TRACKING OF RANDOM NUMBER OF TARGETS WITH RANDOM NUMBER OF SENSORS USING RANDOM FINITE SET THEORY

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ABSTRACT

Variation in the number of targets and sensors needs to be addressed in any realistic sensor system. Targets may come in or out of a region or may suddenly stop emitting detectable signal. Sensors can be subject to failure for many reasons. We derive a tracking algorithm with a model that includes these variations using Random Finite Set Theory (RFST). RFST is a generalization of standard probability theory into the finite set theory domain. This generalization does come with additional mathematical complexity. However, many of the manipulations in RSFT are similar in behavior and intuition to those of standard probability theory.

Index Terms— Tracking, Multisensor systems, Set theory, Array signal processing, Object detection

1. INTRODUCTION

Recently, there has been a continuing increase in popularity and interest to engineer small low cost devices that are capable of sensing, data processing, and communication known as wireless sensors [1, 2]. By virtue of its cost, a wireless sensor network (WSN) allows the possibility of monitoring complex natural environmental states with a finer spatial resolution (i.e., more devices per area). As demands for power efficiency and complexity continue to grow, research and development continue to search for better ways to manage, analyze and combine information that changes rapidly and are very error prone.

Tracking targets is one aspect of monitoring that carries a wide interest from bio-complexity study to military surveillance. In reality, the targets often enter and exit the monitored area and can sometimes fail to transmit (or hide) its signal. The sensors can also fail from lack of power, hardware failure, software failure, or any combinations of these. The difficulties of modeling these variations via standard probability theory are reduced from a random finite set theory (RFST) perspective. RFST is a generalization of standard probability theory into a set theory with finite elements [3–5]. The flexible structure of a set has made RFST easier to model variation in both the number of elements and the value of the elements jointly. For example, a no-target case can easily be modeled by an empty set, which may have non-zero probability.

To simplify design, we will assume the sensor deployment is on a uniform grid over a region of interest (ROI). Each sensor is a self-contained, battery operated, small computer that has a wireless capability for communication to a fusion center. These type of sensors have begun to emerge in the market and will become more readily available over time. Being battery operated, power management becomes a major issue. The overall system will benefit significantly if we can minimize communication as much as possible.

This article is organized into seven sections. The next two sections will define the observation and motion models in terms of random sets. Section 4 and 5 will be dedicated to deriving the set densities. Section 6 will discuss the simulation result for target tracking and the importance of detecting sensor failure in the system, and we will end with a brief conclusion.

2. OBSERVATION MODEL

Let the set Σ denotes the collective observations received at the fusion center. Given N sensors, we can express Σ as $\cup_{i=1}^{N}(\Sigma_i \times i)$, where Σ_i is the observation from sensor i, and i is the index. Since the internal sensor noise is independent between sensors, the probability set density for Σ becomes $f_\Sigma = \prod_{i=1}^{N} f_{\Sigma_i}$. To describe the event for each sensor i, we write

$$\Sigma_i = \Sigma_i' \cap F_i, \tag{1}$$

$$\Sigma_i' = \{Z_i\} \cap D_i, \tag{2}$$

where Σ_i' is the sensor's observation, F_i is the sensor failure state, and D_i is the sensor detection state. $Z_i \in \mathcal{R}$ is the detected signal power with form $Z_i = h_i(\Theta) + W$, where W is a random noise with density $f_W(w)$, and Θ is a random set describing the target geo-kinematic properties. Supposed that M is the measurement space, then we can choose a discrete random subset D_i of M such that $D_i = \emptyset$ with probability $1 - p_{d_i}$ and $D_i = M$ with probability p_{d_i}. The sensor failure state can also be written in the similar manner to describe failing state ($F_i = \emptyset$) or normal state ($F_i = M$).

To conserve power, we define a threshold τ, such that

$$\Sigma_i' = \begin{cases} \emptyset, & \text{if} \quad Z_i < \tau, \\ \{Z_i\}, & \text{if} \quad Z_i \geq \tau. \end{cases} \tag{3}$$

This will limit the sensor's communication to the fusion center only when it is reasonably significant. Then from (2) and (3), D_i is described completely by the thresholding (i.e. $p_d = P(Z_i \geq \tau)$). The function $h(.)$ describes the relationship between the target position and the detected signal amplitude. In our case, we assume that the source emits a signal with power P_0 when measured at a reference distance d_0 in a free space, which is given by

$$h_i(\emptyset) = 0, \quad (4)$$

$$h_i(\theta) = \sqrt{P_0 d_0^2 / d^2(\theta_i, \theta)}, \quad (5)$$

where we denote $h_i(\theta) \triangleq h_i(\{\theta\})$, $\theta = [x, y, u_x, u_y]^T$ is the target position and velocity, $\theta_i = [x_i, y_i]^T$ is sensor i position, and $d(\theta_i, \theta) = \sqrt{(x - x_i)^2 + (y - y_i)^2}$ is the distance between the sensor i and the target. Throughout this article ^{T} will denote the transpose operation, and we will use θ as the shorthand notation for the set $\{\theta\}$ to reduce notation clutter.

For sequential indexing, we will denote the time by augmenting the subscript with a time index t, so $\Sigma_{i,t}$ will denote the observation of sensor i at time t. We will further assume that the signal is received by the farthest sensor before the next epoch.

3. MOTION MODEL

When a target exists, we assume it follows a constant velocity with a small random perturbation and a density of $f_V(v)$. For each epoch, we want to update the Θ and all the F_i. The update equation can then be written as

$$\Theta_{t+1} = S^\Theta(\Theta_t) \cup B^\Theta(\Theta_t) \quad (6)$$

$$S^\Theta(\emptyset) = \emptyset, \quad (7)$$

$$S^\Theta(\theta_t) = \begin{cases} \emptyset, & \text{with probability} \quad 1 - p_s \\ s(\theta_t), & \text{with probability} \quad p_s \end{cases}, \quad (8)$$

$$B^\Theta(\emptyset) = \begin{cases} \emptyset, & \text{with probability} \quad 1 - p_b \\ b, & \text{with probability} \quad p_b \end{cases}, \quad (9)$$

$$B^\Theta(\theta_t) = \emptyset, \quad (10)$$

$$s(\theta_t) = \begin{bmatrix} x + u_x + v_x \\ y + u_y + v_y \\ u_x + v_x \\ u_y + v_y \end{bmatrix}^T, \quad (11)$$

where B^Θ and S^Θ describe the birth and the surviving set function respectively. If a target survives, $s(\theta_t)$ describes its state transition behavior. If a birth occurs, b describes the geo-kinematic probability of the target when it appears with the density $f_b(b)$. x and y denotes the position of the target, and u_x and u_y denotes its component velocities. v_x and v_y are the random perturbation, which is described by $f_V(v)$.

For each $F_{i,t}$, the update equation is given by

$$F_{i,t+1} = S^F(F_{i,t}, \Theta) \cup B^F(\Sigma'_{i,t}), \quad (12)$$

$$B^F(\Sigma'_{i,t}) = \begin{cases} \emptyset, & \text{if} \quad \Sigma'_{i,t} = \emptyset, \\ M, & \text{if} \quad \Sigma'_{i,t} \neq \emptyset, \end{cases} \quad (13)$$

$$S^F(-) = \begin{cases} q_i(\theta), & \text{if } F_{i,t} = M, \Theta = \theta, \\ F_{i,t}, & \text{otherwise}, \end{cases} \quad (14)$$

where S^F models the sensor survival, and B^F is the set indicator modeling the sensor birth (or recovery from failure). $q_i(\theta)$ is a random set that models the decision to tag sensor failure when an estimate exists and sensor i provides no detection. Intuitively, if a target passes very close without any report, the sensor has probably failed, and a sensor far away from the target should not be tagged as failing. To describe this, we set $q_i(\theta) = \emptyset$ if $h_i(\theta) < \tau'$, where $\tau' = \tau + \alpha\sigma$, and $\alpha \geq 0$. Under normal conditions, each sensor's observation is perturbed by an internal noise with variance σ^2. We want to set α so that S^F is insensitive to most internal noise perturbation. If we set $\alpha = 0$, all sensors that failed to report will be incorrectly tagged as failing. α around 2 - 3 are reasonable values to ignore most sensor noise perturbation. In this way, only an unreasonably large deviation will activate the sensor failure tagging.

4. OBSERVATION DENSITIES

Before we can use the model defined in Section 2, we will need to derive the observation set densities. Recall that the belief function is defined as $\beta_Z(C) \triangleq P(Z \subseteq C)$ and the measurement space M is such that $\beta_Z(M) = 1$. We begin by writing the belief function of (1) as

$$\beta_{\Sigma_{i,t}}(C|\Theta_t, \emptyset) = P(\Sigma_{i,t} = \emptyset), \quad (15)$$

$$\beta_{\Sigma_{i,t}}(C|\emptyset, M) = 1 - p_{fa_i} + p_{fa_i} Pa, \quad (16)$$

$$\beta_{\Sigma_{i,t}}(C|\theta_t, M) = 1 - p_{d_i}(\theta_t) + p_{d_i}(\theta_t) Pb, \quad (17)$$

where $\beta_{\Sigma_{i,t}}(C|A_1, A_2) \triangleq \beta_{\Sigma_{i,t}}(C|\Theta_t = A_1, F_{i,t} = A_2)$, $Pa = P(\Sigma_{i,t} \subseteq C | C \neq \emptyset, \Theta = \emptyset, F_{i,t} = M)$, $Pb = P(\Sigma_{i,t} \subseteq C | C \neq \emptyset, \Theta = \theta, F_{i,t} = M)$, and the notation p_{fa}, "false-alarm" probability, is needed to distinguish (16) from (17). By applying set derivatives and evaluating the results at $C = \emptyset$ [5], the set densities are given by

$$f_{\Sigma_i}(\emptyset|\Theta_t, \emptyset) = 1, \quad (18)$$

$$f_{\Sigma_i}(z|\Theta_t, \emptyset) = 0, \quad (19)$$

$$f_{\Sigma_i}(\emptyset|\emptyset, M) = 1 - p_{fa_i}, \quad (20)$$

$$f_{\Sigma_i}(z|\emptyset, M) = p_{fa_i} f_c(z), \quad (21)$$

$$f_{\Sigma_i}(\emptyset|\theta_t, M) = 1 - p_{d_i}(\theta_t), \quad (22)$$

$$f_{\Sigma_i}(z|\theta_t, M) = p_{d_i}(\theta_t) f_s(z), \quad (23)$$

where $f_{\Sigma_i}(C|A_1, A_2) \triangleq f_{\Sigma_i}(C|\Theta_t = A_1, F_{i,t} = A_2)$. If a target exists, $f_s(z)$ is the target's observation probability distribution. Similarly, if a clutter exists, $f_c(z)$ is the clutter's observation probability distribution.

For the sensor noise distribution, we will assume $f_W(w) \sim \mathcal{N}_w(0, \sigma^2)$. Although this is a simplistic assumption for real applications, it helps to reduce equation complexity. Consequently, $p_{fa_i} = Q(\tau/\sigma)$, and $p_{d_i}(\theta) = Q((\tau - h_i(\theta))/\sigma)$, $f_c(z) \sim \mathcal{N}_w(0,\sigma^2)$, and $f_s(z) \sim \mathcal{N}_w(z,\sigma^2)$, where $Q(\gamma) = \int_\gamma^\infty (2\pi\sigma^2)^{-1/2}\exp(-2x^2/\sigma^2)dx$ is the right-tail probability of Gaussian random variable. Clearly, more realistic noise pdf can also be used.

5. MOTION DENSITIES

Similarly, we also need to derive the set densities from the model described in Section 3. The belief function for (6) can be written as

$$\beta_{\Theta_{t+1}}(C|\Theta_t = \emptyset) = 1 - p_b + p_b Pc, \quad (24)$$
$$\beta_{\Theta_{t+1}}(C|\Theta_t = \theta_t) = 1 - p_s + p_s Pd, \quad (25)$$

where $Pc = P(\Theta_{t+1} \subseteq C|C \neq \emptyset, \Theta_t = \emptyset)$, and $Pd = P(\Theta_{t+1} \subseteq C|C \neq \emptyset, \Theta_t = \theta_t)$. Following the same procedures, the set densities can be obtained as

$$f_{\Theta_{t+1}}(\emptyset|\emptyset) = 1 - p_b, \quad (26)$$
$$f_{\Theta_{t+1}}(\theta_{t+1}|\emptyset) = p_b f_b, \quad (27)$$
$$f_{\Theta_{t+1}}(\emptyset|\theta_t) = 1 - p_s, \quad (28)$$
$$f_{\Theta_{t+1}}(\theta_{t+1}|\theta_t) = p_s f_s(\theta_{t+1}|\theta_t), \quad (29)$$

where f_b and f_s are as described in Section 2.

For the sensor failure state, we can write the belief functions as

$$\beta_{F_{i,t+1}}(C|-) = \begin{cases} 1 - p_q + p_q P_e, & \text{if } \begin{cases} F_{i,t} = M, \\ \Theta = \theta, \\ B^F = \emptyset, \end{cases} \\ P(F_{i,t+1} = M), & \text{if } B^F = M, \\ P(F_{i,t+1} = F_{i,t}), & \text{otherwise}, \end{cases} \quad (30)$$

where $P_e = P(F_{i,t+1} \subseteq C|C \neq \emptyset, F_{i,t} = M, \Theta = \theta, B^F = \emptyset)$. Since $F_{i,t+1}$ takes only discrete values, the set probability mass functions are given as

$$f_{F_{i,t+1}}(\emptyset|M,\theta,\emptyset) = p_q(\theta), \quad (31)$$
$$f_{F_{i,t+1}}(M|M,\theta,\emptyset) = 1 - p_q(\theta), \quad (32)$$
$$f_{F_{i,t+1}}(M|F_{i,t},\Theta,M) = 1, \quad (33)$$
$$f_{F_{i,t+1}}(\emptyset|F_{i,t},\Theta,M) = 0, \quad (34)$$
$$f_{F_{i,t+1}}(M|\emptyset,\Theta,\emptyset) = 0, \quad (35)$$
$$f_{F_{i,t+1}}(\emptyset|\emptyset,\Theta,\emptyset) = 1, \quad (36)$$
$$f_{F_{i,t+1}}(M|M,\emptyset,\emptyset) = 1, \quad (37)$$
$$f_{F_{i,t+1}}(\emptyset|M,\emptyset,\emptyset) = 0 \quad (38)$$

where $p_q(\theta) = Q(\tau' - h_i(\theta)/\sigma)$.

6. SIMULATION

In a real system, each sensor has a separate processing unit, so the complexity of the whole system is well distributed. To practically simulate the entire sensor suite, we will use particle filtering approach using the sequential importance resampling method [6].

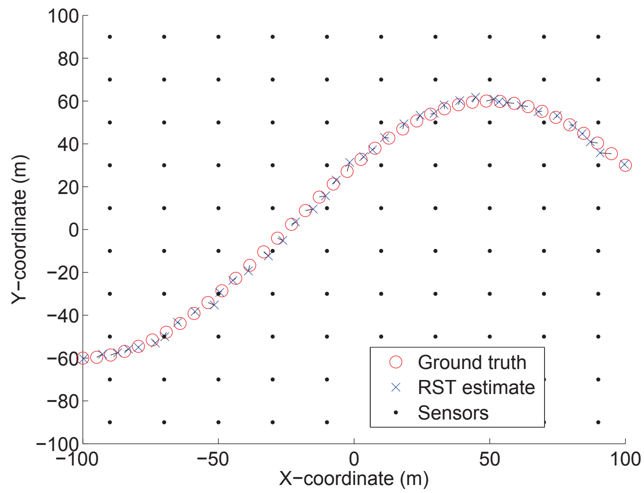


Fig. 1. All sensors behave normally

The region of interest (ROI) is a 200 x 200 square meter with 100 sensors deployed on a uniform grid. For reference, we place the axis $(0, 0)$ at the center of the ROI. The target, when measured at $d_0 = 1$ meter, emits a signal with power intensity of $P_0 = 800$. The noise variance for each sensor is $\sigma^2 = 0.4$, and the τ is set to 1.6. Other parameters are given as $\alpha = 1, p_s = 0.92, p_b = 0.2, f_V(v) \sim \mathcal{N}(0, 1)$

Let's consider a scenario where a target repeatedly pass through the same path. When all sensors behave normally, the system is able to track the target properly (e.g. Fig. 1). If some sensors in the proximity of the path fail, the estimate can be very erroneous (e.g. Fig. 2). Normal non-transmitting sensors carry inherent information that the target is far from its position. Hence, when a nearby sensor fails to transmit, the estimate becomes inaccurate. If the sensor failure detection is employed, the system can avoid large errors by tagging sensors that might be failing. The effect of automatic sensor failure detection can be seen in Fig. 3(a). After the failed sensors are tagged, future tracking efforts can use only the remaining sensors as seen in Fig. 3(b).

Sometimes, the target stops emitting its signal or purposefully hides its presence from the sensors. The analysis on target's behavior is beyond the scope of this article; however, we still want to detect if this event occurs. In Fig. 4, the system begins to track a target that suddenly disappear. Four time instants later, a target appears at a different starting point. Note that the second trace can be a different target from the first. Notice also that sensors that are mistakenly tagged as failing does not degrade much of the system performance. Once a tagged sensor sends information, the system can immediately use it and simultaneously lift the tag off the sensor. Information is lost only if a working sensor does not send the data due

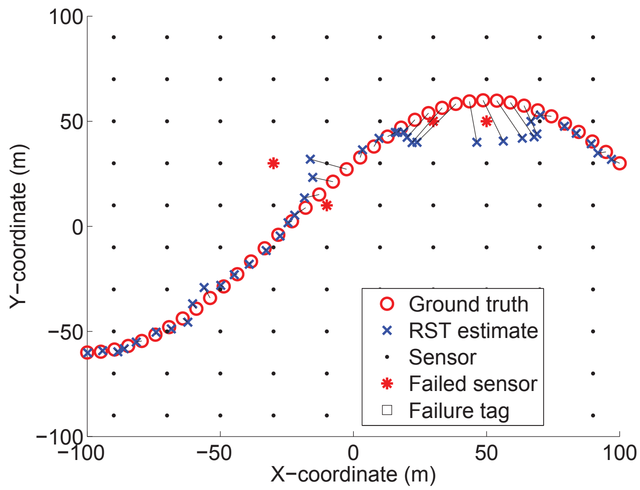
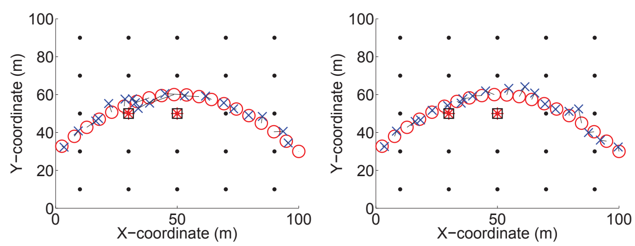


Fig. 2. Undetected sensor failures that cause erroneous tracking results



(a) Unknown sensor failures (b) Known sensor failures

Fig. 3. Tracking behavior with automatic sensor failure detection scheme.

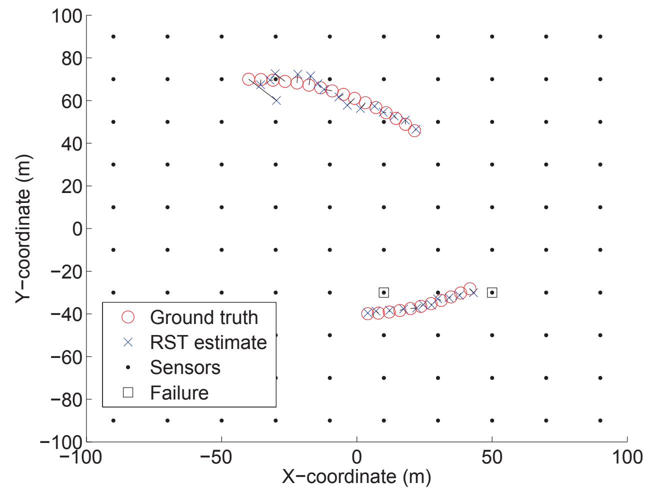
to thresholding, though it is often a negligible amount.

7. CONCLUSION

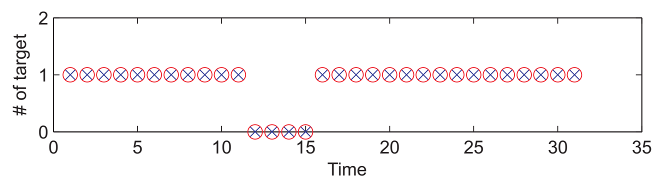
Simultaneous randomness in the number of targets and sensors along with their geo-kinematic information can be modeled methodically using RFST. This modeling system allows a systematic derivation of densities that can be analyzed using bayesian recursive prediction-filter methodologies.

The simulation results have exemplified the importance of sensor failure detection in a real system. When failed sensors incorrectly interpreted as non-reporting working sensor, the tracking estimates show severe degradation. Once the sensor failure detection is operating, the system can quickly avoid large errors by tagging sensor that might be failing. Future targets can then be tracked correctly based on the remaining active sensors.

A randomly missing target can also be detected using the same scheme. We also have shown that the system can track the number of targets and simultaneously estimate the geo-kinematic information of the target within the ROI.



(a) Position tracking



(b) Number of sources

Fig. 4. Tracking appearing and disappearing target

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