M-1} w_i^2\left(t_s + \frac{m}{f_s}\right)$, a more concise energy decay model is obtained, so that

$$y_i = \frac{g_i P}{\|\boldsymbol{x}-\boldsymbol{s}_i\|^\beta} + v_i, \quad i = 1, \ldots, N. \tag{4}$$

According to the central limit theorem, if M is large enough, the energy measurement noise of ith sensor v_i approximately obey a Gaussian distribution, i.e., $v_i \sim \mathcal{N}(\sigma_w^2, 2\sigma_w^4/M)$. According to the distribution, we can see that the larger the M, the smaller the energy noise variance. In the sequel, we will subtract the mean σ_w^2 for simplicity[1], and assume $v_i \sim \mathcal{N}(0, \sigma_v^2)$, where $\sigma_v^2 = 2\sigma_w^4/M$.

Notice that in [5] the authors derived a more accurate energy decay model than that mentioned by (4), where an unknown correlation function $\boldsymbol{R}(\delta)$ is introduced. However, $\boldsymbol{R}(\delta)$ is not involved in the method they proposed in [5]. This means that the model proposed in [5] cannot help improving the localization performance.

3. LOCALIZATION VIA SEMIDEFINITE RELAXATION

According to (4), the ML localization is formulated by

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \sum_{i=1}^{M} \left(y_i - \frac{g_i P}{\|\boldsymbol{x} - \boldsymbol{s}_i\|^\beta} \right)^2. \quad (5)$$

It is obvious that (5) is nonlinear and non-convex. Here we approach it via semidefinite relaxation.

3.1. Self localization of sensor locations

In the cooperative localization, for example, self localization of sensor locations, transmit power P is often known. Here we consider self localization with known P. According to (4), we have

$$y_i - v_i = \frac{g_i P}{\|\boldsymbol{x} - \boldsymbol{s}_i\|^\beta}, \quad (6)$$

which is equivalent to

$$\frac{y_i - v_i}{g_i P} = \frac{1}{\|\boldsymbol{x} - \boldsymbol{s}_i\|^\beta}. \quad (7)$$

Then

$$\frac{1}{\|\boldsymbol{x} - \boldsymbol{s}_i\|^2} = \left(\frac{y_i - v_i}{g_i P} \right)^{\frac{2}{\beta}} \approx \left(\frac{y_i}{g_i P} \right)^{\frac{2}{\beta}} \left(1 - \frac{2}{\beta} \frac{v_i}{y_i} \right), \quad (8)$$

where the approximation follows from the first-order Taylor expansion. By taking simple manipulations, we have

$$y_i \left[\|\boldsymbol{x} - \boldsymbol{s}_i\|^2 - \left(\frac{g_i P}{y_i} \right)^{\frac{2}{\beta}} \right] = \frac{2 v_i}{\beta} \|\boldsymbol{x} - \boldsymbol{s}_i\|^2. \quad (9)$$

Let

$$d_i = \|\boldsymbol{x} - \boldsymbol{s}_i\| \quad \text{and} \quad \varepsilon_i = \frac{2 v_i}{\beta} \|\boldsymbol{x} - \boldsymbol{s}_i\|^2,$$

we have

$$\varepsilon_i \sim \mathcal{N}\left(0, \sigma_i^2\right), \quad (10)$$

where $\sigma_i^2 = \left(\frac{2 d_i^2}{\beta} \right)^2 \sigma_v^2$. Notice that since v_i and v_j are identically independent distributed (i.i.d.), ε_i and $\varepsilon_j (i \neq j)$ are independent.

According to (9), the ML localization by (5) can be written as

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \sum_{i=1}^{N} \frac{y_i^2}{\sigma_i^2} \left(\|\boldsymbol{x} - \boldsymbol{s}_i\|^2 - \left(\frac{g_i P}{y_i} \right)^{\frac{2}{\beta}} \right)^2, \quad (11)$$

[1] Here we assume $y_i > 0$ ($i = 1, \ldots, N$), which is actually assumed in the previous works.

where σ_i^2 is a function of the true sensor location $\boldsymbol{x}$ in the definition of (10). With the approximation $d_i^2 \approx \left(\frac{g_i P}{y_i} \right)^{2/\beta}$ that follows from (4), we can have

$$\sigma_i^2 \approx \left[\frac{2}{\beta} \left(\frac{g_i P}{y_i} \right)^{\frac{2}{\beta}} \right]^2 \sigma_v^2. \quad (12)$$

By expanding $\|\boldsymbol{x} - \boldsymbol{s}_i\|^2$ and denoting $\boldsymbol{X} = \boldsymbol{x}\boldsymbol{x}^T$, (11) can be equivalently written as:

$$\begin{aligned} \min_{\boldsymbol{X}, \boldsymbol{x}} \quad & \sum_{i=1}^{N} \frac{y_i^2}{\sigma_i^2} \left(\mathrm{tr}(\boldsymbol{X}) - 2\boldsymbol{s}_i^T \boldsymbol{x} + \|\boldsymbol{s}_i\|^2 - \left(\frac{g_i P}{y_i} \right)^{\frac{2}{\beta}} \right)^2 \\ \text{subject to} \quad & \boldsymbol{X} = \boldsymbol{x}\boldsymbol{x}^T, \end{aligned} \quad (13)$$

where tr$(\boldsymbol{X})$ denotes the trace of matrix $\boldsymbol{X}$. Notice that the above problem is still non-convex due to the equality constraint. By relaxing $\boldsymbol{X} = \boldsymbol{x}\boldsymbol{x}^T$ into $\boldsymbol{X} \succeq \boldsymbol{x}\boldsymbol{x}^T$ ($\boldsymbol{A} \succeq \boldsymbol{B}$ means $\boldsymbol{A} - \boldsymbol{B}$ is positive semidefinite), and with the equivalence between $\boldsymbol{X} \succeq \boldsymbol{x}\boldsymbol{x}^T$ and $\left[\boldsymbol{X}\ \boldsymbol{x}; \boldsymbol{x}^T\ 1\right] \succeq 0$ [8], we obtain the following SDP formulation:

$$\begin{aligned} \min_{\boldsymbol{X}, \boldsymbol{x}} \quad & \sum_{i=1}^{N} \frac{y_i^2}{\sigma_i^2} \left(\mathrm{tr}(\boldsymbol{X}) - 2\boldsymbol{s}_i^T \boldsymbol{x} + \|\boldsymbol{s}_i\|^2 - \left(\frac{g_i P}{y_i} \right)^{\frac{2}{\beta}} \right)^2 \\ \text{subject to} \quad & \begin{bmatrix} \boldsymbol{X} & \boldsymbol{x} \\ \boldsymbol{x}^T & 1 \end{bmatrix} \succeq 0. \end{aligned} \quad (14)$$

Let $[\hat{\boldsymbol{X}}^*\ \hat{\boldsymbol{x}}^{*T}]$ denote the solution of (14). We apply the following procedure called *randomization*: draw K samples $\hat{\boldsymbol{x}}^k$ from $\mathcal{N}(\hat{\boldsymbol{x}}^*, \hat{\boldsymbol{X}}^* - \hat{\boldsymbol{x}}^* \hat{\boldsymbol{x}}^{*T})$ repeatedly and compute the corresponding value of the objective function in (5). The estimate corresponding to the smallest objective value is taken as the final one.

3.2. Source Localization

In the case of localizing an unknown source, the transmit power P of the source is often unknown. In this case, it is difficult to localize the source using the above-mentioned method. However, eliminating the nuisance variable P will be very helpful. Similar to the formulation in [2], the ratio of energy is given by

$$\frac{(y_i - v_i)/g_i}{(y_1 - v_1)/g_1} = \frac{\|\boldsymbol{x} - \boldsymbol{s}_1\|^\beta}{\|\boldsymbol{x} - \boldsymbol{s}_i\|^\beta}, \quad (15)$$

which is equivalent to

$$\left[\frac{(y_i - v_i)/g_i}{(y_1 - v_1)/g_1} \right]^{\frac{2}{\beta}} = \frac{\|\boldsymbol{x} - \boldsymbol{s}_1\|^2}{\|\boldsymbol{x} - \boldsymbol{s}_i\|^2}. \quad (16)$$

Applying the Taylor expansion to both the numerator and the denominator on the left-hand side of (16) and neglecting the terms of orders higher than 2, we have

$$[(y_i - v_i)/g_i]^{\frac{2}{\beta}} \approx \left(\frac{y_i}{g_i} \right)^{\frac{2}{\beta}} \left(1 - \frac{2}{\beta} \frac{v_i}{y_i} \right). \quad (17)$$

Notice that the two sides are strictly equivalent when $\beta = 2$.

Substituting (17) into (16) yields

$$\frac{\left(\frac{y_i}{g_i} \right)^{\frac{2}{\beta}} \left(1 - \frac{2}{\beta} \frac{v_i}{y_i} \right)}{\left(\frac{y_1}{g_1} \right)^{\frac{2}{\beta}} \left(1 - \frac{2}{\beta} \frac{v_1}{y_1} \right)} = \frac{\|\boldsymbol{x} - \boldsymbol{s}_1\|^2}{\|\boldsymbol{x} - \boldsymbol{s}_i\|^2}. \quad (18)$$

With manipulations, we have

$$\begin{aligned}
&\left(\frac{y_i}{g_i}\right)^{\frac{2}{\beta}}\|\boldsymbol{x}-\boldsymbol{s}_i\|^2-\left(\frac{y_1}{g_1}\right)^{\frac{2}{\beta}}\|\boldsymbol{x}-\boldsymbol{s}_1\|^2\\
&=\frac{2d_i^2 y_i^{\left(\frac{2}{\beta}-1\right)}}{\beta g_i^{\frac{2}{\beta}}}v_i-\frac{2d_1^2 y_1^{\left(\frac{2}{\beta}-1\right)}}{\beta g_1^{\frac{2}{\beta}}}v_1\\
&\approx\frac{2d_i^2 P^{\left(\frac{2}{\beta}-1\right)}}{\beta g_i d_i^{(2-\beta)}}v_i-\frac{2d_1^2 P^{\left(\frac{2}{\beta}-1\right)}}{\beta g_1 d_1^{(2-\beta)}}v_1\\
&=\frac{2d_i^\beta P^{\left(\frac{2}{\beta}-1\right)}}{\beta g_i}v_i-\frac{2d_1^\beta P^{\left(\frac{2}{\beta}-1\right)}}{\beta g_1}v_1\\
&\triangleq\xi_{i1},
\end{aligned}\tag{19}$$

where the approximation follows from $y_i\approx\frac{g_iP}{d_i^\beta}$. Notice that the approximation becomes equality when $\beta=2$. By stacking $\xi_{i1}(i=2,3,\ldots,N)$, the vector $\boldsymbol{\xi}=[\xi_{21},\xi_{31},\ldots,\xi_{N1}]^T$ is joint Gaussian distributed:

$$\boldsymbol{\xi}\sim\mathcal{N}(\mathbf{0},\boldsymbol{Q}),\tag{20}$$

provided that v_i and v_1 are i.i.d. Gaussian variables.

With the definition: $\boldsymbol{Q}\triangleq E[\boldsymbol{\xi}\boldsymbol{\xi}^T]$, we have

$$\boldsymbol{Q}[i-1,j-1]=\begin{cases}\frac{4d_i^{2\beta}P^{\left(\frac{4}{\beta}-2\right)}}{\beta^2 g_i^2}\sigma_i^2+\frac{4d_1^{2\beta}P^{\left(\frac{4}{\beta}-2\right)}}{\beta^2 g_1^2}\sigma_1^2, & i=j;\\ \frac{4d_1^{2\beta}P^{\left(\frac{4}{\beta}-2\right)}}{\beta^2 g_1^2}\sigma_1^2, & i\neq j.\end{cases},$$

$$\text{for } i,j=2,3,\ldots,N.\tag{21}$$

If $y_i/g_i\neq y_1/g_1$ $(i\neq 1)$, then the term of left-hand side in (19) can be written as

$$\begin{aligned}
&\left(\frac{y_i}{g_i}\right)^{\frac{2}{\beta}}\|\boldsymbol{x}-\boldsymbol{s}_i\|^2-\left(\frac{y_1}{g_1}\right)^{\frac{2}{\beta}}\|\boldsymbol{x}-\boldsymbol{s}_1\|^2\\
&=\left[\left(\frac{y_i}{g_i}\right)^{\frac{2}{\beta}}-\left(\frac{y_1}{g_1}\right)^{\frac{2}{\beta}}\right]\left(\|\boldsymbol{x}-\boldsymbol{c}_{i1}\|^2-r_{i1}^2\right),
\end{aligned}\tag{22}$$

where $\boldsymbol{c}_{i1}=\frac{\boldsymbol{s}_i-\varphi_{i1}^2\boldsymbol{s}_1}{1-\varphi_{i1}^2}$ and $r_{i1}=\frac{\varphi_{i1}\|\boldsymbol{s}_i-\boldsymbol{s}_1\|}{1-\varphi_{i1}^2}$ with $\varphi_{i1}=\left[\frac{(y_i/g_i)}{(y_1/g_1)}\right]^{\frac{1}{\beta}}$. Stacking all these terms on the right-hand side of (22) into a vector $\boldsymbol{\theta}_a$, we can have

$$\boldsymbol{\theta}_a\sim\mathcal{N}(\mathbf{0},\boldsymbol{Q}_a),\tag{23}$$

where $\boldsymbol{Q}_a$ is similarly defined as $\boldsymbol{Q}$ in (21).

On the other hand, if $y_i/g_i=y_1/g_1$, then (19) reduces to

$$h_{i1}-\boldsymbol{l}_{i1}^T\boldsymbol{x}=\left[\frac{2d_i^2 y_i^{\left(\frac{2}{\beta}-1\right)}}{\beta g_i^{\frac{2}{\beta}}}\right]v_i-\left[\frac{2d_1^2 y_1^{\left(\frac{2}{\beta}-1\right)}}{\beta g_1^{\frac{2}{\beta}}}\right]v_1,\tag{24}$$

where $\boldsymbol{l}_{i1}=2(y_1/g_1)^{\frac{2}{\beta}}(\boldsymbol{s}_i-\boldsymbol{s}_1)$ and $h_{i1}=(y_1/g_1)^{\frac{2}{\beta}}(\|\boldsymbol{s}_i\|^2-\|\boldsymbol{s}_1\|^2)$. Similarly, stacking the terms on the left-hand side of (24) into the vector $\boldsymbol{\theta}_b$, we have $\boldsymbol{\theta}_b\sim\mathcal{N}(\mathbf{0},\boldsymbol{Q}_b)$, where $\boldsymbol{Q}_b$ is defined as $\boldsymbol{Q}$.

Substituting $d_i^\beta\approx\frac{g_iP}{y_i}$ into (21) and defining

$$\boldsymbol{Q}'\triangleq\frac{1}{P^{(4/\beta)}}\boldsymbol{Q},\boldsymbol{Q}_a'\triangleq\frac{1}{P^{(4/\beta)}}\boldsymbol{Q}_a,\boldsymbol{Q}_b'\triangleq\frac{1}{P^{(4/\beta)}}\boldsymbol{Q}_b,$$

we have

$$\boldsymbol{\theta}_a^T\boldsymbol{Q}_a^{-1}\boldsymbol{\theta}_a+\boldsymbol{\theta}_b^T\boldsymbol{Q}_b^{-1}\boldsymbol{\theta}_b\propto\boldsymbol{\theta}_a^T\boldsymbol{Q}_a'^{-1}\boldsymbol{\theta}_a+\boldsymbol{\theta}_b^T\boldsymbol{Q}_b'^{-1}\boldsymbol{\theta}_b.\tag{25}$$

The ML estimate can thus be obtained by

$$\begin{aligned}\hat{\boldsymbol{x}}&=\arg\min_{\boldsymbol{x}}(\boldsymbol{\theta}_a^T\boldsymbol{Q}_a^{-1}\boldsymbol{\theta}_a+\boldsymbol{\theta}_b^T\boldsymbol{Q}_b^{-1}\boldsymbol{\theta}_b)\\&=\arg\min_{\boldsymbol{x}}(\boldsymbol{\theta}_a^T\boldsymbol{Q}_a'^{-1}\boldsymbol{\theta}_a+\boldsymbol{\theta}_b^T\boldsymbol{Q}_b'^{-1}\boldsymbol{\theta}_b).\end{aligned}\tag{26}$$

Similar to the case in sensor self-localization, we have

$$\|\boldsymbol{x}-\boldsymbol{c}_{i1}\|^2=\mathrm{tr}(\boldsymbol{X})-2\boldsymbol{c}_{i1}^T\boldsymbol{x}+\|\boldsymbol{c}_{i1}\|^2\ \text{ with }\ \boldsymbol{X}=\boldsymbol{x}\boldsymbol{x}^T.\tag{27}$$

Relaxing $\boldsymbol{X}=\boldsymbol{x}\boldsymbol{x}^T$ to $\boldsymbol{X}\succeq\boldsymbol{x}\boldsymbol{x}^T$, we obtain the following SDP formulation:

$$\begin{aligned}\min_{\boldsymbol{X},\boldsymbol{x}}\quad&\boldsymbol{\theta}_a^T\boldsymbol{Q}_a'^{-1}\boldsymbol{\theta}_a+\boldsymbol{\theta}_b^T\boldsymbol{Q}_b'^{-1}\boldsymbol{\theta}_b\\ \text{subject to}\quad&\begin{bmatrix}\boldsymbol{X}&\boldsymbol{x}\\\boldsymbol{x}^T&1\end{bmatrix}\succeq 0.\end{aligned}\tag{28}$$

After solving (28), similar procedure used in the last subsection is employed to get the final estimate. Note that the original objective function in (5) is used here but without knowledge of P. Therefore, we will first estimate P using the linear weighted least squares (LWLS) method after drawing a sample, and then we compute the corresponding value of the objective function.

4. SIMULATION RESULTS

In this section, we will show the performance of the proposed method through simulations. And we will compare the performance of the proposed method with the methods presented in [4] and [5], which are called the weighted least squares (WLS) and the weighed direct least squares with correction (WDC), respectively.

4.1. Self localization of sensor locations

9 randomly and uniformly distributed sensors in a 2-D sensor field of size 25×25 square meters are employed in our simulations. The locations of the sensors are assumed to be known. The data are generated according to (4), where we set $\beta=2$, $P=100$ and $g_i=1$ $(i=1,\ldots,N)$. In the simulations, P is known. The number of the samples drawn in the procedure of randomization is $K=1000$. The performance is evaluated by root mean square error (RMSE) defined by

$$\text{RMSE}=\sqrt{\sum_{m=1}^{M_c}\frac{\|\hat{\boldsymbol{x}}_m-\boldsymbol{x}\|^2}{M_c}}\tag{29}$$

where M_c is the number of Monte Carlo (MC) runs and $\hat{\boldsymbol{x}}_m$ is the estimate of source location in the mth run. In the simulations, we set $M_c=3000$. In each run, the location of the sensor to be estimated is generated randomly and uniformly in the sensor field. The Cramer-Rao bound (CRB) is computed the same as that in [4]. Note that the WDC method can be modified and be used to self-localization. For fair comparison, in the simulations we apply the improving technique used in Section V of [4] to the WDC method although there is no such a technique in the method presented in [5]. The RMSEs are shown in Fig. 1, where we see that the performance of the proposed method is better than that of the WDC method, and most importantly, the proposed method can achieve the CRB in a moderately noisy environment.

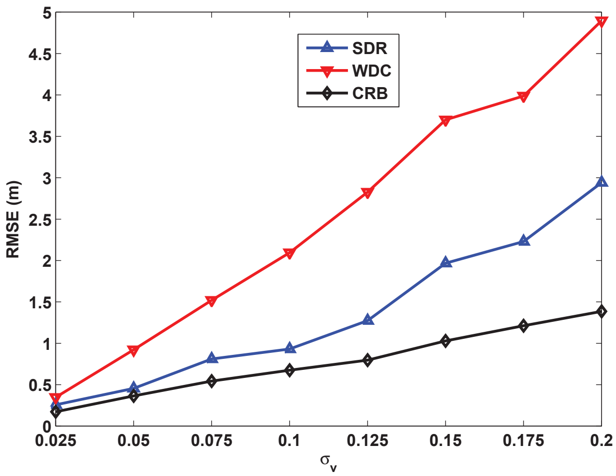

Fig. 1. Comparison of different methods in self localization

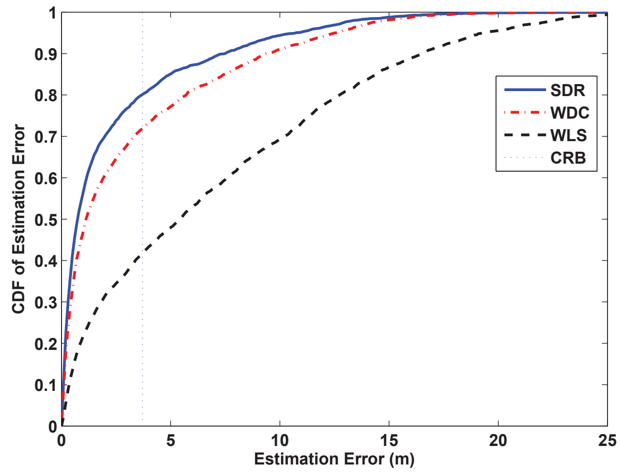

Fig. 3. The CDF of the estimation errors when $\sigma_v = 0.2$

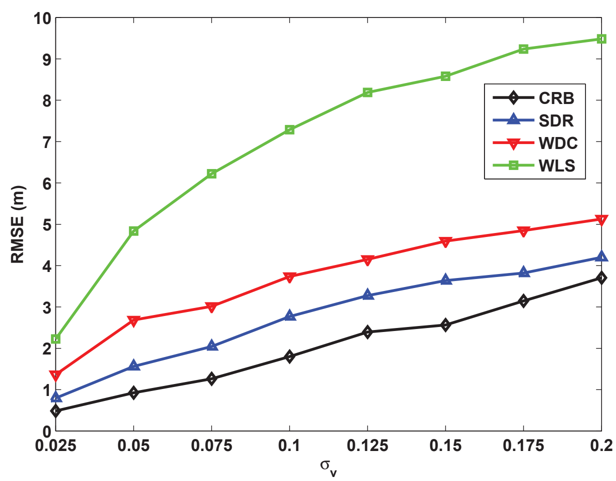

Fig. 2. Comparison of different methods in source localization

4.2. Source Localization

The simulation parameters are set as the same as those in last subsection, except that P is unknown in the simulations. In Fig. 2, the RMSEs obtained by WLS, WDC and the proposed method are plotted for comparison. It is seen that Fig. 2 shares the similar phenomenon to that of Fig. 1. To further show the performance of the proposed method when the noise is large, we plot the cumulative distribution function (CDF) of the estimation errors ($\|\hat{\boldsymbol{x}}_m - \boldsymbol{x}\|, m = 1, 2, \ldots, M_c$) of all the MC runs in Fig. 3 when $\sigma_v = 0.2$. From Fig. 3, we see that the estimation errors in over 85% of the simulated runs are close to the CRB, which indicates that the proposed method is very efficient.

5. CONCLUSIONS

In this paper, we have proposed a new method for energy-based source localization by SDR. Using the solution of the SDP, we apply the randomization procedure to find a better estimate. Simulation results show the proposed method outperforms the existing methods.

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