# Progressive Distributed Estimation over Noisy Channels in Wireless Sensor Networks

Li Zhang, Xian-Da Zhang
Department of Automation, Tsinghua University
Beijing 100084, China
Email: zhang-li04@mails.thu.edu.cn
zxd-dau@mail.thu.edu.cn

Tracey Ho and Theodoros Dikaliotis
Department of Electrical Engineering
California Institute of Technology
Pasadena CA 91125, USA
Email: {tho,tdikal}@caltech.edu

***Abstract*—This paper presents a new progressive distributed estimation scheme (DES) along with the power scheduling among sensors under AWGN channels. The progressive DES consists of a transmission bit allocation scheme and a quasi best linear unbiased estimate (BLUE) of the unknown parameter at each sensor. This scheme is shown to outperform the traditional progressive DES. Moreover, the power scheduling among sensors, which minimizes the total transmission power subject to the desired MSE performance tolerance, is formulated as a convex problem and the optimal solution is derived analytically.**

***Index Terms*—Distributed estimation, incremental, quantization, distributed signal processing, sensor networks.**

## I. Introduction

Parameter estimation by a set of distributed sensors and a fusion center (FC) is frequently encountered in the wireless sensor network (WSN) applications. Some of the previous works present the centralized distributed estimation schemes (DESs), which assume communication is only between sensors and the FC and rely on severely quantized digital data from sensors to form an estimate of the unknown parameter at the FC, e.g. [1]-[4]. The other schemes, which are much more efficient in terms of energy and communications than centralized schemes, are the progressive DESs, developed in [5]-[8]. Assuming there is a multihop routing tree from sensors to the FC, in the progressive DESs the parameter estimate is circulated through the network, and along the way each sensor makes an adjustment to the estimate based on its observations.

The data transmitted through the network is quantized in [5]-[6] and raw in [7]-[8], respectively. This paper focuses on the progressive DESs based on quantized data. In [5], each sensor first performs a quasi-best linear unbiased estimate (BLUE) of the unknown parameter based on the data received from its immediate upstream sensor and the local observation, and forwards the quantized version of the estimate to its immediate downstream sensor over AWGN channels. Assuming perfect transmission, [6] proposes a distributed adaptive quantization and estimation scheme, in which each sensor dynamically adjusts the threshold of its quantizer based on the earlier transmissions from its immediate upstream sensor.

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Note that demodulation errors occur probably for transmission bits under AWGN channels. The reconstruction mean squared error (MSE) of each quantized data more depends on the demodulation accuracy of the leading quantization bits, and less on the accuracy of the trailing bits. Therefore, it is meaningful to allocate more transmission bits to the leading quantization bits. However, both previous centralized and progressive DESs in [2]-[5] have not considered the transmission bit allocation among quantization bits at each sensor, and 1 transmission bit is allocated to each quantization bit. In addition, the impact of channel noise has not been considered when determining the fusion weight of the received data in the quasi-BLUE of the unknown parameter in [2]-[5].

This paper presents a new progressive DES based on quantized data over AWGN channels. Firstly, motivated by the fact more transmission bits should be allocated to the leading quantization bits, a transmission bit allocation scheme is presented for each sensor, which improves the reconstruction MSE of the received data at its immediate downstream sensor. Secondly, a quasi-BLUE of the unknown parameter is constructed using the data received from its immediate upstream sensor and the local observation at each sensor. The weight of the received data in the quasi-BLUE is based on its MSE as an estimate of the unknown parameter, in which the impact of channel noise has been considered. Thirdly, a MSE upper bound of the quasi-BLUE is given, and the target estimate MSE is ensured by imposing this upper bound to be within the desired performance tolerance. Finally, the power scheduling among sensors, which minimizes the total transmission power subject to the performance constraint, is formulated as a convex problem and the optimal solution is derived analytically. Through this paper, $\mathrm{E}(\cdot)$ represents the mathematical expectation.

## II. Progressive DES

Consider a 1-D sensor network with $N$ sensors, in which the sensor nearest to the FC is indexed by $n = N$ and the sensor farthest from the FC is indexed by $n = 1$. Each sensor makes a noisy observation of an unknown parameter $\theta$. The observation of sensor $n$ is modeled as

$$x_n = \theta + w_n, \quad n = 1, \ldots, N \tag{1}$$

where we assume $\{w_n\}_{n=1}^N$ are zero mean and spatially uncorrelated noise with variance $\{\sigma_n^2\}_{n=1}^N$.

The input to sensor $n$ consists of $x_n$ and $m_{n-1}^{'}$, where $m_{n-1}^{'}$ is the demodulated result of the data received from sensor $n-1$. Sensor $n$ forms a quasi-BLUE estimate of $\theta$ as

$$\hat{\theta}_n = \left(\frac{1}{E(m_{n-1}^{'}-\theta)^2}+\frac{1}{\sigma_n^2}\right)^{-1}\left(\frac{m_{n-1}^{'}}{E(m_{n-1}^{'}-\theta)^2}+\frac{x_n}{\sigma_n^2}\right). \quad (2)$$

Here, $\hat{\theta}_n$ is quasi-BLUE instead of a BLUE since the mean of $\hat{\theta}_n$ may not be $\theta$.

Due to the bandwidth and energy constraint, $\hat{\theta}_n$ should be quantized to $m_n$. Suppose $\hat{\theta}_n$ is bounded within $[-T, T]$ and is expressed as

$$\hat{\theta}_n = 2T\sum_{k=1}^{\infty} b_{n,k}2^{-k} - T,$$

where $\{b_{n,k}\}_{k=1}^{\infty}$ are quantization bits. We assume that sensor $n$ quantizes its estimate into $\varphi_n$ bits and transmits the resulting bits using $k_n$ number of transmission bits, $k_n \geq \varphi_n$, which means that some quantized bits may be transmitted more than once to get diversity effect. The proposed scheme can be considered as a combination of quantization and repetition coding. Then we have

$$m_n = 2T\sum_{k=1}^{\varphi_n} b_{n,k}2^{-k} - T. \quad (3)$$

It has been shown in [5] that

$$\mathrm{E}(m_n - \hat{\theta}_n)^2 \leq \frac{T^2}{3}2^{-2\varphi_n} \quad (4)$$

The wireless links among sensors are modeled as independent AWGN channels with noise variance $\{\delta_n^2\}_{n=1}^N$. Assume the channel between sensor $n$ and sensor $n+1$ experiences a pathloss proportional to $a_n = d_n^{\alpha_n}$, where $d_n$ is the transmission distance and $\alpha_n$ is the pathloss exponent. The binary phase shift keying (BPSK) modulation is adopted, and the transmission energy for each bit at sensor $n$ is set as $w_n = a_n E_b$. The received signal-to-noise ratio (SNR) per bit at sensor $n+1$ is defined as $\gamma_n \triangleq E_b/(2\delta_n^2)$. Denote $b_{n,k}^{'}$ as the decoding result of $b_{n,k}$ at sensor $n+1$. Then $m_n$ is reconstructed as

$$m_n^{'} = 2T\sum_{k=1}^{\varphi_n} b_{n,k}^{'}2^{-k} - T. \quad (5)$$

From (3) and (5), it holds

$$\mathrm{E}(m_n^{'} - m_n)^2 = 4T^2\mathrm{E}\left[\sum_{k=1}^{\varphi_n}\left(b_{n,k}^{'} - b_{n,k}\right)2^{-k}\right]^2. \quad (6)$$

Obviously, the reconstruction MSE in (6) is more dependent on the decoding accuracy of the leading bits of $m_n$, and less on the accuracy of the trailing bits. It is reasonable that more bits should be allocated to transmit the leading quantization bits, less for the trailing bits, especially when the available bits are enough and the channel is in bad condition.

Assume $k_n$ transmission bits are available at sensor $n$. Motivated by this fact, a transmission bit allocation scheme is presented: $1/2$ of the $k_n$ bits are allocated to transmit the first bit of $m_n$, $1/4$ of the $k_n$ bits are allocated to transmit the second bit, and so on. Since $k_n$ may not be a power of 2, we allocate $\lceil 2^{-k}k_n \rceil$ transmission bits to $b_{n,k}$. With the proposed transmission bit allocation scheme, it is easy to prove that the number of quantization levels, $\varphi_n$, satisfies

$$k_n < 2^{\varphi_n} \leq 2k_n. \quad (7)$$

Then, according to (4) and (7), it holds

$$\mathrm{E}(m_n - \hat{\theta}_n)^2 \leq \frac{T^2}{3}k_n^{-2}. \quad (8)$$

At sensor $n+1$, the soft decoding algorithm is adopted [9]. Denote $p_n^k$ as the bit error rate of $b_{n,k}$, then it holds $p_n^k = Q\left(\sqrt{2\lceil 2^{-k}k_n\rceil\gamma_n}\right)$, where $Q(\cdot)$ is the Gaussian tail function. Then the reconstruction MSE in (6) can be bounded by the following lemma.

**Lemma 1.** *Suppose $\varphi_n$-bit quantized message $m_n$ is transmitted over the AWGN channel from sensor $n$ to sensor $n+1$ after BPSK modulation with the proposed transmission bit allocation scheme. The reconstructed MSE of $m_n$ at sensor $n+1$ is bounded by*

$$\mathrm{E}(m_n^{'} - m_n)^2 < \frac{16}{3}T^2 Q\left(\sqrt{\gamma_n}\right) \quad (9)$$

*Proof*: let $A_{n,l}$ denote the event that the first bit decoded incorrectly at sensor $n+1$ is $b_{n,l}$. Then for $1 \leq l \leq \varphi_n$,

$$P(A_{n,l}) = \prod_{k=1}^{l-1}(1-p_n^k)p_n^l \quad (10)$$

Recall that $p_n^k = Q\left(\sqrt{2\lceil 2^{-k}k_n\rceil\gamma_n}\right)$ and $Q(x)$ is a decreasing function with respect to $x$, then, $p_n^{\varphi_n} > p_n^k > p_n^1, \forall 1 < k < \varphi_n$. Thus, $P(A_{n,l})$ is bounded by

$$P(A_{n,l}) < (1-p_n^1)^{l-1}p_n^l \leq (1-p_n^1)^{l-1}p_n^{\varphi_n}. \quad (11)$$

When the event $A_{n,l}$ happens, it holds that [c.f. (6)]

$$\begin{aligned}\left|m_n^{'} - m_n\right|^2 &= 4T^2\left|\sum_{k=l}^{\varphi_n}\left(b_{n,k}^{'} - b_{n,k}\right)2^{-k}\right|^2 \\ &\leq 4T^2\left(\sum_{k=l}^{\varphi_n}2^{-k}\right)^2 \\ &= 4T^2\left[2^{1-l}\left(1-2^{l-\varphi_n-1}\right)\right]^2 \\ &\leq 4T^2\left(2^{1-l}\right)^2 \\ &= T^2 2^{4-2l}.\end{aligned} \quad (12)$$

From (11) and (12), it follows that

$$\begin{aligned}\mathrm{E}(|m_n^{'} - m_n|^2|m_n) &= \sum_{l=1}^{\varphi_n}P(A_{n,l})\mathrm{E}(|m_n^{'} - m_n|^2|A_{n,l}) \\ &< T^2 p_n^{\varphi_n}\sum_{l=1}^{\varphi_n}(1-p_n^1)^{l-1}2^{4-2l} \\ &= 16T^2 p_n^{\varphi_n}\frac{1-\left(\frac{1-p_n^1}{4}\right)^{\varphi_n}}{3+p_n^1} \\ &< \frac{16T^2}{3}p_n^{\varphi_n}.\end{aligned}$$

Since the above bound is independent of $m_n$, we have

$$\mathrm{E}(|m_n^{'} - m_n|^2|) < \frac{16T^2}{3}p_n^{\varphi_n}. \quad (13)$$

Using $p_n^{\varphi_n} \leq Q\left(\sqrt{2^{1-\varphi_n}k_n\gamma_n}\right)$ and $2^{\varphi_n} \leq 2k_n$, we obtain $p_n^{\varphi_n} \leq Q(\sqrt{\gamma_n})$, which with (13) leads to (9). □

Recall $E(m_n^{'} - \theta)^2$ is needed to construct the quasi-BLUE at sensor $n+1$. Based on the Cauchy-Schwartz inequality,

$$\begin{aligned}\mathrm{E}(m_n^{'} - \theta)^2 &= \mathrm{E}(m_n^{'} - m_n + m_n - \hat{\theta}_n + \hat{\theta}_n - \theta)^2 \\ &\leq 3\mathrm{E}(m_n^{'} - m_n)^2 + 3\mathrm{E}(m_n - \hat{\theta}_n)^2 + 3\mathrm{E}(\hat{\theta}_n - \theta)^2.\end{aligned}$$

Then from (2) (8) and (9) , it follows

$$\begin{aligned}\mathrm{E}(m_n^{'} - \theta)^2 &\leq 16T^2 Q\left(\sqrt{\gamma_n}\right) + T^2 k_n^{-2} + 3\left(\frac{1}{\mathrm{E}(m_{n-1}^{'} - \theta)^2} + \frac{1}{\sigma_n^2}\right)^{-1} \\ &:= \Upsilon_n,\end{aligned} \tag{14}$$

where $\Upsilon_n$ denotes the upper bound. We use $\Upsilon_n$ in the quasi-BLUE at sensor $n+1$ instead of $\mathrm{E}(m_n^{'} - \theta)^2$, then

$$\hat{\theta}_{n+1} = \left(\frac{1}{\Upsilon_n} + \frac{1}{\sigma_n^2}\right)^{-1}\left(\frac{m_n^{'}}{\Upsilon_n} + \frac{x_{n+1}}{\sigma_n^2}\right), \tag{15}$$

where the MSE of $\hat{\theta}_{n+1}$ is bounded by

$$\mathrm{E}(\hat{\theta}_{n+1} - \theta)^2 \leq \left(\frac{1}{\Upsilon_n} + \frac{1}{\sigma_{n+1}^2}\right)^{-1}. \tag{16}$$

Using (14) and (16), we have

$$\Upsilon_n = 16T^2 Q\left(\sqrt{\gamma_n}\right) + T^2 k_n^{-2} + 3\left(\frac{1}{\Upsilon_{n-1}} + \frac{1}{\sigma_n^2}\right)^{-1}. \tag{17}$$

In the progressive DES, $\{\Upsilon_n\}_{n=1}^N$ should be computed using (17) in advance and become available at the corresponding sensors before the distributed estimate of $\theta$ is conducted in the network. Finally, $m_N^{'}$ is received at the FC and as the final estimate of $\theta$ in the network.

## III. Power Scheduling

This section considers the power scheduling among sensors, which minimizes the total transmit power while insuring a given MSE performance. According to the Cauchy-Schwartz inequality, we can further bound (14) as

$$\mathrm{E}(m_n^{'} - \theta)^2 \leq 16T^2 Q\left(\sqrt{\gamma_n}\right) + T^2 k_n^{-2} + \frac{3}{4}\mathrm{E}(m_{n-1}^{'} - \theta)^2 + \frac{3}{4}\sigma_n^2.$$

Then the recursion leads to

$$\begin{aligned}\mathrm{E}(m_N^{'} - \theta)^2 &\leq \sum_{n=1}^N 4^{n-N} T^2\left(16Q\left(\sqrt{\gamma_n}\right) + k_n^{-2}\right) + 3\sum_{n=2}^N 4^{n-N+1}\sigma_n^2 \\ &\quad + 4^{1-N}\sigma_1^2 := \Gamma_N,\end{aligned} \tag{18}$$

where $\Gamma_N$ denotes the MSE upper bound of the final estimate received at the FC.

Denote $P_n$ as the transmit power consumption of sensor $n$. Recall $w_n = a_n E_b$ is the transmit energy for each bit at sensor $n$ and there are $k_n$ transmission bits available for sensor $n$, then $P_n = k_n a_n E_b$. The power scheduling subject to an allowable MSE performance can be formulated as

$$\begin{aligned}\min \quad & \sum_{n=1}^N k_n a_n, \\ \text{s.t.} \quad & \mathrm{E}(m_N^{'} - \theta)^2 \leq D,\end{aligned} \tag{19}$$

where $D$ is the given MSE.

To ensure $E(m_N^{'} - \theta)^2 \leq D$, we constrain $\Gamma_N \leq D$. Then (19) is reformulated as

$$\begin{aligned}\min \quad & \sum_{n=1}^N k_n a_n, \\ \text{s.t.} \quad & \Gamma_N \leq D, \quad k_n \in \mathbb{Z}^+, \quad n = 1, \dots, N.\end{aligned} \tag{20}$$

Define $n_0 = \max\{1 \leq n \leq N | k_n = 0\}$. Then, the upstream sensors of sensor $n_0$, which are indexed by $n < n_0$, should not be allocated any bits to save unnecessary energy consumption. Sensor $n_0 + 1$ is the first active sensor in the 1-D network and there are $\bar{N} = N - n_0$ active sensors. Define the new sets $\{\bar{\gamma}_n\}_{n=1}^{\bar{N}}$, $\{\bar{a}_n\}_{n=1}^{\bar{N}}$, $\{\bar{k}_n\}_{n=1}^{\bar{N}}$ and $\{\bar{\sigma}_n^2\}_{n=1}^{\bar{N}}$, where $\bar{\gamma}_{n-n_0} = \gamma_n$, $\bar{a}_{n-n_0} = a_n$, $\bar{k}_{n-n_0} = k_n$ and $\bar{\sigma}_{n-n_0}^2 = \sigma_n^2$.

The problem (20) can be reformulated to be convex by defining $D_1$ as

$$D1 := T^{-2}\left(4^N D - \sum_{n=1}^{\bar{N}} 4^{n+2} T^2 Q\left(\sqrt{\bar{\gamma}_n}\right) - 3\sum_{n=2}^{\bar{N}} 4^{n-1}\bar{\sigma}_n^2 - 4\bar{\sigma}_1^2\right)$$

and relaxing the constraint $\bar{k}_n \in \mathbb{Z}^+$ to $\bar{k}_n > 0$. Then we have

$$\begin{aligned}\min \quad & \sum_{n=1}^{\bar{N}} \bar{k}_n \bar{a}_n, \\ \text{s.t.} \quad & \sum_{n=1}^{\bar{N}} 4^n \bar{k}_n^{-2} \leq D_1, \quad \bar{k}_n > 0, \quad n = 1, \dots, \bar{N}.\end{aligned} \tag{21}$$

It is easy to prove the optimal problem (21) is convex with respect to $\bar{\boldsymbol{k}} = (\bar{k}_1, ..., \bar{k}_{\bar{N}})$ and leads to a close form solution.

First, the Lagrangian function of (21) is

$$L(\bar{\boldsymbol{k}}, \lambda, \boldsymbol{\mu}) = \sum_{n=1}^{\bar{N}} \bar{k}_n \bar{a}_n + \lambda\left(\sum_{n=1}^{\bar{N}} 4^n \bar{k}_n^{-2} - D_1\right) - \mu_n \bar{k}_n \tag{22}$$

The corresponding KKT conditions are

$$\bar{a}_n - \lambda 2^{2n+1}\bar{k}_n^{-3} - \mu_n = 0, \quad n = 1, \dots, \bar{N} \tag{23}$$

$$\lambda\left(\sum_{n=1}^{\bar{N}} 4^n \bar{k}_n^{-2} - D_1\right) = 0 \tag{24}$$

$$\sum_{n=1}^{\bar{N}} 4^n \bar{k}_n^{-2} - D_1 \leq 0 \tag{25}$$

$$\mu_n \bar{k}_n = 0, \quad n = 1, \dots, \bar{N} \tag{26}$$

$$\lambda \geq 0, \quad \bar{k}_n > 0, \quad \mu_n \geq 0, \quad n = 1, \dots, \bar{N} \tag{27}$$

From (26) and the fact $\bar{k}_n > 0$, it follows $\mu_n = 0, n = 1, \dots, \bar{N}$. Using (23) and the fact $\bar{a}_n > 0$, we have $\lambda > 0$, implying that

$$\sum_{n=1}^{\bar{N}} 4^n \bar{k}_n^{-2} = D_1. \tag{28}$$

Then solving (23) and (28), we obtain the solution of (21) as

$$\bar{k}_n^{opt} = D_1^{-\frac{1}{2}}\left(\frac{4^n}{\bar{a}_n}\right)^{\frac{1}{3}}\left(\sum_{n=1}^{\bar{N}} 4^{\frac{n}{3}}\bar{a}_n^{\frac{2}{3}}\right)^{\frac{1}{2}}. \tag{29}$$

Remind that the optimal problem in (21) is based on the assumption that the first active sensor in the network is sensor $n_0 + 1$. Thus, the optimal power scheduling can be achieved by repeating the computation of (29) for $N-1$ times assuming $n_0 = 0, 1, ..., N-1$, respectively. The optimal power scheduling suggests that the number of quantization levels and transmission power of each sensor is determined jointly by the individual channel path loss and channel noise, the local observation noise and the targeted MSE performance.

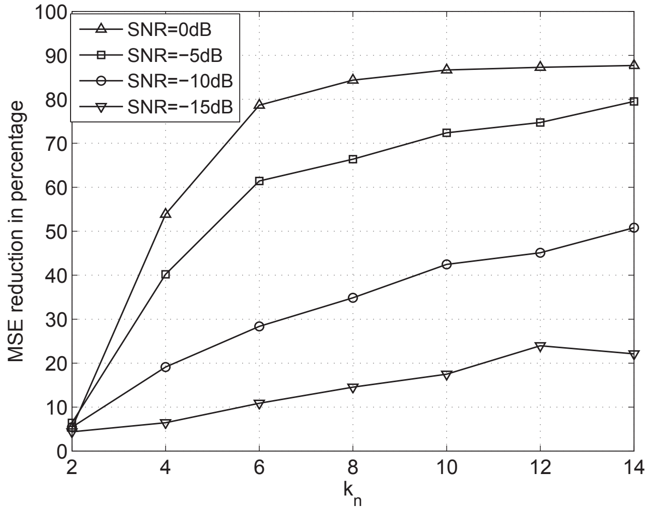


Fig. 1. Estimation MSE reduction in percentage of the proposed progressive DES with the transmission bit allocation scheme compared with the traditional progressive DES.

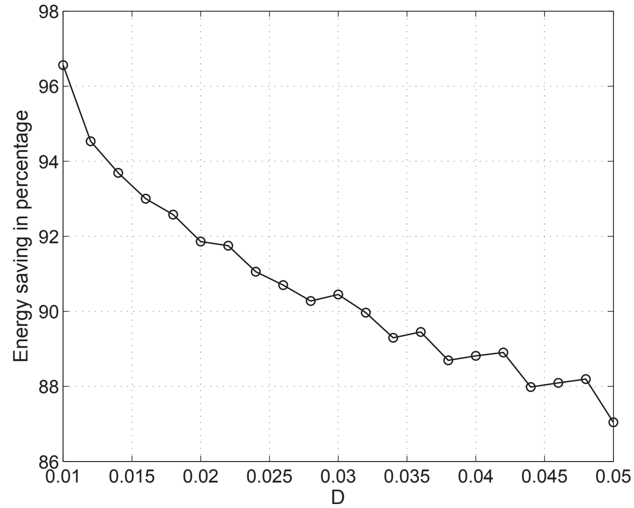


Fig. 2. Energy saving of the power scheduling compared with the uniform transmission bit allocation scheme versus the target MSE.

## IV. Simulation Results

This section presents some simulation results and all simulation results are averaged from 50,000 independent runs.

Firstly, a homogeneous 1-D network with $N = 100$ sensors is simulated under AWGN channels. Observation noise is Gaussian distributed with mean zero and variance 1. The number of transmission bits per sensor, $k_n$, is equal. Fig. 1 plots the estimation MSE reduction of our proposed progressive DES with transmission bit allocation compared with the DES [5] versus different number of transmission bits per sensor with different received SNRs per bit. Fig. 1 shows our proposed DES with transmission bit allocation scheme achieves large MSE reduction compared with the DES [5], and the amount of the reduction becomes more significant when the total received SNR becomes larger.

Secondly, we evaluate the performance of the proposed power scheduling (29). We assume $N = 100$, $r = 4$, $\sigma_n^2 = 0.01$, $\gamma_n = 20$ and the distance between sensor $n$ and sensor $n+1$ is generated from a uniform distribution $d_n \in [0.5, 1.5]$. Since $\bar{k}_n^{opt}$ takes the real values in (29), we use $\lceil \bar{k}_n^{opt} \rceil$ in our simulations. Fig. 2 shows the energy saving of the proposed power scheduling compared with the uniform transmission bit allocation scheme, in which equal number of transmission bits are allocated to each sensor computed by (20). We can see that the proposed power scheduling is much more energy-efficient than the uniform transmission bit allocation scheme, especially when the target MSE is low.

## V. Conclusions

This paper considers the progressive distributed estimate of a parameter based on noisy quantized versions of noisy sensor observations. A transmission bit allocation scheme is presented, which improves the reconstruction performance of the data transmitted among sensors. At each sensor, a quasi-BLUE of the unknown parameter is constructed based on the received data from the upstream sensor and local observations. The power scheduling among sensors, which minimizes the total transmission power subject to the desired tolerance, is formulated as a convex problem and the optimal solution is derived analytically. Simulation results show the efficiency of the proposed progress DES as well as the power scheduling.

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