

# NEW POLARIMETRIC SIGNAL SUBSPACE DETECTORS FOR SAR PROCESSORS

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## ABSTRACT

This paper deals with three new polarimetric SAR processors based on subspace detectors. These algorithms aim at using models with physical and polarimetric scattering properties not exploited by the isotropic point model. These processors are implemented by computing the corresponding target subspaces. Results on simulated data with realistic targets show the interest of these new processors.

**Index Terms**— SAR Processors, Subspace Detectors, Polarimetry

## 1. INTRODUCTION

The detection of Man Made Targets (MMT) embedded in noise, clutter or speckle with a polarimetric SAR system is a current issue for both the signal processing and the SAR communities. Most of SAR detection algorithms consider targets as isotropic point and focus on noise properties [1, 2]. However, the target scattering properties can be used to have a more suitable model. We previously developed new SAR processors based on subspace detectors to take into account the scattering properties of MMT in only one polarimetric channel [3, 4]. In this paper, we consider, in addition to the scattering properties of the target, all the polarization channels. For this purpose, we develop polarimetric SAR processors exploiting a target model based on its physical properties; the techniques used in [4] are extended to take in account polarimetric properties. We assume that the target scattering belongs to a low polarimetric subspace. Considering all the configurations of a plate for the co-polarized channels (HH and VV), we construct processors based on an appropriate Generalized Likelihood Ratio (GLR) [5]. Since several possibilities are available to organize the polarimetric vectors, we develop three new processors based on different preprocessing polarimetric decompositions; we show that these polarimetric preprocessing treatments are equivalent to classical post processing treatments in polarimetry [6]. Compared to a single

polarization channel processor, these new polarimetric processors show important improvements in terms of probability of detection and robustness.

This paper is organized as follows. The detection problem and the description of the polarimetric model are presented in section 2. Three cases depending on polarimetric decompositions are determined and related to their physical meaning. Section 3 describes the polarimetric target subspaces. In section 4, we discuss about the expressions of these new processors. Finally, in section 5, we compare the performance of the three processors. Two targets with different polarimetric properties are considered to test the ability and the robustness of the three processors.

The following convention is adopted: italic indicates a scalar quantity, lower case boldface indicates a vector quantity and upper case boldface a matrix.  $T$  denotes the transpose operator and  $\dagger$  the transpose conjugate.

## 2. POLARIMETRIC DETECTION PROBLEM FOR A DIRECTIVITY OBJECT

Assuming that the scattering of a man-made-target (MNT) is directive, the isotropic point model used in classical SAR detectors is not suitable. A canonical element for the model is more adapted to detect realistic targets. As in [3, 4], the canonical element is chosen here to be a perfectly conducting plate whose backscattering is computed by means of physical optics approximation (PO). Moreover, all polarization channels are considered in this paper. For a plate, the PO approximation gives rise to the nullity of the cross-polarization (HV) and the equality of the co-polarization (HH, VV) of the target backscattering. Hence, only the HH and VV channels will be considered in our study. (The subscripts H and V will refer to HH and VV channels).

We denote by  $\mathbf{z}_{pi} \in \mathbb{C}^K$  ( $p = H$  or  $p = V$ ) the received signal samples at every  $u_i$  position of the antenna in channel  $p$ ; the total received signal  $\mathbf{z}_p$  for one polarization channel is the concatenation of the  $N$  vectors  $\mathbf{z}_{pi}$  (see [4]).

$$\mathbf{z}_p \in \mathbb{C}^{NK}, \quad \mathbf{z}_p = [\mathbf{z}_{p1}^T \quad \mathbf{z}_{p2}^T \quad \dots \quad \mathbf{z}_{pN}^T]^T \quad (1)$$

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The total polarimetric received signal  $\mathbf{z}$  is then the concatenation of  $\mathbf{z}_H$  and  $\mathbf{z}_V$ :

$$\mathbf{z} \in \mathbb{C}^{2NK}, \quad \mathbf{z} = [\mathbf{z}_H^T \quad \mathbf{z}_V^T]^T \quad (2)$$

The detection problem at position  $(x, y)$  can be stated as the following binary hypothesis test:

$$\begin{cases} H_0 : \mathbf{z} = \mathbf{n} \\ H_1 : \mathbf{z} = \begin{pmatrix} a_H \mathbf{y}(\alpha, \beta) \\ a_V \mathbf{y}(\alpha, \beta) \end{pmatrix} + \mathbf{n} \end{cases}, \quad (3)$$

where  $\mathbf{n} \in \mathbb{C}^{2NK}$  is a complex Gaussian white noise vector of known variance  $\sigma^2$  (for the unknown variance case, see [4]),  $\mathbf{y}(\alpha, \beta)$  is the backscattering of the target model at position  $(x, y)$  with orientation  $(\alpha, \beta)$ . The PO approximation yields for a plate model [7]:

$$\mathbf{y}(\alpha, \beta) = \mathbf{y}_H(\alpha, \beta) = \mathbf{y}_V(\alpha, \beta) \quad (4)$$

$(a_H, a_V)$  are two unknown complex attenuation coefficients.

Some assumptions on the relationship between the polarization channels of the target can be formulated through those coefficients. We distinguish three cases:

1. ‘‘Decorrelated’’ case,  $a_H \neq a_V$ : the HH and VV channels are not related, no assumption is made on the polarimetric properties of the model;
2. ‘‘Correlated +’’ case,  $a_H = a_V$ : the attenuations in the HH and VV channels are equal in magnitude and in phase. Physically this corresponds to single bounce scattering [6];
3. ‘‘Correlated -’’ case,  $a_H = -a_V$ : the attenuations in the HH and VV channels have the same magnitude but are opposite in phase; this corresponds to double bounce scattering [6].

These three cases lead us to develop three different processors.

Since the orientation  $(\alpha, \beta)$  of the target is unknown, we make the following assumptions:

$$\forall(\alpha, \beta) \in [\alpha_{min}, \alpha_{max}] \times [\beta_{min}, \beta_{max}], \mathbf{y}(\alpha, \beta) \in \langle H_{xy} \rangle \quad (5)$$

where  $\langle H_{xy} \rangle$  is a polarimetric signal subspace of dimension  $D_H$ . We denote  $\mathbf{H}_{xy}$  a  $2KN \times D_H$  basis of this subspace. The detection problem may then be rewritten as follows:

$$\begin{cases} H_0 : \mathbf{z} = \mathbf{n} \\ H_1 : \mathbf{z} = \mathbf{H}_{xy} \lambda + \mathbf{n} \end{cases}, \quad (6)$$

where  $\lambda \in \mathbb{C}^{D_H \times 1}$  is an unknown coordinate vector. As in [5, 8], we propose to form the SAR image by choosing

for the pixel value  $I(x, y)$  a monotonic function of the Generalized Likelihood Ratio Test (GLRT) [3, 4]:

$$I(x, y) = \frac{\|\mathbf{H}_{xy}^\dagger \mathbf{z}\|^2}{\sigma^2} \quad (7)$$

The intensity  $I(x, y)$  depends directly on the projection of  $\mathbf{z}$  onto the signal subspace  $\langle H_{xy} \rangle$ ; for the three detection problems defined above we will have three different subspaces and so, three different processors.

### 3. BASIS COMPUTATION

We propose in this section a method to compute the  $\mathbf{H}_{xy}$  basis for each polarimetric hypothesis described above. This computation is performed from the matrix  $\mathbf{S}_{xy}$  containing all the target model responses in the polarimetric channels. The different constructions of this matrix are firstly described and the resulting basis are next presented.

#### 3.1. Construction of Signal Matrix

For one polarimetric channel, each column of  $\mathbf{S}_{xy}^p \in \mathbb{C}^{NK \times M}$  is the concatenation of responses of the plate for each position of the radar with one known orientation of the plate [3, 4]:

$$\mathbf{S}_{xy}^p = [\mathbf{y}(\alpha_1, \beta_1) \dots \mathbf{y}(\alpha_i, \beta_j) \dots \mathbf{y}(\alpha_P, \beta_Q)] \quad (8)$$

where  $(\alpha_i, \beta_j)$  span  $[0, \pi] \times [0, \pi]$  and  $M = PQ$ .

Since in Eq.4,  $\mathbf{y}(\alpha_i, \beta_j)$  are equal for the HH and VV channels,  $\mathbf{S}_{xy}$  is the same for the HH and the VV polarization channels:

$$\mathbf{S}_{xy}^H = \mathbf{S}_{xy}^V = \mathbf{S}_{xy}^p \quad (9)$$

From detection problem in Eq.3 and the polarimetric hypothesis, we have three polarimetric signal matrices:

1. ‘‘In the decorrelated’’ case, the two polarization channels are independent:

$$\mathbf{S}_{xy}^{deco} = \begin{pmatrix} \mathbf{S}_{xy}^p & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_{xy}^p \end{pmatrix} \quad (10)$$

2. ‘‘In the correlated +’’ case, we consider that the HH and VV channels are positively correlated; thus the signal matrix is defined as follows:

$$\mathbf{S}_{xy}^{co+} = \begin{pmatrix} \mathbf{S}_{xy}^p \\ \mathbf{S}_{xy}^p \end{pmatrix} \quad (11)$$

3. ‘‘In the correlated -’’ case, we consider that the HH and VV channels are negatively correlated; thus the signal matrix is defined as follows:

$$\mathbf{S}_{xy}^{co-} = \begin{pmatrix} \mathbf{S}_{xy}^p \\ -\mathbf{S}_{xy}^p \end{pmatrix} \quad (12)$$

### 3.2. Construction of Signal Basis

For one polarization channel, the basis  $\mathbf{H}_{xy}^p$  is obtained from the SVD of  $\mathbf{S}_{xy}^p$  [9] defined in Eq.8:

$$\mathbf{S}_{xy}^p = \mathbf{U}_{xy} \mathbf{\Sigma}_{xy} \mathbf{V}_{xy}^\dagger \quad (13)$$

where  $\mathbf{U}_{xy}$  and  $\mathbf{V}_{xy}$  are the left and right singular vectors, and  $\mathbf{\Sigma}_{xy}$  is a diagonal matrix containing the singular values. Then the basis  $\mathbf{H}_{xy}^p$  corresponds to the  $D_H$  first singular vectors of  $\mathbf{U}_{xy}$  associated to the  $D_H$  highest singular values. Now, we will see how the basis  $\mathbf{H}_{xy}^{deco}$  and  $\mathbf{H}_{xy}^{co\pm}$  are obtained.

For the “decorrelated” case, the matrices  $\mathbf{S}_{xy}^p$  representing the HH and VV channels, are orthogonally arranged in  $\mathbf{S}_{xy}^{deco}$  as shown by Eq.10. We compute the SVD of  $\mathbf{S}_{xy}^{deco}$ :

$$\mathbf{S}_{xy}^{deco} = \mathbf{U}_{xy}^{deco} \mathbf{\Sigma}_{xy}^{deco} \mathbf{V}_{xy}^{deco\dagger} \quad (14)$$

$$= \begin{pmatrix} \mathbf{U}_{xy} \mathbf{\Sigma}_{xy} \mathbf{V}_{xy}^\dagger & \mathbf{0} \\ \mathbf{0} & \mathbf{U}_{xy} \mathbf{\Sigma}_{xy} \mathbf{V}_{xy}^\dagger \end{pmatrix} \quad (15)$$

where,

$$\mathbf{U}_{xy} \in \mathbb{C}^{NK \times M}, \quad \mathbf{U}_{xy} = [\mathbf{u}_{1xy} \quad \mathbf{u}_{2xy} \quad \dots \quad \mathbf{u}_{Mxy}] \quad (16)$$

Thereby, after arranging the singular values in Eq.15 in descending order,  $\mathbf{U}_{xy}^{deco}$  has the form:

$$\mathbf{U}_{xy}^{deco} = \begin{pmatrix} \mathbf{u}_{1xy} & \mathbf{0}_{[0, NK]} & \mathbf{u}_{2xy} & \dots & \mathbf{u}_{Mxy} & \mathbf{0}_{[0, NK]} \\ \mathbf{0}_{[0, NK]} & \mathbf{u}_{1xy} & \mathbf{0}_{[0, NK]} & \dots & \mathbf{0}_{[0, NK]} & \mathbf{u}_{Mxy} \end{pmatrix} \quad (17)$$

The basis  $\mathbf{H}_{xy}^{deco}$  consists in the  $2D_H$  first columns of  $\mathbf{U}_{xy}^{deco}$ ; by the structure of  $\mathbf{H}_{xy}^{deco}$ , we see that in the polarimetric subspace  $\langle H_{xy}^{deco} \rangle$  the polarimetric information is treated separately.

In the “correlated” cases, the SVD of  $\mathbf{S}_{xy}^{co\pm}$  is expressed as:

$$\mathbf{S}_{xy}^{co\pm} = \begin{pmatrix} \mathbf{S}_{xy}^p \\ \pm \mathbf{S}_{xy}^p \end{pmatrix} \quad (18)$$

We deduce that,

$$\mathbf{S}_{xy}^{co\pm} = \mathbf{U}_{xy}^{co\pm} \sqrt{2} \mathbf{\Sigma}_{xy} \mathbf{V}_{xy}^\dagger \quad (19)$$

with

$$\mathbf{U}_{xy}^{co\pm} = \begin{pmatrix} \frac{1}{\sqrt{2}} \mathbf{U}_{xy} \\ \pm \frac{1}{\sqrt{2}} \mathbf{U}_{xy} \end{pmatrix} \quad (20)$$

The basis  $\mathbf{H}_{xy}^{co\pm}$  consist then in the  $D_H$  first columns of  $\mathbf{U}_{xy}^{co\pm}$ .

Contrary to  $\mathbf{H}_{xy}^{deco}$ , each vector of  $\mathbf{H}_{xy}^{co\pm}$  describes the two polarization channels. Moreover the subspaces  $\langle H_{xy}^{co\pm} \rangle$  have a lower dimension than  $\langle H_{xy}^{deco} \rangle$ .

Actually, these basis  $\mathbf{H}_{xy}^{deco}$  and  $\mathbf{H}_{xy}^{co\pm}$  have to be computed for all pixels of the detection image. A method for using only one SVD is presented in [4] and allows to perform these computations in realistic SAR configurations.

## 4. DISCUSSION ABOUT NEW POLARIMETRIC SAR PROCESSORS

As we have seen in Sec 2, the pixel intensity at  $(x, y)$  of the SAR processors depends on the projection of the received signal  $\mathbf{z}$  onto the polarimetric signal subspace; from  $\mathbf{H}_{xy}^{deco}$  and  $\mathbf{H}_{xy}^{co\pm}$ , we compute the structures of the different detectors. For the “decorrelated” case, the intensity of a pixel is expressed as follow:

$$I_{deco}(x, y) = \frac{\|\mathbf{H}_{xy}^{deco\dagger} \mathbf{z}\|^2}{\sigma^2} \quad (21)$$

$$= \frac{\|\mathbf{H}_{xy}^{H\dagger} \mathbf{z}_H\|^2}{\sigma^2} + \frac{\|\mathbf{H}_{xy}^{V\dagger} \mathbf{z}_V\|^2}{\sigma^2} \quad (22)$$

$$= I_H(x, y) + I_V(x, y) \quad (23)$$

The application of the “decorrelated” detector leads to apply the processor to the HH and VV channels separately. From a polarimetric point of view, the sum of  $I_H(x, y)$  and  $I_V(x, y)$  has no physical signification because the phase information between the two channels is lost.

For the “correlated” cases, the intensities are:

$$I_{co+}(x, y) = \frac{\|\mathbf{H}_{xy}^{co+\dagger} \mathbf{z}\|^2}{\sigma^2} \quad (24)$$

$$= \frac{1}{2} \frac{\|\mathbf{H}_{xy}^{H\dagger} \mathbf{z}_H + \mathbf{H}_{xy}^{V\dagger} \mathbf{z}_V\|^2}{\sigma^2} \quad (25)$$

$$I_{co-}(x, y) = \frac{\|\mathbf{H}_{xy}^{co-\dagger} \mathbf{z}\|^2}{\sigma^2} \quad (26)$$

$$= \frac{1}{2} \frac{\|\mathbf{H}_{xy}^{H\dagger} \mathbf{z}_H - \mathbf{H}_{xy}^{V\dagger} \mathbf{z}_V\|^2}{\sigma^2} \quad (27)$$

We clearly see that the intensities of the “correlated” processors are the sum or the difference of the projections of  $\mathbf{z}_H$  and  $\mathbf{z}_V$ . In polarimetry, the sum of the HH and VV channels corresponds to a single bounce scattering and the difference, to double bounce scattering; in fact we retrieve the Pauli decomposition [6] used in radar image post processing. Thus, targets with different polarimetric properties will be treated differently with the two “correlated” processors. In the next section we will see that for realistic targets, only the “correlated -” processor will be useful.

## 5. SIMULATIONS

We are interested in evaluating and comparing the performance of the polarimetric detectors; the three processors are

tested on simulated data. For the target, we choose two objects with different polarimetric signatures and which can describe a MMT: a perfectly conducting plate in free space (single bounce scattering) whose scattering is computed from PO approximation [7, 4], and a perfectly conducting box with ground (double bounce) whose scattering is simulated using Feko [10]. Both targets are embedded in a white Gaussian noise. We represent in figure 1 and figure 2 the probability of detection for the new detectors and for detectors using a single polarization channel (HH and VV).

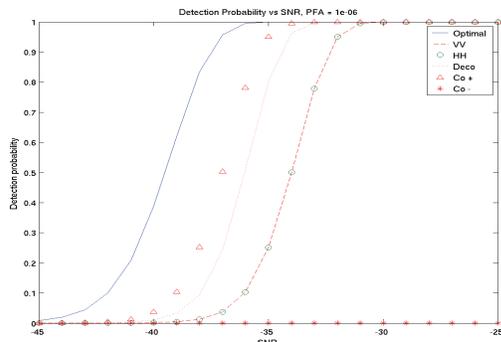


Fig. 1. Detection Probability versus SNR. for a plate

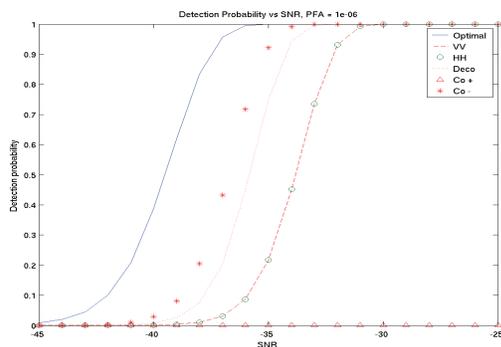


Fig. 2. Detection Probability versus SNR. for a box with ground

In figure 1, the “decorrelated” detector allows a 2dB gain compared to a single channel detector; since the main scattering mechanism for the plate is single bounce, only the “correlated +” detector is able to detect it. This detector provides a 1dB gain compared to the “decorrelated” one and a 3dB gain compared to a single channel detector. In figure 2, we have the same results as for the plate, except that we have to use the “correlated-” detector to detect the box on ground (because of the double bounce); moreover, the scattering mechanisms involved for this target correspond to those for realistic targets. Thus, only the “decorrelated” and the “correlated -” detectors will be used in practice, knowing that the “correlated -” detec-

tor is optimal. Finally, in addition to an increased probability of detection, these two processors are robust since they are able to detect any kind of MMT.

## 6. CONCLUSION

We presented in the paper, three new polarimetric SAR processors based on subspace detectors and polarimetric decomposition. By using other models than isotropic point and by taking in consideration the target polarimetric information, these three processors increase the probability of detection compared to processors using one polarization channel. Furthermore, by a good arrangement of the polarimetric data, the “correlated” processors outperform the “decorrelated” one by using the polarimetric properties of the model; these results show firstly, that the separate use of polarimetric channels in detection provides improvements and secondly, that the use of polarimetric information between HH and VV channels yields additional detection gain.

The extension of our model to the cross-polarization channels and the study of a optimal polarimetric detectors seem to be a promising future work.

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