# JOINT ESTIMATION OF SIGNAL AND NOISE CORRELATION MATRICES AND ITS APPLICATION TO INVERSE FILTERING

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## ABSTRACT

Noise suppression by linear filters for a time series is discussed. We propose a method for jointly estimating signal and noise correlation matrices by incorporating steering vectors of the noise or eigenvectors of the noise correlation matrix as well as steering vectors of the target signals. Our estimates bring us two significant advantages. One is reduction of computational cost in obtaining the Wiener filter since the Wiener post filter, which is combined to the minimum variance distortionless response filter (MVDRF), is no longer needed with the estimates of signal and noise correlation matrices. The other is an improvement of the performance of the MVDRF since we can construct the regularized version of it with an estimate of the noise correlation matrix.

*Index Terms*— noise suppression, linear filter, correlation matrix, Wiener filter

### 1. INTRODUCTION

Noise suppression for a time series is one of important topics in the field of speech and acoustic signal processing. As well known, one of popular methods for suppressing additive noise is the spectral subtraction (SS). Although this method effectively suppresses the noise, we have some difficulties to use it since it requires an estimate of the power of the noise and may generate a particular noise so-called 'a musical noise'. A sensor-array-based noise suppression scheme is known as one of resolutions for these problems. The minimum variance distortionless response filter (MVDRF) and the Wiener filter (WF), which is implemented as the combination of the MVDRF and the Wiener post filter (WPF), are representative ones in the scheme. One of common and remarkable features of these methods is that they do not require the correlation structure of the noise.

Recently, Ono et al. clarified that eigenvectors of noise correlation matrices for diffuse noise fields are invariant with specific crystal-shape arrays [1, 2, 3]. On the basis of the knowledge, they also succeeded in improving the performance of the WF. Moreover, recent progress in DOA estimation enables us to obtain accurate steering vectors for isolated noise

sources. As shown above, there exist cases where we have partial information of noise correlation matrices.

In this paper, we propose a method for jointly estimating signal and noise correlation matrices by using the partial information of the noise correlation matrix such as the eigenvectors of the noise correlation matrix or the steering vectors of the noise. We also discuss two significant advantages that these estimates may bring us. One is reduction of computational cost in obtaining the WF since we can directly design the WF with these estimates, which implies that the WPF is no longer needed. The other is an improvement of the performance of the MVDRF since we can construct the regularized version of it with an estimate of the noise correlation matrix.

## 2. INVERSE FILTERING

Let  $n, m \ (m \leq n)$ , and t be the number of observations, the number of target signals, and the time index (or the frame index in the short time Fourier domain), respectively. Let  $s(t) \in \mathbf{C}^m$ ,  $n(t) \in \mathbf{C}^n$ , and  $A \in \mathbf{C}^{n \times m}$  be an unknown target signal vector, an observation noise vector, and an given observation matrix consisting of steering vectors of s(t) (or corresponding to a mixing matrix related with impulse responses between the sources and the sensors) with rank(A) = m, where  $\mathbf{C}^n$  and  $\mathbf{C}^m$  are *n*-dimensional and *m*-dimensional unitary spaces called the observation signal space and the original signal space. Note that we omit the frequency bin index since the following contents does not depend on it. We assume that an observation vector  $\mathbf{x}(t) \in \mathbf{C}^n$  is given by the following model:

$$\boldsymbol{x}(t) = A\boldsymbol{s}(t) + \boldsymbol{n}(t). \tag{1}$$

The aim of the inverse filtering is to obtain the signal  $\boldsymbol{y}(t)$  written as

$$\boldsymbol{y}(t) = W\boldsymbol{x}(t),\tag{2}$$

that is as closer to s(t) as possible, where the matrix  $W \in \mathbb{C}^{m \times n}$  denotes an inverse filter of A.

The MVDRF is one of popular inverse filters and is defined as

$$W_{MVDRF} = \operatorname{argmin}_{W} E_{\boldsymbol{n}} ||W\boldsymbol{x}(t)||^2 \tag{3}$$

subject to  $WA = I_m$ , where  $E_n$ ,  $|| \cdot ||$ , and  $I_m$  denote the mathematical expectation over n, the Euclidean norm of a vector, and the identity matrix of degree m, respectively. The closed-form solution of the MVDRF is written as

$$W_{MVDRF} = (A^* X^{-1} A)^{-1} A^* X^{-1}, \qquad (4)$$

where X denotes the correlation matrix of the observations  $\boldsymbol{x}(t)$  and the superscript \* denotes the adjoint (conjugate and transposition) operator. In general, the observation noise  $\boldsymbol{n}(t)$  is usually uncorrelated with the target signals  $\boldsymbol{s}(t)$ . In such a case, the MVDRF is identical to the best linear unbiased estimator[4](BLUE) written as

$$W_{BLUE} = (A^* Q^{-1} A)^{-1} A^* Q^{-1}, (5)$$

where Q denotes the correlation matrix of the noise, which means that the MVDRF can minimize the variance of the noise without the information of the noise correlation matrix when the noise is uncorrelated with the target signals. Since the MVDRF is an unbiased estimator, the variance of the noise in the restored signal can not fall below the Cramer-Rao bound[5]. Thus in order to suppress the variance of the noise more effectively, we have to relax the unbiasedness of the solution. The parametric projection filter[6](PPF) may be one of the resolutions for the problem, which is a regularized version of the MVDRF in which the unbasedness of the solution is relaxed. The PPF is defined as

$$W_{PPF} = \operatorname{argmin}_{W}[\operatorname{tr}\{(WA - I_m)(WA - I_m)^*\} + \gamma E \boldsymbol{n} ||W \boldsymbol{n}(t)||^2], \quad (6)$$

where  $\gamma$  denotes a positive parameter that controls the tradeoff between two terms in the criterion, and its closed-form solution is written as

$$W_{PPF} = A^* (AA^* + \gamma Q)^{-1}.$$
 (7)

The closed-form solution Eq.(7), however, requires the estimate of the noise correlation matrix.

The WF is also a popular inverse filter and is defined as

$$W_{WF} = \operatorname{argmin}_{W} E_{\boldsymbol{s},\boldsymbol{n}} ||W\boldsymbol{x}(t) - \boldsymbol{s}(t)||^{2}, \qquad (8)$$

where  $E_{s,n}$  denotes the mathematical expectation over s and n. The closed-form solution of the WF is written as

$$W_{WF} = RA^* (ARA^* + Q)^{-1} = RA^* X^{-1}, \qquad (9)$$

where R denotes the correlation matrix of the target signals. In this paper, we assume that target signals are mutually uncorrelated, which implies that R is diagonal. The WF can be also represented by the combination of the MVDRF and the Wiener post filter (WPF) written as

$$W_{WF} = W_{WPF} W_{VMDRF}, \tag{10}$$

where the WPF is written as

$$W_{WPF} = RY^{-1} \tag{11}$$

with Y denoting the correlation matrix of the output of the MVDRF. We can effectively reduce the computational cost by this representation when we apply the WF adaptively to non-stationary target signals in case of m < n. As shown in Eqs.(9), (10) and (11), the WF requires the estimate of the correlation matrix of the target signal vector. In case of m = 1, Zelinski estimated the power of the target signal by the non-diagonal elements of the correlation matrix of the observations in [7], incorporating the fact that the cross-power spectral of the noise is nearly equal to zero when the distance between two sensors are far enough. However, this requirement for the sensors prevents us from adopting the method for signals including high-frequency components. On the other hand, Ono et al. clarified that eigenvectors of noise correlation matrices for diffuse noise fields are invariant with specific crystal-shape arrays [1, 2, 3]. On the basis of the knowledge, they succeeded in accurately estimating the power of the target signal in case of m = 1 even if the distance between sensors are comparatively small.

The WF has one more representation written as

$$W_{WF} = (A^*Q^{-1}A + R^{-1})^{-1}A^*Q^{-1}$$
(12)

as shown in [8]. Please refer to [8] for more details of the relationship between Eqs.(9) and (12). The computational cost for obtaining this representation is less than the representation of the combination of the MVDRF and the WPF since the computational cost to obtain Eq.(12) is nearly equal to that of the MVDRF and we do not have to calculate the signal correlation matrix Y of the output of the MVDRF. However, the representation Eq.(12) requires the estimate of the noise correlation matrix while the former two representations do not require it. A simple and straight-forward way to obtain an estimate of Q is calculating

$$\hat{Q} = X - A\hat{R}A^*,\tag{13}$$

where  $\hat{R}$  denotes an estimate of R. However, this estimate may be unreliable since all the errors are integrated to  $\hat{Q}$ .

## 3. JOINT ESTIMATION OF SIGNAL AND NOISE CORRELATION MATRICES

As shown in the previous section, Ono et al. proposed a method for estimating the power of the target signal in case of m = 1, incorporating the given eigenvectors of the noise correlation matrix with crystal-shape arrays. Moreover, recent progress in DOA estimation enables us to obtain accurate steering vectors for isolated noise sources. Thus, there exist cases where we have partial information of noise correlation matrices. In this section, we discuss the joint estimation

of signal and noise correlation matrices with the partial information of the noise correlation matrices as well as the steering vectors of the target signals.

We assume that the noise correlation matrix  $\boldsymbol{Q}$  can be written as

$$Q = P\Lambda P^*, \tag{14}$$

where P is a given matrix and  $\Lambda$  denotes a diagonal matrix. In the case of the crystal-shape arrays by Ono et al., P is the unitary matrix consisting of the eigenvectors of the noise correlation matrix. In the case where we have all steering vectors of the noise, P is the matrix consisting of the steering vectors. It is obvious that the correlation matrix of the observations can be written as

$$X = ARA^* + P\Lambda P^*, \tag{15}$$

where A and P are known *a priori*. Here, we introduce some useful notations and a proposition.

**Definition 1** [9] Let  $A = [a_1, ..., a_m]$ ,  $a_i \in \mathbb{C}^n$ , then the 'vec'-ed version of A is defined as

$$\operatorname{vec}(A) = [\boldsymbol{a}_1^*, \dots, \boldsymbol{a}_m^*]^* \in \mathbf{C}^{mn}.$$
 (16)

**Definition 2** [9] Let  $A \in \mathbb{C}^{p \times q}$  and  $B \in \mathbb{C}^{m \times n}$  be arbitrary matrices and  $B = (b_{ij})$ , then the Kronecker product of B and A is defined as

$$B \otimes A = \begin{bmatrix} b_{11}A & \cdots & b_{1n}A \\ \vdots & \ddots & \vdots \\ b_{m1}A & \cdots & b_{mn}A \end{bmatrix} \in \mathbf{C}^{pm \times qn}.$$
(17)

**Theorem 1** [9] Let A, B, and C be arbitrary matrices such that the product ABC is defined. Then,

$$\operatorname{vec}(ABC) = (C' \otimes A)\operatorname{vec}(B)$$
 (18)

holds, where the superscript ' denotes the transposition operator.

Applying Theorem 1 to Eq.(15) yields

$$\operatorname{vec}(X) = (\overline{A} \otimes A)\operatorname{vec}(R) + (\overline{P} \otimes P)\operatorname{vec}(\Lambda),$$
 (19)

where  $\overline{X}$  denotes the conjugate of X. Let  $Z_m$  denotes the linear operator that extracts only non-diagonal elements from 'vec'-ed version of an *m*-dimensional square matrix. For instance  $Z_3$  is written as

Since R and  $\Lambda$  are diagonal, they must satisfy

$$Z_m \operatorname{vec}(R) = \mathbf{0}, \qquad (21)$$

$$Z_n \operatorname{vec}(\Lambda) = \mathbf{0}, \qquad (22)$$

where **0** denotes the zero vector with an appropriate size. Thus, the unknown vectors vec(R) and  $vec(\Lambda)$  must satisfy the equation

$$G\begin{bmatrix} \operatorname{vec}(R)\\ \operatorname{vec}(\Lambda) \end{bmatrix} = \begin{bmatrix} \operatorname{vec}(X)\\ \mathbf{0}\\ \mathbf{0} \end{bmatrix}, \quad (23)$$

where

$$G = \begin{bmatrix} (\overline{A} \otimes A) & (\overline{P} \otimes P) \\ Z_m & O \\ O & Z_n \end{bmatrix}$$
(24)

and O denotes the zero matrix with an appropriate size. The least-squares minimum norm solution of Eq.(23) is given by

$$\begin{bmatrix} \operatorname{vec}(\hat{R}) \\ \operatorname{vec}(\hat{\Lambda}) \end{bmatrix} = G^+ \begin{bmatrix} \operatorname{vec}(X) \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad (25)$$

where the superscript <sup>+</sup> denotes the Moore-Penrose generalized inverse[4]. Note that the matrix G is usually of full column rank except the special cases such as A and P share the same steering vectors. Thus, Eq.(25) gives an unbiased estimator in many practical cases. Finally  $\hat{R}$  gives an estimate of R and  $P\hat{\Lambda}P^*$  gives an estimate of Q.

### 4. COMPUTER SIMULATIONS

In this section, we numerically investigate the accuracy of the proposed estimates and the performance of inverse filters using our estimates. Let m = 1, n = 4, and we randomly generate the matrices A, Q, and R. The condition m = 1 is needed for comparison with the Ono's method[3], in which the power of the target signal is estimated by the least-squares scheme using non-diagonal elements of the correlation matrix of the observations transformed by the given eigenvectors of the noise correlation matrix. We also generate 1-dimensional and 4-dimensional *i.i.d.* zero-mean white noises with 10,000 samples, written as  $\varepsilon_1(t)$  and  $\varepsilon_4(t)$  with the unit variance and generate s(t) and n(t) by

$$\boldsymbol{s}(t) = R^{1/2} \boldsymbol{\varepsilon}_1(t), \tag{26}$$

$$\boldsymbol{n}(t) = Q^{1/2} \boldsymbol{\varepsilon}_4(t). \tag{27}$$

Note that the correlation matrices of s(t) and n(t) calculated by using Eqs.(26) and (27) are not exactly identical to R and Q due to a small deviation of the white noises.

In case of m = 1, the constraint Eq.(21) vanishes. Thus the accuracy of the estimated R by the proposed method is identical to that based on Ono's method. We show the averaged accuracy (with the standard deviation) of the estimate  $\hat{Q}$ evaluated by

$$J(\hat{Q}) = \frac{\sqrt{\text{tr}[(Q - \hat{Q})^2]}}{\sqrt{\text{tr}[Q^2]}}$$
(28)

SNR of obs.	Conventional	Proposed
-15(dB)	$0.0167 \pm 0.0074$	$0.0144 \pm 0.0074$
-12(dB)	$0.0171 \pm 0.0075$	$0.0146 \pm 0.0075$
-9(dB)	$0.0178 \pm 0.0076$	$0.0151 \pm 0.0077$
-6(dB)	$0.0190 \pm 0.0081$	$0.0161 \pm 0.0081$
-3(dB)	$0.0210 \pm 0.0092$	$0.0176 \pm 0.0090$
0(dB)	$0.0245 \pm 0.0114$	$0.0203 \pm 0.0109$
3(dB)	$0.0300 \pm 0.0153$	$0.0245 \pm 0.0142$
6(dB)	$0.0386 \pm 0.0213$	$0.0311 \pm 0.0195$
9(dB)	$0.0515 \pm 0.0301$	$0.0411 \pm 0.0274$
12(dB)	$0.0703 \pm 0.0427$	$0.0558 \pm 0.0387$
15(dB)	$0.0975 \pm 0.0606$	$0.0771 \pm 0.0547$

**Table 1**. Estimation accuracy of  $\hat{Q}$  with respect to the SNR of the observations.

**Table 2**. Performance of inverse filters with the proposed estimates.

SNR of obs.	$s_{WF_P} - s_{WF_C},$	$s_{WF_T} - s_{WF_C}$
-15(dB)	$0.0030 \pm 0.0617$	$0.6224 \pm 2.4118$
-12(dB)	$0.0019 \pm 0.0920$	$0.5306 \pm 2.2378$
-9(dB)	$0.0062 \pm 0.1587$	$0.4870 \pm 2.1565$
-6(dB)	$0.0195 \pm 0.2696$	$0.4950 \pm 2.1807$
-3(dB)	$0.0424 \pm 0.4003$	$0.5603 \pm 2.3172$
0(dB)	$0.0753 \pm 0.5265$	$0.6916 \pm 2.5693$
3(dB)	$0.1216 \pm 0.6709$	$0.9049 \pm 2.9383$
6(dB)	$0.1780 \pm 0.8704$	$1.2234 \pm 3.4211$
9(dB)	$0.2458 \pm 1.1298$	$1.6767 \pm 4.0088$
12(dB)	$0.2864 \pm 1.8839$	$2.2995 \pm 4.6865$
15(dB)	$0.3249 \pm 2.5902$	$3.1312 \pm 5.4321$

over 1,000 trials in Table 1 for the proposed estimate and the conventional estimate shown in Eq.(13) with respect to the SNR of observations. According to Table 1, we can confirm that the proposed estimate of Q is better than the simple subtraction shown in Eq.(13).

Next, we investigate the performance of the inverse filters using the proposed estimates. Due to the page limitation, we concentrate the performance of the WF. Let  $s_{WF_C}$ ,  $s_{WF_P}$ , and  $s_{WF_T}$  be the SNRs of the Wiener filters implemented by the combination of the MVDRF and the WPF, Eq.(12) with our estimates, and Eq.(12) with the true R and the true Q. Table 2 shows the averaged values (with the standard deviation) of  $s_{WF_P} - s_{WF_C}$  and  $s_{WF_T} - s_{WF_C}$  over 1,000 trials with respect to the SNR of the observations. According to Table 2, the restored result by the WF with our estimates slightly outperforms the WF with conventional estimates. The difference from the WF with true correlation matrices is still large. However, this result seems to be fair since we can adopt Eq.(12) whose computational cost is comparatively small as the WF with the estimate of Q.

#### 5. CONCLUSION

In this paper, we proposed a method for jointly estimating the signal and the noise correlation matrices for a linear inversion problem of a time series, incorporating the partial information of the noise correlation matrix as well as the steering vectors of the target signals. We also investigated the performance of the estimates by computer simulations and confirmed that the results are fair.

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