ROBUST TARGET LOCALIZATION IN MOVING RADAR PLATFORM THROUGH SEMIDEFINITE RELAXATION

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ABSTRACT

Accurate target localization is an important task in various commercial and military applications. One way to achieve this goal is to use the time-of-arrival (TOA) or time-delayof-arrival (TDOA) information observed at multiple distributed sensors. On the other hand, there is a great need to use moving sensors to form a radar platform with synthetic apertures. In this paper, we consider the problem of target localization based on the range information estimated from two-way time-of-flight (TW-TOF) at multiple synthetic array locations, where the position of these synthetic array locations is subject to certain random errors. The nonconvex estimation problem is approximated by a convex optimization problem using the semidefinite relaxation (SDR) approach. Simulation results show that the proposed estimator provides mean square position error performance close to the Cramer-Rao lower bound.

Keywords: Radar signal processing, position estimation, optimization methods, semidefinite relaxation.

1. INTRODUCTION

Accurate target localization is an important task in various applications, including wireless communications, navigation, and surveillance. In some applications such as cellular systems and sensor networks, this goal can be achieved by utilizing time-of-arrival (TOA) or time-delay-of-arrival (TDOA) information observed at multiple fixed sensors that are distributed over a region [1, 2, 3]. In many surveillance applications, however, it is rather feasible to use a moving radar platform that estimates the range from the twoway time-of-flight (TW-TOF) at each time, and the target location information is then estimated using the range estimates at multiple locations. Examples of such applications include energy-based localization, target tracking in urban canyons, and through-the-wall radar systems [4, 5]. When there is no error in the sensor positions, e.g., when the sensors have fixed locations and their positions can be precisely estimated, such problem can be solved using least squares (LS) estimation methods [2, 3]. On the other hand, in many situations, particularly when a moving platform is involved, there may be random errors in the sensor positions that would introduce bias in target locations. For example, sensor platform mounted on autonomous aerial or ground vehicles may not have precise location information due to their maneuvering and limitations on the GPS accuracy, which degrades especially in high multipath and jamming environments. In [6], robust target localization for a sensor network platform is considered to exploit TDOA information. In this paper, we address the estimation problem of the target location in such situations based on the TW-TOF or TOA information. We derive the nonlinear and nonconvex maximum likelihood (ML) formulation, and then convert it to a convex optimization problem by using the semidefinite relaxation (SDR) approach.

2. PROBLEM FORMULATION

Consider a moving mono-static radar system that measures the range of a target at M different positions through direct line-of-sight measurements. The position of each radar measurement is assumed to be known up to certain accuracy. That is, the location information is subject to some bounded random errors. Denote \mathbf{s}_i and $\tilde{\mathbf{s}}_i$, respectively, as vectors representing the estimated and true position of the radar at the *i*th measurement, $1 \leq i \leq M$. Depending on applications, the position information can be described using either two-dimensional (2-D) or three-dimensional (3-D) coordination system. The relationship between the estimated and true location information of the *i*th radar position is described by

$$\tilde{\mathbf{s}}_i = \mathbf{s}_i + \Delta \mathbf{s}_i \in \mathcal{A}_i(\epsilon),\tag{1}$$

where

$$\mathcal{A}_i(\epsilon) = \{ \mathbf{s}_i + \mathbf{e}_i, \|\mathbf{e}_i\| \le \epsilon \},\tag{2}$$

and ϵ is the maximum error in the radar locations.

Define the location of the target of interest as \mathbf{x} , and the velocity of wave propagation as c. Then, the true value of

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the TW-TOF between the ith radar location and the target is determined by

$$\tilde{\tau}_i = \frac{2}{c} \tilde{r}_i = \frac{2}{c} \|\mathbf{x} - \tilde{\mathbf{s}}_i\|,\tag{3}$$

where

$$\tilde{r}_i = \|\mathbf{x} - \tilde{\mathbf{s}}_i\| \tag{4}$$

is the true one-way distance between the ith radar position and the target. In the presence of noise, the observed TW-TOF becomes

$$\tau_i = \frac{2}{c} \|\mathbf{x} - \tilde{\mathbf{s}}_i\| + n_i, \tag{5}$$

where the noise components n_1, \dots, n_M are assumed to be independent and identically distributed (i.i.d.) Gaussian processes with mean zero and variance matrix $\sigma_{\tau}^2 \mathbf{I}_M$ with \mathbf{I}_M being the $M \times M$ identity matrix. Denote $\tilde{\mathbf{s}} =$ $[\tilde{\mathbf{s}}_1^T, \dots, \tilde{\mathbf{s}}_M^T]^T$, where T denotes transpose of a matrix or a vector. Then, the ML estimate of the target and sensor locations is expressed as

$$[\mathbf{x}, \tilde{\mathbf{s}}] = \arg\min_{\mathbf{x}, \tilde{\mathbf{s}}} \sum_{i=1}^{M} \left[\frac{2}{c} \| \mathbf{x} - \tilde{\mathbf{s}}_{i} \| - \tau_{i} \right]^{2}.$$
 (6)

The above equation can be equivalently written as

$$\min_{\mathbf{x},\mathbf{\tilde{s}}} \sum_{i=1}^{M} \left[\tilde{r}_i - r_i \right]^2, \tag{7}$$

where

$$r_i = \frac{\tau_i c}{2} \tag{8}$$

is the estimated range between the *i*th radar position and the target, $i = 1, \dots, M$. Denote

$$\tilde{\mathbf{r}} = [\tilde{r}_1, \cdots, \tilde{r}_M]^T$$
, and $\mathbf{r} = [r_1, \cdots, r_M]^T$, (9)

Then, (7) can be simplified as

$$\min_{\mathbf{x},\tilde{\mathbf{s}}} \|\tilde{\mathbf{r}} - \mathbf{r}\|^2,$$
s.t. $\tilde{r}_i^2 = \|\mathbf{x} - \tilde{\mathbf{s}}_i\|^2 = (\mathbf{x} - \tilde{\mathbf{s}}_i)^T (\mathbf{x} - \tilde{\mathbf{s}}_i).$
(10)

3. OPTIMIZATION APPROACHES

3.1. Optimization Without Sensor Position Error

When there is no error in the radar positions, i.e., $\tilde{\mathbf{s}}$ is known, the above problem can be solved using semidefinite relaxation (SDR) [2, 3, 6]. Note that

$$\|\tilde{\mathbf{r}} - \mathbf{r}\|^2 = \begin{bmatrix} \mathbf{r} \\ -1 \end{bmatrix}^T \begin{bmatrix} \mathbf{I}_M & \tilde{\mathbf{r}} \\ \tilde{\mathbf{r}}^T & \tilde{\mathbf{r}}^T \tilde{\mathbf{r}} \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ -1 \end{bmatrix}.$$
(11)

To linearize the formulations, we note that

$$\tilde{r}_i^2 = \|\mathbf{x} - \tilde{\mathbf{s}}_i\|^2 = \begin{bmatrix} \tilde{\mathbf{s}}_i \\ -1 \end{bmatrix}^T \begin{bmatrix} \mathbf{I}_K & \mathbf{x} \\ \mathbf{x}^T & \mathbf{x}^T \mathbf{x} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{s}}_i \\ -1 \end{bmatrix}$$
(12)

for $i = 1, \dots, M$, where K = 2 or 3 is the dimension of the coordinate system, and define

$$\tilde{w} = \tilde{\mathbf{r}}^T \tilde{\mathbf{r}} = \sum_{i=1}^M \tilde{r}_i^2 = \sum_{i=1}^M \begin{bmatrix} \tilde{\mathbf{s}}_i \\ -1 \end{bmatrix}^T \begin{bmatrix} \mathbf{I}_K & \mathbf{x} \\ \mathbf{x}^T & \mathbf{x}^T \mathbf{x} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{s}}_i \\ -1 \end{bmatrix}.$$
(13)

We further introduce $z = \mathbf{x}^T \mathbf{x}$ to linearize the above expression. Thus, the ML problem expressed in (10) can be relaxed as a linear and convex problem in $(\mathbf{x}, \tilde{\mathbf{r}}, \tilde{w}, z)$ as

$$\min_{\mathbf{x},\tilde{\mathbf{r}},\tilde{w},z} \begin{bmatrix} \mathbf{r} \\ -1 \end{bmatrix}^{T} \begin{bmatrix} \mathbf{I}_{M} & \tilde{\mathbf{r}} \\ \tilde{\mathbf{r}}^{T} & \tilde{w} \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ -1 \end{bmatrix},$$
s.t. $\tilde{w} = \sum_{i=1}^{M} \begin{bmatrix} \tilde{\mathbf{s}}_{i} \\ -1 \end{bmatrix}^{T} \begin{bmatrix} \mathbf{I}_{K} & \mathbf{x} \\ \mathbf{x}^{T} & z \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{s}}_{i} \\ -1 \end{bmatrix}, \quad (14)$

$$\begin{bmatrix} \mathbf{I}_{M} & \tilde{\mathbf{r}} \\ \tilde{\mathbf{r}}^{T} & \tilde{w} \end{bmatrix} \succeq 0, \quad \begin{bmatrix} \mathbf{I}_{K} & \mathbf{x} \\ \mathbf{x}^{T} & z \end{bmatrix} \succeq 0,$$

where the notation $\mathbf{A} \succeq 0$ means that \mathbf{A} is positive semidefinite. Note that, it was shown in [2] that the optimal solution meets the requirement $r_i > 0$ and thus such constraint is not necessary.

3.2. Optimization With Sensor Position Error

Now we consider the localization in the presence of radar position errors. By assuming observations with i.i.d. noise, the log-likelihood function of the target location can be expressed as

$$L(\mathbf{x}) = -\frac{1}{2\sigma_n^2} (\mathbf{\tilde{r}} - \mathbf{r})^T (\mathbf{\tilde{r}} - \mathbf{r}), \qquad (15)$$

where $\sigma_n = \sigma_\tau c/2$. The above expression can be rewritten as

$$L(\mathbf{x}) = -\begin{bmatrix} \tilde{\mathbf{r}} \\ -1 \end{bmatrix}^{T} \mathbf{F} \begin{bmatrix} \tilde{\mathbf{r}} \\ -1 \end{bmatrix}, \qquad (16)$$

where

$$\mathbf{F} = \frac{1}{2\sigma_n^2} \begin{bmatrix} \mathbf{I}_M & \mathbf{r} \\ \mathbf{r}^T & v \end{bmatrix}$$
(17)

with

Because the constant factor $(2\sigma_n^2)^{-1}$ in (17) does not affect the optimization, it can be ignored for formulation simplicity. That is,

 $v = \mathbf{r}^T \mathbf{r}.$

$$\mathbf{F} = \begin{bmatrix} \mathbf{I}_M & \mathbf{r} \\ \mathbf{r}^T & v \end{bmatrix}$$
(19)

(18)

is used hereafter. Thus, robust target localization by maximizing the worst-case likelihood function leads to the following formulation,

$$\min_{\mathbf{x}} \sup \left\{ \begin{bmatrix} \tilde{\mathbf{r}} \\ -1 \end{bmatrix}^T \mathbf{F} \begin{bmatrix} \tilde{\mathbf{r}} \\ -1 \end{bmatrix} \middle| \|\Delta \mathbf{s}_i\| \le \epsilon, i = 1, \cdots, M \right\}.$$
(20)

By using the first-order Taylor expansion, \tilde{r}_i can be expressed as

$$\tilde{r}_i = \|\mathbf{x} - \tilde{\mathbf{s}}_i\| = \|\mathbf{x} - \mathbf{s}_i\| - \frac{\Delta \mathbf{s}_i^T (\mathbf{x} - \mathbf{s}_i)}{\|\mathbf{x} - \mathbf{s}_i\|} + o(\|\Delta \mathbf{s}_i\|).$$
(21)

Define

$$e_i = \frac{\Delta \mathbf{s}_i^T (\mathbf{x} - \mathbf{s}_i)}{\|\mathbf{x} - \mathbf{s}_i\|}.$$
(22)

From (1) and (2), it is obvious that $|e_i| \leq \epsilon$. Let

$$\hat{r}_i = \|\mathbf{x} - \mathbf{s}_i\|, \quad \hat{\mathbf{r}} = [\hat{r}_1, \cdots, \hat{r}_M], \quad \mathbf{e} = [e_1, \cdots, e_M].$$
 (23)

Then, we can reformulate (20) as

$$\min_{\mathbf{x}} \sup \left\{ \begin{bmatrix} \hat{\mathbf{r}} - \mathbf{e} \\ -1 \end{bmatrix}^T \mathbf{F} \begin{bmatrix} \hat{\mathbf{r}} - \mathbf{e} \\ -1 \end{bmatrix} \middle| |e_i| \le \epsilon, i = 1, \cdots, M \right\},$$

s.t. $\hat{r}_i = \|\mathbf{x} - \mathbf{s}_i\|.$ (24)

To facilitate numerical optimization, we replace the box constraints $|e_i| \leq \epsilon, i = 1, \dots, M$, with an ellipsoid constraint $||\mathbf{e}|| \leq \rho = \epsilon \sqrt{M}$. Thus, the above expression becomes

$$\min_{\mathbf{x}} \sup \left\{ \begin{bmatrix} \hat{\mathbf{r}} - \mathbf{e} \\ -1 \end{bmatrix}^T \mathbf{F} \begin{bmatrix} \hat{\mathbf{r}} - \mathbf{e} \\ -1 \end{bmatrix} \middle| \|\mathbf{e}\| \le \rho \right\}, \quad (25)$$
s.t. $\hat{r}_i = \|\mathbf{x} - \mathbf{s}_i\|.$

This can be further represented by

$$\min_{\mathbf{x}} \eta$$
s.t.
$$\sup \left\{ \begin{bmatrix} \hat{\mathbf{r}} - \mathbf{e} \\ -1 \end{bmatrix}^T \mathbf{F} \begin{bmatrix} \hat{\mathbf{r}} - \mathbf{e} \\ -1 \end{bmatrix} \middle| \|\mathbf{e}\| \le \rho \right\} \le \eta, \quad (26)$$

$$\hat{r}_i = \|\mathbf{x} - \mathbf{s}_i\|.$$

The first constraint in (26) can be written as

$$\mathbf{e}^{T}\mathbf{e} - \rho^{2} \leq 0 \Rightarrow$$

$$\mathbf{e}^{T}\mathbf{e} - 2\mathbf{e}^{T}(\mathbf{\hat{r}} - \mathbf{r}) + \mathbf{\hat{r}}^{T}\mathbf{\hat{r}} - 2\mathbf{r}^{T}\mathbf{\hat{r}} + v - \eta \leq 0.$$
(27)

By using the S-procedure [7], the above implication holds if and only if there exists a $\lambda > 0$ such that

$$\begin{bmatrix} (\lambda - 1)\mathbf{I}_M & \mathbf{\hat{r}} - \mathbf{r} \\ (\mathbf{\hat{r}} - \mathbf{r})^T & \eta - \mathbf{\hat{r}}^T \mathbf{\hat{r}} + 2\mathbf{r}^T \mathbf{\hat{r}} - v - \lambda\rho^2 \end{bmatrix} \succeq 0.$$
(28)

Similar to the approaches used in Section 3-1, we linearize the terms $\hat{\mathbf{r}}^T \hat{\mathbf{r}}$ in (28) and the second constraint in (26) by introducing

$$\hat{w} = \sum_{i=1}^{M} \hat{r}_{i}^{2} = \sum_{i=1}^{M} \|\mathbf{x} - \mathbf{s}_{i}\|^{2}$$

$$= \sum_{i=1}^{M} \begin{bmatrix} \mathbf{s}_{i} \\ -1 \end{bmatrix}^{T} \begin{bmatrix} \mathbf{I}_{K} & \mathbf{x} \\ \mathbf{x}^{T} & \mathbf{x}^{T} \mathbf{x} \end{bmatrix} \begin{bmatrix} \mathbf{s}_{i} \\ -1 \end{bmatrix},$$
(29)

and $z = \mathbf{x}^T \mathbf{x}$. Then, the SDR approach in the presence of

sensor position errors is formulated as

$$\begin{split} \min_{\mathbf{x}, \hat{\mathbf{f}}, \hat{w}, z, \lambda} \eta \\ \text{s.t.} & \begin{bmatrix} (\lambda - 1) \mathbf{I}_M & \hat{\mathbf{r}} - \mathbf{r} \\ (\hat{\mathbf{r}} - \mathbf{r})^T & \eta - \hat{w} - 2\mathbf{r}^T \hat{\mathbf{r}} + v - \lambda \rho^2 \end{bmatrix} \succeq 0, \\ \hat{w} &= \sum_{i=1}^M \begin{bmatrix} \mathbf{s}_i \\ -1 \end{bmatrix}^T \begin{bmatrix} \mathbf{I}_K & \mathbf{x} \\ \mathbf{x}^T & z \end{bmatrix} \begin{bmatrix} \mathbf{s}_i \\ -1 \end{bmatrix}, \\ \begin{bmatrix} \mathbf{I}_M & \hat{\mathbf{r}} \\ \hat{\mathbf{r}}^T & \hat{w} \end{bmatrix} \succeq 0, \begin{bmatrix} \mathbf{I}_K & \mathbf{x} \\ \mathbf{x}^T & z \end{bmatrix} \succeq 0, \\ \eta \ge 0, \quad \lambda \ge 0. \end{split}$$

$$(30)$$

4. CRAMER-RAO LOWER BOUND

Under the assumption that the TOA measurement noise and the radar location errors follow independent Gaussian distributions, the log-likelihood function of the joint target and node location is given by

$$L(\mathbf{x}, \tilde{\mathbf{s}}) = -\frac{1}{2\sigma_n^2} (\tilde{\mathbf{r}} - \mathbf{r})^T (\tilde{\mathbf{r}} - \mathbf{r}) - \frac{1}{2\sigma_s^2} (\tilde{\mathbf{s}} - \mathbf{s})^T (\tilde{\mathbf{s}} - \mathbf{s})$$
$$= -\frac{1}{2\sigma_n^2} \sum_{i=1}^M (\tilde{\mathbf{r}}_i - \mathbf{r}_i)^2 - \frac{1}{2\sigma_s^2} \sum_{i=1}^M (\tilde{\mathbf{s}}_i - \mathbf{s}_i)^T (\tilde{\mathbf{s}}_i - \mathbf{s}_i),$$
(31)

where $\mathbf{s} = [\mathbf{s}_1^T, \cdots, \mathbf{s}_M^T]^T$.

The Fisher information matrix can be defined as

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_{\mathbf{x}\mathbf{x}} & \mathbf{J}_{\mathbf{x}\tilde{\mathbf{s}}} \\ \mathbf{J}_{\mathbf{x}\tilde{\mathbf{s}}}^T & \mathbf{J}_{\tilde{\mathbf{s}}\tilde{\mathbf{s}}} \end{bmatrix}.$$
(32)

By noting (4), (9), and the fact $E(\tilde{r}_i - r_i) = 0$, where $E[\cdot]$ denotes the statistical expectation operator, we have

$$\mathbf{J}_{\mathbf{xx}} = -E\left[\frac{\partial^{2}L(\mathbf{x},\tilde{\mathbf{s}})}{\partial\mathbf{x}\partial\mathbf{x}^{T}}\right] = \sum_{i=1}^{M} \Delta_{i}, \\
\mathbf{J}_{\mathbf{x}\tilde{\mathbf{s}}} = -E\left[\frac{\partial^{2}L(\mathbf{x},\tilde{\mathbf{s}})}{\partial\mathbf{x}\partial\tilde{\mathbf{s}}^{T}}\right] = [\Delta_{1}, \cdots, \Delta_{M}] \\
\mathbf{J}_{\tilde{\mathbf{s}x}} = -E\left[\frac{\partial^{2}L(\mathbf{x},\tilde{\mathbf{s}})}{\partial\tilde{\mathbf{s}}\partial\mathbf{x}^{T}}\right] = \mathbf{J}_{\mathbf{x}\tilde{\mathbf{s}}}^{T}, \\
\mathbf{J}_{\tilde{\mathbf{s}}\tilde{\mathbf{s}}} = -E\left[\frac{\partial^{2}L(\mathbf{x},\tilde{\mathbf{s}})}{\partial\tilde{\mathbf{s}}\partial\tilde{\mathbf{s}}^{T}}\right] = \begin{bmatrix}\Delta_{1} + \frac{1}{\sigma_{s}^{2}}\mathbf{I}_{2} & \mathbf{O} \\ & \ddots \\ \mathbf{O} & \Delta_{M} + \frac{1}{\sigma_{s}^{2}}\mathbf{I}_{2} \end{bmatrix}, \\$$
(33)

where

$$\Delta_i = \frac{1}{\sigma_n^2} \frac{(\mathbf{x} - \tilde{\mathbf{s}}_i)(\mathbf{x} - \tilde{\mathbf{s}}_i)^T}{\tilde{r}_i^2}.$$
 (34)

Using the partitioned matrix inversion, the Cramer-Rao lower bound (CRB) of the variance of the target location estimation is found as

$$\mathbf{c}_{\mathbf{x}} = \operatorname{diag}\left\{ \left(\mathbf{J}_{\mathbf{x}\mathbf{x}} - \mathbf{J}_{\mathbf{x}\tilde{\mathbf{s}}} \mathbf{J}_{\tilde{\mathbf{s}}\tilde{\mathbf{s}}}^{-1} \mathbf{J}_{\mathbf{x}\tilde{\mathbf{s}}}^{T} \right)^{-1} \right\}.$$
 (35)

5. SIMULATION RESULTS

Computer simulations are conducted to verify the effectiveness of the proposed method. We consider a moving radar platform mounted on a ground vehicle. As such, we use a 2-D coordinate system. As illustrated in Fig. 1, a stationary or inanimate target is located at the origin, and the ground vehicle moves around the target in two crossed roads. The shortest distance from each road to the target is L = 30m. The vehicle moves at a constant speed of 18 km/h (or 5 m/s). Assume that the time required for data acquisition and processing is 2 s per visit. The effect of radar displacement within each data set is ignored. We consider two scenarios. In the first scenario, the vehicle moves from west to east over the interval of length 2L = 60 m. In this case, the overall observation time period is 12 s, yielding observations at 7 different positions. In the second scenario, the vehicle moves from west to east, and then turns to south, yielding an interval of length 4L = 120 m. In this case, the overall observation time period is 24 s, and observations at 13 different positions are made. Each observed data set is contaminated by the error in the radar positions and measurement noise. The radar position error is described by a zero-mean Gaussian distribution with variance σ_s^2 , and the maximum position error at each position is bounded by $\epsilon = 4\sigma_s$. The measurement noise follows a zero-mean Gaussian distribution with variance σ_n^2 .

Fig. 2 shows the root mean square error (RMSE) of the estimated target position, overlayed with the CRB. The SeDuMi toolbox [8, 9] was used to carry out the simulations, and the results are further improved via local optimization by applying a standard nonlinear optimization routine, e.g., SOLNP [10]. For each plot, we vary σ_n from 0 to 1 m, and σ_s was chosen to be 0 m, 0.5 m, and 1 m, respectively. Each result is obtained from 400 independent trials. All the results obtained from the proposed method are very close to the CRB. For Scenario I, the x and y directions are asymmetric, and the estimation error in the y direction appears to be higher. Scenario II is symmetric in the two directions, and the increased number of radar positions help reduce the estimation error in both directions.

6. CONCLUSIONS

We have investigated the problem of localizing a target using the two-way time-of-flight measurements operated in a moving radar platform, which observes the target from different positions. In particular, we have formulated the problem in the context of robust worst-case target localization with the consideration of errors in the radar observation positions. The nonlinear and nonconvex problem is reformulated into a convex optimization problem through semidefinite relaxation. Simulation results verified that the target localization performance is very close to the Cramer-Rao lower bound.

REFERENCES

[1] Y. Huang, J. Benesty, G. W. Elko, and R. M. Mersereati,



Figure 1: Scene of the simulations.



Figure 2: RMSE (marks) and CRB (lines) of the estimated target position.

"Real-time passive source localization: a practical linearcorrection least-squares approach," *IEEE Trans. Speech Audio Processing*, vol. 9, no. 8, pp. 943–956, Nov. 2001.

- [2] K. W. Cheung, W. K. Ma, H. C. So, "Accurate approximation algorithm for TOA-based maximum likelihood mobile location using semidefinite programming," *IEEE ICASSP*, May 2004.
- [3] A. Beck, P. Stoica, and J. Li, "Exact and approximate solutions of source localization problems," *IEEE Trans. Signal Processing*, vol. 56, no. 5, pp. 1770–1778, May 2008.
- [4] D. Li and Y. H. Hu, "Least square solutions of energy based acoustic source localization problems," *Int. Conf. Parallel Processing Workshops*, pp. 443–446, 2004.
- [5] Y. Zhang, M. G. Amin, and A. Fauzia, "Localization of inanimate moving targets using dual-frequency synthetic aperture radar and time-frequency analysis, *IEEE Int. Geoscience & Remote Sensing Symp.*, Boston, MA, July 2008.
- [6] K. Yang and Z.-Q. Luo, "Robust target localization with multiple sensors using time difference of arrivals," *IEEE Radar Conf.*, Rome, Italy, pp. 1080–1085, May 2008.
- [7] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge Univ. Press, 2004.
- [8] J. F. Sturm, "Using SeDuMi 1.02, a Matlab toolbox for optimization over symmetric cones," *Optimization Methods & Software*, vol. 11-12, pp. 625–653, Aug. 1999.
- [9] http://sedumi.mcmaster.ca/
- [10] Y. Ye, SOLNP Users' Guide A Nonlinear Optimization Program in MATLAB. [online] http://www.stanford.edu/ ~yyye/matlab/manual.ps.