

DECENTRALIZED VARIATIONAL FILTERING FOR SIMULTANEOUS SENSOR LOCALIZATION AND TARGET TRACKING IN BINARY SENSOR NETWORKS

Jing Teng, Hichem Snoussi and Cédric Richard

ICD/LM2S, University of Technology of Troyes, 12 rue Marie Curie, 10000, France

ABSTRACT

Resource limitations in wireless sensor networks have put stringent constraints on distributed signal processing. In this paper, we propose a cluster-based decentralized variational filtering algorithm with minimum resource allocation for simultaneous sensor localization and target tracking. At each sampling instant, only one cluster of sensors is activated according to the prediction of the target state. Slave sensors employ a binary proximity observation model to reduce energy consumption and minimize communication cost. Based on the binary measurements between sensors and the target, activated sensors and target location estimates are interdependently improved. By adopting the variational method, the inter-cluster information exchange is reduced to one single Gaussian statistic, further minimizing resource consumption in the network. Since the measurement incorporation and the approximation of the filtering distribution are jointly performed by variational calculus, an effective and lossless compression is achieved compared to the classical Particle Filtering. Effectiveness of the proposed approach is evaluated in terms of tracking accuracy and localization precision.

Index Terms— Cluster-based, variational method, localization, tracking, binary proximity sensor

1. INTRODUCTION

Wireless sensor networks (WSN) are "data centric". The data sensed by WSN, such as range, bearing, temperature or humidity, are meaningless without supplementary sensor locations information [1, 2, 3]. Sensor localization has thus received considerable attention in literature [4]. Target tracking is one of the typical location-dependent WSN applications. The moving target can be well tracked if the sensor positions and orientations are known exactly [5]. However, it is not always possible to deploy or localize the sensors precisely. Sensor location refinement/calibration based on known positions of a moving target has been proposed in [6, 5]. In this work, by incorporating measurement information between sensors and a moving target, we consider the simultaneous sensor localization and target tracking problem. This attractive solution poses no restriction on the mobile target, whose timely position is estimated in the presence of sensor localization errors, without additional hardware configuration requirement on the sensor. Furthermore, it allows a continuous improvement of sensor localization, even during the tracking phase, since each observation adds a geometric constraint and leads to an improvement in estimation over time. The problem was defined by Taylor et al. [2] as simultaneous localization and tracking (SLAT). In earlier works [7, 8], the target to be located is a mobile robot, whose control input is known a priori. In addition, the incorporated observations are assumed to be range-bearing measurements, which require a special antenna configuration and omnidirectional signals. In this paper, we consider a much more general situation, where the tar-

get moves arbitrarily through the environment, with no constraint on its direction or velocity. Concerning the sensors, a hierarchical WSN is formed. Cluster heads (CHs), with high computation and communication capabilities, are sparsely placed to fuse data from their slave sensors and perform the SLAT algorithm. They are triggered according to the prediction of the target location. Further information on the cluster-activating protocol is stated in [9]. Slave sensors are randomly and densely deployed through the span of the network. They belong to clusters with singular cluster head. By employing a binary proximity observation model [10], they report their observation in one bit to corresponding CH. A general state evolution model is proposed to describe the locations of the target and the activated sensors by a joint probability distribution. We adopt the Bayesian framework to estimate the joint probability distribution. By incorporating the binary observation received in the activated CH, the joint probability distribution is updated on-line. To avoid the representational complexity, we use the variational method to approximate the joint state during the observation incorporation phase. To sum up, a decentralized variational filtering algorithm for SLAT (DVaSLAT) in binary sensor networks (BSN) is proposed, ensuring the tracking accuracy and the localization precision with minimum resource allocation. We will formulate the SLAT problem by a general state evolution model (GSEM) and a binary proximity observation model (BPOM) in Section 2. Section 3 is dedicated to a detailed description of the DVaSLAT algorithm. In Section 4, performance of the proposed algorithm is studied by computer simulations. Section 5 concludes the paper.

2. PROBLEM FORMULATION

2.1. General State Evolution Model

Since the mobile target travels arbitrarily in the sensor field, instead of a traditional kinematic parameter model [10], we employ the general state evolution model (GSEM) [11, 12, 13]. The model is more adaptive to practical situation and has no restriction on the velocity and moving direction of the target. At instant t , the hidden state to be estimated contains the target position x_t and a set of activated sensor locations $S_t = \{s_t^1, s_t^2, \dots, s_t^m\}$, where m denotes the number of sensors in the activated cluster. The sensor position s^i is assumed to be a Gaussian variable, whose expectation is its latest estimate value \hat{s}^i , and the precision matrix is η^i . The initial value of \hat{s}^i is the assumed deployment position \bar{s}^i , and η^i indicates the position offset due to deployment error and other spatial factors. The target x_t is assumed to follow an extended Gaussian model, where the expectation μ_t and the precision matrix λ_t are both random, with a

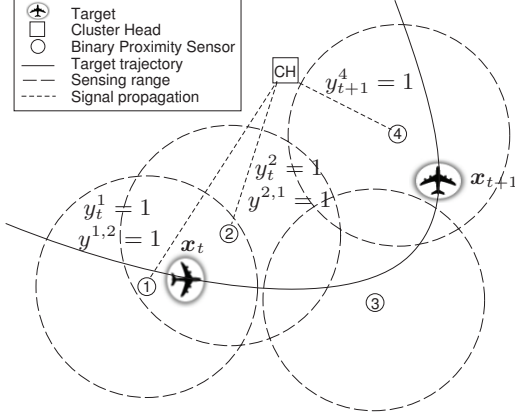


Fig. 1. The Binary Proximity Observation Model is described by a simple example. With respect to the 1st sensor, the target and the 2nd sensor are within its sensing range at instant t . Observation $y_t^1 = 1$ and $y^{1,2} = 1$ is thus transmitted to the CH. The same principle holds true for the 2nd sensor. Concerning the 3rd and the 4th sensors, they keep silence at instant t . The CH then assign a "zero" to the observation of them after waiting a given time slot. The situation at instant $t + 1$ can be similarly deduced.

Gaussian distribution and a Wishart distribution respectively:

$$\begin{cases} \mathbf{s}^i & \sim \mathcal{N}(\hat{\mathbf{s}}^i, \boldsymbol{\eta}^i) \\ \mathbf{x}_t & \sim \mathcal{N}(\boldsymbol{\mu}_t, \boldsymbol{\lambda}_t) \\ \boldsymbol{\mu}_t & \sim \mathcal{N}(\boldsymbol{\mu}_{t-1}, \bar{\boldsymbol{\lambda}}) \\ \boldsymbol{\lambda}_t & \sim \mathcal{W}_d(\bar{\mathbf{V}}, \bar{n}) \end{cases}, \quad \boldsymbol{\alpha}_t \equiv \{\mathbf{x}_t, \boldsymbol{\mu}_t, \boldsymbol{\lambda}_t, \mathbf{S}_t\} \quad (1)$$

where $\bar{\boldsymbol{\lambda}}$ is the initial precision matrix reflecting the uncertainty of the target position expectation at instant t with respect to the previous one. The target state precision matrix $\boldsymbol{\lambda}_t$ is modeled by a d dimensional Wishart distribution, with $\bar{\mathbf{V}}$ and \bar{n} denoting respectively its precision matrix and degree of freedom. Notice that $\bar{\cdot}$ denotes initial fixed parameter. We use $\boldsymbol{\alpha}_t$ to denote the extended hidden state.

2.2. Binary Proximity Observation Model

We investigate the SLAT problem using binary proximity sensors. As shown in Fig. 1, such simple sensors only provide one single bit per instant, which indicates the presence or absence of a target within their detection range. The binary signal y_t^i is constructed and transmitted in the following form:

$$y_t^i = \begin{cases} 1, & \text{if } \|\mathbf{x}_t - \mathbf{s}^i\| \leq \gamma \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

where γ is the sensing range of sensors, \mathbf{x}_t is the location of the target at instant t , and \mathbf{s}^i is the location of the i^{th} sensor. In order to minimize energy and bandwidth consumption, only those slave sensors that detected the presence of the target transfer their binary proximity information and identify themselves to their CH. Due to the noisy wireless link, the signal received at the CH is distributed according to $p(z_t^{i,x} | y_t^i) \sim \mathcal{N}(\beta^i y_t^i, \sigma_\epsilon^2)$, where β^i is the attenuation coefficient associated with the i^{th} sensor, and σ_ϵ^2 is the noise covariance. Assuming the noise samples ϵ_t^i are independently and

identically distributed, we have

$$p(z_t^{i,x} | \mathbf{x}_t, \mathbf{s}^i) = \sum_{y_t^i} p(z_t^{i,x} | y_t^i) P(y_t^i | \mathbf{x}_t, \mathbf{s}^i). \quad (3)$$

As shown in the formulation (2), the mapping between \mathbf{x}_t , \mathbf{s}^i and y_t^i is deterministic. Therefore, $p(z_t^{i,x} | \mathbf{x}_t, \mathbf{s}^i) = p(z_t^{i,x} | y_t^i)$. Similarly, if the j^{th} activated sensor is in the sensing range of the i^{th} one, $y^{i,j} = 1$, else $y^{i,j} = 0$. The binary observations received at the activated CH from the i^{th} sensor is thus defined as follows:

$$\begin{aligned} p(z_t^{i,x} | \mathbf{x}_t, \mathbf{s}^i) &= \mathcal{N}(\beta^i y_t^i, \sigma_\epsilon^2) \\ p(z_t^{i,j} | \mathbf{s}^i, \mathbf{s}^j) &= \mathcal{N}(\beta^i y^{i,j}, \sigma_\epsilon^2). \end{aligned} \quad (4)$$

Defining m as the number of slave sensors in the activated cluster, the observations gathered in the CH at instant t are denoted by $\mathbf{z}_t \equiv \{\mathbf{z}_t^i\}_{i=1, \dots, m}$, where $\mathbf{z}_t^i \equiv \{z_t^{i,x}, \{z_t^{i,j}\}_{j=1, \dots, m}^{j \neq i}\}$.

3. DVASLAT ALGORITHM

The SLAT problem can be viewed as an optimal estimation problem, consisting of recovering the unobserved hidden state $\boldsymbol{\alpha}_t$ from a set of observations \mathbf{z}_t . In the Bayesian context, it can be formulated as recursively calculating the predictive distribution $p(\boldsymbol{\alpha}_t | \mathbf{z}_{1:t-1})$ and the posterior distribution $p(\boldsymbol{\alpha}_t | \mathbf{z}_{1:t})$.

$$\begin{aligned} p(\boldsymbol{\alpha}_t | \mathbf{z}_{1:t-1}) &= \int p(\boldsymbol{\alpha}_t | \boldsymbol{\alpha}_{t-1}) p(\boldsymbol{\alpha}_{t-1} | \mathbf{z}_{1:t-1}) d\boldsymbol{\alpha}_{t-1}; \\ p(\boldsymbol{\alpha}_t | \mathbf{z}_{1:t}) &= p(\mathbf{z}_t | \boldsymbol{\alpha}_t) p(\boldsymbol{\alpha}_t | \mathbf{z}_{1:t-1}) / p(\mathbf{z}_t | \mathbf{z}_{1:t-1}). \end{aligned} \quad (5)$$

The non-linear and non-Gaussian aspects of the GSEM in Eq. (1) lead to intractable integrals, when calculating the marginal distributions above. We propose a Variational Filtering to approximate the density distribution $p(\boldsymbol{\alpha}_t | \mathbf{z}_{1:t})$ by a separable distribution $q(\boldsymbol{\alpha}_t)$ in minimizing the Kullback-Leibler (KL) divergence error:

$$D_{\text{KL}}(q || p) = \int q(\boldsymbol{\alpha}_t) \log \frac{q(\boldsymbol{\alpha}_t)}{p(\boldsymbol{\alpha}_t | \mathbf{z}_{1:t})} (d\boldsymbol{\alpha}_t), \quad (6)$$

$$\text{where } q(\boldsymbol{\alpha}_t) = \prod_i q(\boldsymbol{\alpha}_t^i) = q(\mathbf{x}_t) q(\boldsymbol{\mu}_t) q(\boldsymbol{\lambda}_t) q(\mathbf{S}_t),$$

$$\text{and } q(\mathbf{S}_t) = \prod_{i=1}^m q(\mathbf{s}_t^i).$$

Since $\int q(\boldsymbol{\alpha}_t^i) d\boldsymbol{\alpha}_t^i = 1$, by using a Lagrange multiplier, the following approximate distribution yields [14],

$$q(\boldsymbol{\alpha}_t^i) \propto \exp\langle \log p(\mathbf{z}_{1:t}, \boldsymbol{\alpha}_t) \rangle_{\prod_{j \neq i} q(\boldsymbol{\alpha}_t^j)}, \quad (7)$$

where $\langle \cdot \rangle_q$ denotes the expectation operator relative to the distribution q . Taking into account the separable approximate distribution at time $t - 1$, that is, $\hat{p}(\boldsymbol{\alpha}_{t-1} | \mathbf{z}_{1:t-1}) = q(\boldsymbol{\alpha}_{t-1})$, the filtering distribution at time t is deduced,

$$\begin{aligned} \hat{p}(\boldsymbol{\alpha}_t | \mathbf{z}_{1:t}) &= \frac{p(\mathbf{z}_t | \boldsymbol{\alpha}_t) \int p(\boldsymbol{\alpha}_t | \boldsymbol{\alpha}_{t-1}) q(\boldsymbol{\alpha}_{t-1}) d\boldsymbol{\alpha}_{t-1}}{p(\mathbf{z}_t | \mathbf{z}_{1:t-1})} \\ &\propto p(\mathbf{z}_t | \mathbf{x}_t, \mathbf{S}_t) p(\mathbf{x}_t | \boldsymbol{\mu}_t, \boldsymbol{\lambda}_t) p(\boldsymbol{\lambda}_t) p(\mathbf{S}_t) q_p(\boldsymbol{\mu}_t), \end{aligned} \quad (8)$$

where $q_p(\boldsymbol{\mu}_t) = \int p(\boldsymbol{\mu}_t | \boldsymbol{\mu}_{t-1}) q(\boldsymbol{\mu}_{t-1}) d\boldsymbol{\mu}_{t-1}$.

Thanks to the separable form of $q(\boldsymbol{\alpha}_t)$, the filtering distribution $p(\boldsymbol{\alpha}_t | \mathbf{z}_{1:t})$ is sequentially updated by a simple integration with respect to $\boldsymbol{\mu}_{t-1}$. Considering the GSEM proposed in (1), the evolution

of μ_{t-1} is Gaussian, namely $p(\mu_t|\mu_{t-1}) \sim \mathcal{N}(\mu_{t-1}, \bar{\lambda})$. Defining $q(\mu_{t-1}) \sim \mathcal{N}(\mu_{t-1}^*, \lambda_{t-1}^*)$, $q_p(\mu_t)$ is also Gaussian [15], namely $q_p(\mu_t) \sim \mathcal{N}(\mu_t^p, \lambda_t^p)$. Therefore, the filtering distribution is jointly updated and approximated, yielding a natural and adaptive compression, which is propagated without lossy compression. As the location estimates of sensors are locally stored in the activated CH, the temporal dependence on the past is hence reduced to incorporate only one component approximation $q(\mu_{t-1})$. Accordingly, communication between two successive active CH is then reduced to sending the mean and the precision matrix of it. Equation (7) gives a Gaussian distribution for μ_t and a Wishart distribution for λ_t , namely $q(\mu_t) \sim \mathcal{N}(\mu_t^*, \lambda_t^*)$, $q(\lambda_t) \sim \mathcal{W}_d(V_t^*, n^*)$, where the parameters are iteratively updated until convergence, according to the following scheme:

$$\begin{aligned}\mu_t^p &= \mu_{t-1}^*, \lambda_t^p = ((\lambda_{t-1}^*)^{-1} + (\bar{\lambda})^{-1})^{-1} \\ \mu_t^* &= (\lambda_t^*)^{-1} (\langle \lambda_t \rangle \langle x_t \rangle + \lambda_t^p \mu_t^p) \\ \lambda_t^* &= \langle \lambda_t \rangle + \lambda_t^p, n^* = \bar{n} + 1 \\ V_t^{*-1} &= \langle x_t x_t^T \rangle - \langle x_t \rangle \langle \mu_t \rangle^T - \langle \mu_t \rangle \langle x_t \rangle^T + \langle \mu_t \mu_t^T \rangle + \bar{V}^{-1} \\ \langle \mu_t \rangle &= \mu_t^*, \langle \lambda_t \rangle = n_t^* V_t^*, \langle \mu_t \mu_t^T \rangle = \lambda_t^{*-1} + \mu_t^* \mu_t^{*T}\end{aligned}$$

However, the target state distribution $q(x_t)$ and the activated sensors positions distribution $q(S_t)$ do not have closed forms. In order to compute their means and precision matrices (required for the iteration update above), we resort to the importance sampling (IS) method, where samples are drawn from Gaussian distributions and are weighted according to their likelihoods. Combining the equation (7) and (8), we have the likelihood expression for $q(x_t)$ and $q(s_t^i)$ as follows:

$$\begin{aligned}q(x_t) &\propto \prod_{i=1}^m p(z_t^{i,x} | x_t, \hat{s}_t^i) \mathcal{N}(\langle \mu_t \rangle, \langle \lambda_t \rangle) \\ &\approx \sum_{k=1}^N w_t^{(k)} \delta_{x_t^{(k)}}(x_t) / \sum_{k=1}^N w_t^{(k)},\end{aligned}$$

where $x_t^{(k)} \sim \mathcal{N}(\langle \mu_t \rangle, \langle \lambda_t \rangle)$, $w_t^{(k)} \propto \prod_{i=1}^m p(z_t^{i,x} | x_t^{(k)}, \hat{s}_t^i)$;

$$\begin{aligned}q(s_t^i) &\propto p(z_t^{i,x} | \hat{x}_t, s_t^i) \prod_{j \neq i}^{m-1} p(z_t^{i,j} | s_t^i, \hat{s}_t^j) \mathcal{N}(\hat{s}^i, \eta^i) \\ &\approx \sum_{k=1}^N w_t^{i,(k)} \delta_{s_t^{i,(k)}}(s_t^i) / \sum_{k=1}^N w_t^{i,(k)},\end{aligned}\quad (9)$$

where $s_t^{i,(k)} \sim \mathcal{N}(\hat{s}^i, \eta^i)$,

$$w_t^{i,(k)} \propto p(z_t^{i,x} | \hat{x}_t, s_t^{i,(k)}) \prod_{j \neq i}^{m-1} p(z_t^{i,j} | s_t^{i,(k)}, \hat{s}_t^j),$$

where $\delta_{x_t^{(k)}}$ and $\delta_{s_t^{i,(k)}}$ denote the Dirac delta functions located at $x_t^{(k)}$ and $s_t^{i,(k)}$, respectively. By minimizing mean square errors (MMSE), estimations of the target state and the activated sensors positions are interdependently updated as follows:

$$\begin{aligned}\hat{x}_t &= E_{q(x_t)}[x_t | \{z_t^i\}_{i=1, \dots, m}] \\ \hat{s}_t^i &= E_{q(s_t^i)}[s_t^i | \{z_t^i\}_{i=1, \dots, m}].\end{aligned}\quad (10)$$

4. EVALUATION AND SIMULATION

The performance of the proposed DVaSLAT algorithm is shown on a synthetic example, the purpose of which is to establish a base-

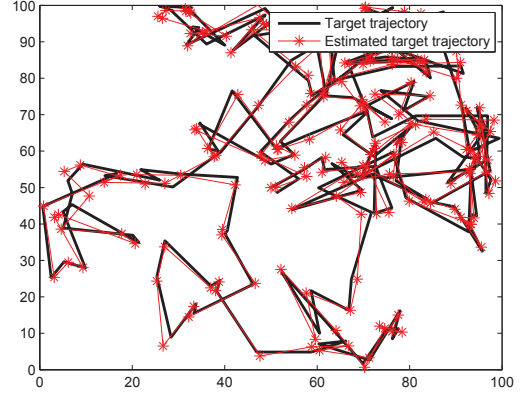


Fig. 2. Target tracking result.

line performance on a relatively difficult problem. Thus no constraint is put on the target velocity or moving direction (see Fig. 2). Concerning the sensors, 400 binary sensors belonging to 16 clusters were uniformly deployed in a 2 dimensional field ($100 \times 100 \text{ m}^2$), with sensing ranges identically fixed to 15 m. Due to the spatially varying environment factors and deployment errors, sensors were in fact randomly distributed around their initially set locations \bar{s}^i , with precision η^i identical for all the sensors (see Fig. 3, where the red lines denote the distances between the true positions of sensors and their deployment values). The parameters involved were set as, $\bar{V} = \text{diag}([5 \ 5])$, $\eta_i = \text{diag}([1/4 \ 1/4])$, $\bar{\lambda} = \text{diag}([1/900 \ 1/900])$, $\bar{n} = 10$, $\sigma_e = 0.1$. The low state precision $\bar{\lambda}$ and the high degree of freedom \bar{n} allow a general non informative prior. Performance of the DVaSLAT algorithm is shown in Fig. 2 and Fig. 3. Fig. 3 demonstrated the central part of the network to clearly show the improvement in sensor localization. As the central part happens to be the high traffic area, the sensors located there are thus frequently re-located. Fig. 4 quantifies the tracking accuracy and the localization precision in Root Mean Square Error (RMSE). Because of the cluster-based scheme, only the sensors that have been activated are localized. The peak points in Fig. 4-(b) reflect corresponding resting sensors. One can notice that accurate tracking performance and sensor localization is achieved, despite the absence of exact a priori information and the lack of accurate observation.

5. CONCLUSION

A decentralized variational filtering solution to simultaneously localize sensors and track mobile target was proposed in the context of BSN. To minimize resource consumption, the algorithm is executed on a fully decentralized cluster scheme. Furthermore, the BPOM quantifies the detected signal to a single bit, not only reducing the energy consumption of sensors but also the communication cost. The variational method allows an implicit compression of the exchanged statistics during the observation incorporation phase. In conclusion, as the target move freely in BSN, a number of observations are generated, which facilitate both the activated sensors localization and the target tracking. By incorporating these measurements into the DVaSLAT algorithm, estimations of sensors and that of the target are interdependently and continuously improved.

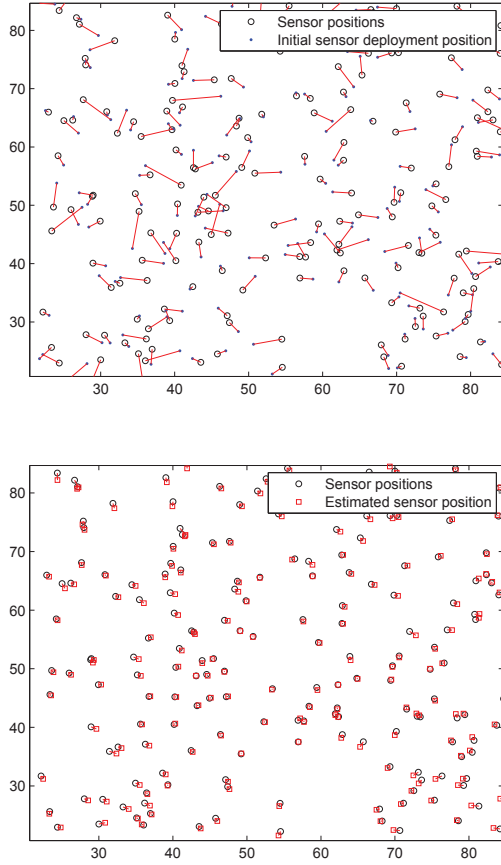
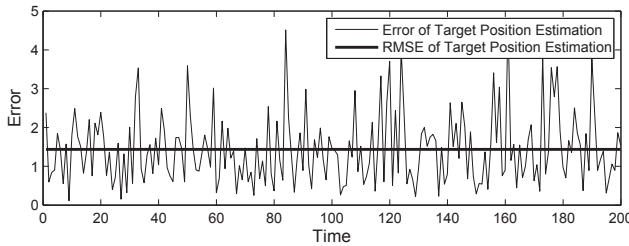
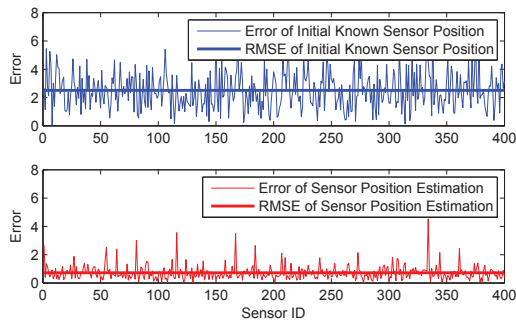


Fig. 3. Initial deployment vs. sensor localization



(a) Tracking performance



(b) Performance of sensor localization vs. initial deployment

Fig. 4. Performance of the DVaSLAT algorithm

6. REFERENCES

- [1] Junaid Ansari, Janne Riihijarvi, and Petri Mahonen, "Combining particle filtering with cricket system for indoor localization and tracking services," in *IEEE 18th International Symposium on Personal, Indoor and Mobile Radio Communications, PIMRC 2007.*, 2007, pp. 1–5.
- [2] Christopher Taylor, Ali Rahimi, Jonathan Bachrach, Howard Shrobe, and Anthony Grue, "Simultaneous localization, calibration, and tracking in an ad hoc sensor network," in *Information Processing in Sensor Networks (IPSN '06)*, 2006.
- [3] Jehoshua Bruck, Jie Gao, and Anxiao Jiang, "Localization and routing in sensor networks by local angle information," in *Proceedings of the 6th ACM international symposium on Mobile ad hoc networking and computing*, 2005.
- [4] Koen Langendoen and Niels Reijers, "Distributed localization in wireless sensor networks: a quantitative comparison," *Computer Networks: The International Journal of Computer and Telecommunications Networking*, vol. 43, pp. 499–518, 2003.
- [5] Volkan Cevher and James H. McClellan, "Acoustic node calibration using a moving source," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 42, pp. 585–600, 2006.
- [6] A. Galstyan, B. Krishnamachari, K. Lerman, and S. Pattem, "Distributed online localization in sensor networks using a moving target," in *Information Processing in Sensor Networks (IPSN '04)*, 2004.
- [7] Hugh Durrant Whyte and Tim Bailey, "Simultaneous localization and mapping: part i," *Robotics & Automation Magazine, IEEE*, vol. 13, pp. 99–110, 2006.
- [8] Tim Bailey and Hugh Durrant Whyte, "Simultaneous localization and mapping (slam) part ii," *IEEE Robotics and Automation Magazine.*, vol. 13, pp. 108–117, 2006.
- [9] Jing Teng, Hichem Snoussi, and Cédric Richard, "Prediction-based proactive cluster target tracking protocol for binary sensor networks," in *The 7th IEEE International Symposium on Signal Processing and Information Technology*, 2007.
- [10] Petar M. Djurić, Mahesh Vemula, and Mónica F. Bugallo, "Target tracking by particle filtering in binary sensor networks," *IEEE Transaction on Signal Processing*, vol. 56, pp. 2229–2238, 2008.
- [11] Hichem Snoussi and Cédric Richard, "Ensemble learning online filtering in wireless sensor networks," in *IEEE ICCS International Conference on Communications Systems*, 2006.
- [12] Jing Teng, Hichem Snoussi, and Cédric Richard, "Binary variational filtering for target tracking in sensor networks," in *IEEE Workshop on Statistical Signal Processing*, 2007.
- [13] Jing Teng, Hichem Snoussi, and Cédric Richard, "Variational filtering algorithm for interdependent target tracking and sensor localization in wireless sensor network," in *The 16th European Signal Processing Conference*, 2008.
- [14] Dimitris G. Tzikas, Aristidis C. Likas, and Nikolaos P. Galatsanos, "The variational approximation for bayesian inference," *IEEE Signal Processing Magazine*, vol. 25, pp. 131–146, 2008.
- [15] J. Vermaak, N.D. Lawrence, and P Perez, "Variational inference for visual tracking," in *Conf. Computer Vision and Pattern Recog, CVPR '03*, June 2003.