

SUBSPACE BASED DOA ESTIMATION IN THE PRESENCE OF CORRELATED SIGNALS AND MODEL ERRORS

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ABSTRACT

High-resolution subspace based direction-of-arrival (DOA) estimation requires a number of assumptions about the signal and antenna. In our application automotive radar, among many practical problems, signals are often correlated and the antenna is not calibrated. This difficult combination has been seldom addressed in the literature. In this paper, we study their simultaneous impact on DOA estimation, describe a so called “coherent model error interference” phenomenon, propose a prewhitening scheme for algebraic subspace based DOA estimation, and show some simulation results.

Index Terms— automotive radar, high-resolution DOA estimation, calibration, model error, decorrelation

1. INTRODUCTION

Automotive radar sensors are used for many driver assistance and safety systems such as adaptive cruise control (ACC), lane change monitoring, brake assistant, collision warning and prevention. In present automotive radar systems, only targets in different distance and velocity cells can be resolved. The increasing demand on safety requirements leads to efforts improving the DOA estimation to allow resolution of targets even in the same distance-velocity cell. The DOA resolution using beamformers is poor since automotive radar have typically a low antenna aperture due to size and cost restriction. A natural solution is thus the use of well known high-resolution methods for DOA estimation, in particular the family of subspace based methods [1] because they are relatively easy to implement. These methods, however, require a number of assumptions about the signal and antenna which are, unfortunately, not always satisfied in automotive applications. At least for the radar sensor we currently develop, we are facing a number of practical problems: a small number of snapshots, colored radar clutter, sometimes strongly correlated signals (for standing ego and target vehicle), and model errors.

In this paper, we address a difficult situation, the simultaneous occurrence of correlated signals and signal model errors. Using calibration [2, 3, 4], we can correct the model errors, but at the expense of colored noise. Correlated signals can be dealt with decorrelation algorithms [1]. They rely on assumptions about the array structure, e.g. a uniform linear array (ULA). If these are violated, e.g. due to model errors, the performance degrades. We study this effect. The colored noise is also transformed by the decorrelation algorithms. We propose a prewhitening procedure for subspace estimation in combination with calibration and decorrelation.

The paper is organized as follows. Section 2 briefly reviews the concept of subspace based high-resolution DOA estimation. In

section 3, signal model errors and the concept of calibration are introduced. In section 4, we briefly review different decorrelation algorithms. In section 5, we study the effect of residual calibration errors when using decorrelation algorithms. A prewhitening procedure is proposed in section 6 to enable even algebraic subspace based methods for DOA estimation in combination with calibration and decorrelation. Section 7 shows some simulation results.

2. SUBSPACE BASED DOA ESTIMATION

We assume p far field narrow band source signals $\mathbf{s}(t)$ impinging on a ULA. The received signal $\mathbf{x}(t)$ can be modeled as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \quad (1)$$

where $\mathbf{A} = [\mathbf{a}(\theta_1) \dots \mathbf{a}(\theta_p)]$ is the steering matrix, $\mathbf{a}(\theta) = [1, e^{j\tau(\theta)}, \dots, e^{j(M-1)\tau(\theta)}]^T$ is the steering vector for the DOA θ , $\mathbf{s}(t)$ denotes the radar signals, and $\mathbf{n}(t)$ describes the sensor noise. In our automotive application, a ULA of $M = 8$ sensors is used and only a fairly small number of 12 snapshots is available for DOA estimation [5].

Let $\mathbf{R} = E[\mathbf{x}(t)\mathbf{x}^H(t)]$ be the sensor spatial correlation matrix. Under ideal assumptions of spatially i.i.d. sensor noise as well as uncorrelated source signals and sensor noise, efficient subspace based methods can be applied for high-resolution DOA estimation. Using an eigenvalue decomposition of \mathbf{R} , the signal subspace \mathbf{U}_s and noise subspace \mathbf{U}_n can be obtained:

$$\mathbf{R} = \mathbf{A}\mathbf{R}_s\mathbf{A}^H + \sigma^2\mathbf{I} = \mathbf{U}_s\mathbf{\Lambda}_s\mathbf{U}_s^H + \mathbf{U}_n\mathbf{\Lambda}_n\mathbf{U}_n^H. \quad (2)$$

$\mathbf{R}_s = E[\mathbf{s}(t)\mathbf{s}^H(t)]$ is the signal correlation matrix, $\mathbf{\Lambda}_s = \text{diag}(\lambda_1, \dots, \lambda_p)$ contains the p dominant signal eigenvalues and $\mathbf{\Lambda}_n = \text{diag}(\lambda_{p+1}, \dots, \lambda_M)$ the remaining noise eigenvalues. By exploiting the signal subspace matrix \mathbf{U}_s or noise subspace matrix \mathbf{U}_n with orthonormal columns, efficient subspace based DOA estimators like MUSIC and ESPRIT can be applied. For coherent signals, \mathbf{R}_s is rank deficient and \mathbf{U}_s has a rank smaller than p . By using decorrelation algorithms such as spatial smoothing and forward backward averaging [1], the original rank can be restored. This will be discussed in more details in section 4.

3. CALIBRATION

For a real antenna array, the true steering vector $\hat{\mathbf{a}}(\theta)$ deviates from the ideal one $\mathbf{a}(\theta)$. The array imperfection can be modeled by

$$\hat{\mathbf{a}}(\theta) = \mathbf{Q}\mathbf{a}(\theta) \quad (3)$$

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where \mathbf{Q} is a square calibration matrix. Gain and phase mismatches between different antenna elements can be modeled by a complex diagonal matrix \mathbf{Q}_{GP} . The mutual coupling between antenna elements is typically described by a full but diagonally dominant matrix \mathbf{Q}_{C} . Both \mathbf{Q}_{GP} and \mathbf{Q}_{C} do not depend on DOA. Angle-dependent model errors are due to sensor position inaccuracy [6] or a nonideal dielectric lens as in our case [7]. For simplicity, we first consider angle-independent model errors.

The signal model becomes then

$$\mathbf{x}(t) = \mathbf{Q}\mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) = \hat{\mathbf{A}}\mathbf{s}(t) + \mathbf{n}(t). \quad (4)$$

The first task is to estimate \mathbf{Q} from a number of calibration measurements, which are sensor signals for a single emitter at given calibration DOAs. After the estimation of the calibration matrix $\hat{\mathbf{Q}}$ [2, 3, 4], the second task is DOA estimation for new sensor signals based on $\hat{\mathbf{Q}}$.

There are two different ways to use the estimated calibration matrix $\hat{\mathbf{Q}}$ [7]. The first approach is to process the received sensor vector $\mathbf{x}(t)$ as usual:

- 1) estimate the noise subspace matrix $\hat{\mathbf{U}}_n$ from the correlation matrix of $\mathbf{x}(t)$.
- 2) use $\hat{\mathbf{Q}}$ to correct the array manifold $\hat{\mathbf{Q}}\mathbf{a}(\theta)$.
- 3) use the corrected array manifold $\hat{\mathbf{Q}}\mathbf{a}(\theta)$ in a DOA spectrum like MUSIC

$$m(\theta) = \frac{\|\hat{\mathbf{Q}}\mathbf{a}(\theta)\|^2}{\|\hat{\mathbf{U}}_n^H \hat{\mathbf{Q}}\mathbf{a}(\theta)\|^2}. \quad (5)$$

This approach is called “manifold-correction”. Its main advantage is that the noise $\mathbf{n}(t)$ remains spatially white. This simplifies the subspace discrimination and order estimation. The main drawback is that decorrelation algorithms and algebraic DOA estimators like rooting methods and ESPRIT cannot be applied since we lose the ULA property due to $\hat{\mathbf{Q}}\mathbf{a}(\theta)$ even if the ideal array is a ULA.

The second approach is to restore the ideal steering vector by using the inverse calibration matrix:

- 1) apply the inverse calibration matrix to the received data vector:

$$\begin{aligned} \tilde{\mathbf{x}}(t) &= \hat{\mathbf{Q}}^{-1}\mathbf{x}(t) = \hat{\mathbf{Q}}^{-1}\mathbf{Q}\mathbf{A}\mathbf{s}(t) + \hat{\mathbf{Q}}^{-1}\mathbf{n}(t) \\ &\approx \mathbf{A}\mathbf{s}(t) + \tilde{\mathbf{n}}(t) \end{aligned} \quad (6)$$

- 2) apply decorrelation algorithm to (6) if necessary.
- 3) estimate the signal or noise subspace from the new correlation matrix

$$\tilde{\mathbf{R}} = \mathbb{E}(\tilde{\mathbf{x}}(t)\tilde{\mathbf{x}}^H(t)) \approx \mathbf{A}\mathbf{R}_s\mathbf{A}^H + \tilde{\mathbf{R}}_n \quad (7)$$

where $\tilde{\mathbf{n}}(t)$ is now a colored noise and apply DOA spectrum or algebraic methods for DOA estimation, see section 5.

This calibration method is referred to as “data-correction”. Its main advantage is the restored ULA property of \mathbf{A} .

4. DECORRELATION

Decorrelation algorithms such as spatial smoothing (SS) and forward backward averaging (FBA) [1] are used to restore the rank of the signal correlation matrix \mathbf{R}_s when signals are highly correlated or coherent. They make use of array symmetries such as shifting and centrosymmetry. For a ULA, both methods are applicable. Without loss of generality, we study one group of coherent signals below, i.e.

$\text{rank}(\mathbf{R}_s) = 1$ regardless of p . The correlation matrix using FBA to the ideal signal model (2) is

$$\begin{aligned} \mathbf{R}_{\text{fb}} &= \frac{1}{2}(\mathbf{R} + \mathbf{J}\mathbf{R}^*\mathbf{J}) \\ &\stackrel{\text{ULA}}{=} \frac{1}{2}\mathbf{A} \left(\mathbf{R}_s + \mathbf{D}^{-(M-1)}\mathbf{R}_s^*(\mathbf{D}^{-(M-1)})^H \right) \mathbf{A}^H + \sigma^2\mathbf{I} \end{aligned} \quad (8)$$

where \mathbf{J} is an M -by- M exchange matrix, whose components are zero except for ones on the anti-diagonal and $\mathbf{D} = \text{diag}(e^{j\tau(\theta_1)}, \dots, e^{j\tau(\theta_p)})$. Obviously, the rank of the smoothed signal correlation matrix can be increased at most by one, allowing a maximum of two coherent signals to be decorrelated.

Using SS with K overlapping subarrays containing each $L = M - K + 1$ sensor elements, the correlation matrix is obtained as

$$\begin{aligned} \mathbf{R}_{\text{ss}} &= \frac{1}{K} \sum_{k=1}^K \mathbf{R}_k \\ &\stackrel{\text{ULA}}{=} \mathbf{A}_L \left(\frac{1}{K} \sum_{k=1}^K \mathbf{D}^{(k-1)} \mathbf{R}_s (\mathbf{D}^{(k-1)})^H \right) \mathbf{A}_L^H + \sigma^2\mathbf{I} \end{aligned} \quad (9)$$

where \mathbf{R}_k is the correlation matrix of the k -th subarray and \mathbf{A}_L is the steering matrix for a ULA with L elements. With SS, the rank of the smoothed signal correlation matrix can be increased by at most $K - 1$, which is also the size of the dimension reduction of the smoothed correlation matrix.

Using a combination of both methods, called forward backward spatial smoothing (FBSS), the correlation matrix is computed as

$$\begin{aligned} \mathbf{R}_{\text{fbss}} &= \frac{1}{2K} \sum_{k=1}^K (\mathbf{R}_k + \mathbf{J}\mathbf{R}_k^*\mathbf{J}) \\ &\stackrel{\text{ULA}}{=} \mathbf{A}_L \frac{1}{2K} \sum_{k=1}^K \left(\mathbf{D}^{(k-1)} \mathbf{R}_s (\mathbf{D}^{(k-1)})^H + \right. \\ &\quad \left. \mathbf{D}^{(-M+k)} \mathbf{R}_s^* (\mathbf{D}^{(-M+k)})^H \right) \mathbf{A}_L^H + \sigma^2\mathbf{I}. \end{aligned} \quad (10)$$

Here, the rank of the smoothed signal correlation matrix can be increased by at most $2K - 1$. Table 1 summarizes the maximum number of coherent signals that can be decorrelated.

Algorithm	Max. no. coherent signals
FBA	2
SS	K
FBSS	$2K$

Table 1. Properties of decorrelation algorithms

5. PREWHITENING FOR DOA ESTIMATION AFTER DATA-CORRECTION CALIBRATION

One problem after the data-correction calibration in (6) is the colored noise. The sensor correlation matrix is given in (7) with

$$\tilde{\mathbf{R}}_n = \hat{\mathbf{Q}}^{-1} \mathbb{E}(\mathbf{n}(t)\mathbf{n}^H(t)) \hat{\mathbf{Q}}^{-H} = \tilde{\mathbf{R}}_n^{1/2} \tilde{\mathbf{R}}_n^{H/2} \quad (11)$$

where $\tilde{\mathbf{R}}_n^{1/2}$ is the matrix square root of $\tilde{\mathbf{R}}_n$. $\tilde{\mathbf{R}}_n$ is given if we know $\mathbb{E}(\mathbf{n}(t)\mathbf{n}^H(t))$, e.g. if $\mathbf{n}(t)$ is spatially white. Below we propose one prewhitening procedure which enables even the application of algebraic subspace DOA estimators after calibration and decorrelation:

- 1) estimate the calibration matrix $\hat{\mathbf{Q}}$ from calibration measurements and apply data-correction calibration as in (6).
- 2) apply decorrelation to (6) as described in section 4. The new sensor correlation matrix has the same structure as in (7) with modified \mathbf{A} , \mathbf{R}_s , and $\tilde{\mathbf{R}}_n$, see section 4. The new $\tilde{\mathbf{R}}_n$ after decorrelation can be computed by inserting (11) into the first line of (8), (9) or (10). Let $\tilde{\mathbf{R}}_n^{1/2}$ be the matrix square root of this new noise correlation matrix after calibration and decorrelation.
- 3) perform a prewhitening with $\mathbf{W} = \tilde{\mathbf{R}}_n^{-1/2}$ on (7):

$$\begin{aligned}\mathbf{W}\tilde{\mathbf{R}}\mathbf{W}^H &\approx \mathbf{W}\mathbf{A}\mathbf{R}_s\mathbf{A}^H\mathbf{W}^H + \mathbf{W}\tilde{\mathbf{R}}_n\mathbf{W}^H \\ &= \mathbf{W}\mathbf{A}\mathbf{R}_s\mathbf{A}^H\mathbf{W}^H + \mathbf{I}\end{aligned}\quad (12)$$

- 4) compute the signal subspace $\tilde{\mathbf{U}}_s$ of $\mathbf{W}\tilde{\mathbf{R}}\mathbf{W}^H$ which spans the same column space as $\mathbf{W}\mathbf{A}$.
- 5) compute $\tilde{\mathbf{U}}_s = \mathbf{W}^{-1}\tilde{\mathbf{U}}_s = \tilde{\mathbf{R}}_n^{1/2}\tilde{\mathbf{U}}_s$ to restore the ULA structure of \mathbf{A} .
- 6) apply a DOA estimator (e.g. ESPRIT) to $\tilde{\mathbf{U}}_s$ for DOA estimation.

6. COHERENT MODEL ERROR INTERFERENCE

We now study the simultaneous occurrence of model errors and correlated signals. Since the estimated calibration matrix is not perfect, we can write

$$\hat{\mathbf{Q}}^{-1}\mathbf{Q}\mathbf{A} = \mathbf{A} + \mathbf{E} \quad (13)$$

where \mathbf{E} accounts for the residual errors after calibration. As the steering vectors of a ULA are linearly independent for $d \leq \lambda/2$, it is always possible to rewrite each of the p columns of the error matrix \mathbf{E} as a linear combination of the same M linearly independent steering vectors $\mathbf{a}(\theta_{E,\nu})$ at some fictitious DOAs $\theta_{E,\nu}$ ($1 \leq \nu \leq M$),

$$\mathbf{E} = \left[\sum_{\nu=1}^M \alpha_{1,\nu} \mathbf{a}(\theta_{E,\nu}), \dots, \sum_{\nu=1}^M \alpha_{p,\nu} \mathbf{a}(\theta_{E,\nu}) \right]. \quad (14)$$

The choice of $\theta_{E,\nu}$ is arbitrary. Therefore, we can rewrite (6) as

$$\begin{aligned}\tilde{\mathbf{x}}(t) &= (\mathbf{A} + \mathbf{E})\mathbf{s}(t) + \hat{\mathbf{Q}}^{-1}\mathbf{n}(t) \\ &= \sum_{k=1}^p \left(\mathbf{a}(\theta_k) + \sum_{\nu=1}^M \alpha_{k,\nu} \mathbf{a}(\theta_{E,\nu}) \right) s_k(t) + \hat{\mathbf{Q}}^{-1}\mathbf{n}(t).\end{aligned}\quad (15)$$

Obviously, the residual calibration error can be interpreted as additional coherent signals because the steering vectors $\mathbf{a}(\theta_{E,\nu})$ and $\mathbf{a}(\theta_k)$ in (15) share the same random signal $s_k(t)$. Without decorrelation, the residual calibration error just represents a small additional error leading to a subspace estimate with small errors. With decorrelation, however, the residual calibration error is equivalent to additional coherent interferences. As the number of coherent interferences is now as high as the number of sensor elements (assuming $\alpha_{k,\nu} \neq 0$), we get the maximum possible number of decorrelated signals according to Table 1. Of course, the strength and the “direction” of these artificial interferences depend on the residual calibration error \mathbf{E} which itself is a function of the steering matrix \mathbf{A} . This phenomenon is called coherent model error interference. It only happens if we apply decorrelation algorithms *and* the signal model suffers from model or, after calibration, residual calibration errors. It will limit the performance of DOA estimation depending on the magnitude of \mathbf{E} .

To illustrate this effect, the eigenvalues of a simulated correlation matrix are depicted in Fig. 1. There is only one strong signal

with a signal to noise ratio (SNR) of 40dB in Fig. 1(a) and 90dB in Fig. 1(b). There is also a small model error \mathbf{E} with $20 \log_{10} \|\mathbf{E}\| / \|\mathbf{A}\| = -18.2\text{dB}$. We used 12 snapshots to estimate the correlation matrix. The eigenvalues are shown in logarithmic scale for the original correlation matrix \mathbf{R} and for the FBSS correlation matrix \mathbf{R}_{fbss} using two subarrays ($K = 2$). For 40dB SNR, the sensor noise is stronger than the coherent model error interferences and there is only one large signal eigenvalue in both \mathbf{R} and \mathbf{R}_{fbss} . If SNR = 90dB, \mathbf{R} shows only one dominant eigenvalue, while \mathbf{R}_{fbss} shows four eigenvalues considerably larger than the sensor noise level, which is the maximum number of coherent signals that can be decorrelated according to Table 1. These “new” signal eigenvalues (no. 2 to 4 in

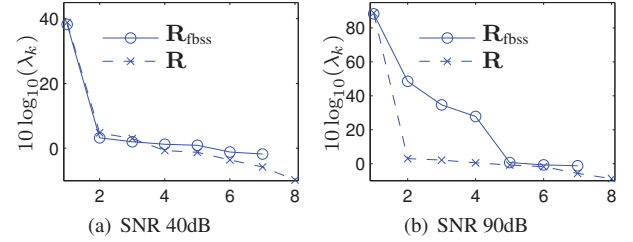


Fig. 1. Eigenvalues for a single target plus residual calibration errors with and without FBSS decorrelation ($K = 2$).

Fig. 1(b)) are called “artificial model error eigenvalues”. They are the result of applying decorrelation algorithms to the model errors. If one of the two factors, decorrelation due to correlated signals and calibration due to array imperfection, is missing, there will be no artificial model error eigenvalues. If those artificial eigenvalues are higher than the sensor noise level, they will degrade the subspace discrimination and DOA estimation. In particular, we can roughly estimate the maximum SNR difference of the wanted signals that still can be resolved. In Fig. 1(b), for example, the artificial model error eigenvalues are about -40dB below the strongest signal eigenvalue. If the DOA estimator is capable to resolve two signals when both of them have a SNR of at least 15dB , then the maximum SNR difference between both signals is 25dB , no matter how high the absolute SNR of both signals is with respect to sensor noise. Therefore, the system performance regarding SNR difference and the DOA resolution capability are greatly limited by model errors or residual calibration errors. Note that angle-dependent model errors can also be modeled as residual calibration error like in (13).

7. SIMULATIONS AND RESULTS

We use MATLAB to simulate calibration measurements, calibration, and DOA estimation under typical automotive radar conditions. The antenna is an eight element ULA with element spacing $d = \lambda$ for a DOA range of interest $[-8^\circ, 8^\circ]$ (long range radar). The array imperfection is described by the calibration matrix $\mathbf{Q} = \mathbf{Q}_{\text{GP}}\mathbf{Q}_{\text{C}}$. \mathbf{Q}_{GP} is a diagonal matrix modeling the gain and phase mismatch. While the gain mismatch is modeled by a lognormal distribution with zero mean and standard deviation 1dB , the phase mismatch is uniformly distributed over $[0, 2\pi]$. The coupling matrix \mathbf{Q}_{C} contains ones on the main diagonal. All other elements are lognormally distributed in amplitude and uniformly distributed over $[0, 2\pi]$ in phase. The mean of lognormal distribution is -10dB for coupling between direct neighbours and -15dB for the rest. The standard deviation is

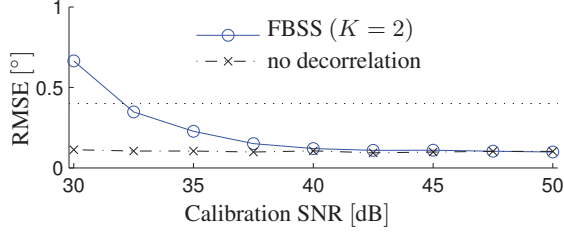


Fig. 2. Decorrelation treats residual calibration errors as coherent signals and degrades the DOA estimation

2dB in both cases. We use 12 snapshots in both calibration measurements and DOA estimation. The simulated calibration measurements are taken in the DOA range $[-20^\circ, 20^\circ]$ with the DOA step 1° . The calibration algorithm by Kortke [4], which outperforms the calibration algorithm by Pierre and Kaveh [2] as well as by See [3], is used to estimate the calibration matrix \mathbf{Q} [7]. We perform two experiments for DOA estimation. In each experiment, we calculate the root mean squared error (RMSE) from all DOA estimates of 250 trials. Table 2 summarizes the signal conditions and algorithm details.

	Experiment 1	Experiment 2
no. of signals	2	2
DOA	$\theta_1 = -8^\circ, -7^\circ, \dots, 8^\circ, \theta_2 = \theta_1 + 3^\circ$	$\theta_1 = -8^\circ, -7^\circ, \dots, 8^\circ, \theta_2 = \theta_1 + 3^\circ$
SNR	60dB, 20dB	equal, x-axis
correlation	uncorrelated	coherent
sensor noise	spatially white	spatially white
calibration SNR	x-axis	50dB
calibration	data-correction	data-correction
decorrelation	no / FBSS ($K = 2$)	FBSS ($K = 2$)
prewhitening	yes	no/yes
DOA estimator	TLS-ESPRIT	TLS-ESPRIT

Table 2. Signal conditions and algorithm details

In the first experiment, we study the performance degradation of artificial model error interference on DOA estimation (Fig. 2). As the SNR in calibration measurements varies, the accuracy of the estimation calibration matrix $\hat{\mathbf{Q}}$ and hence the residual calibration error changes. Without decorrelation, there is no problem in subspace discrimination and DOA estimation of two uncorrelated signals. If we, however, apply decorrelation to the same uncorrelated signals, the decorrelation treats the residual calibration errors as coherent signals and raises some noise eigenvalues to artificial model error eigenvalues, making the DOA estimation of the weak signal (20dB) difficult. The conclusions from this experiment are: 1) For strongly correlated or coherent signals, we of course have to apply decorrelation. But in the presence of model errors or residual calibration errors, the decorrelation will introduce disturbing model error eigenvalues and limit the performance of DOA estimation. Hence no decorrelation should be applied if the signals are uncorrelated. 2) If both correlated signals with different SNR and model errors occur simultaneously, we have to either enhance the calibration accuracy or switch to more complex DOA estimators like maximum-likelihood (ML) methods, which are better capable of dealing with correlated signals. However, the model errors will also limit the performance of ML methods.

In the second experiment, we study the effect of prewhitening

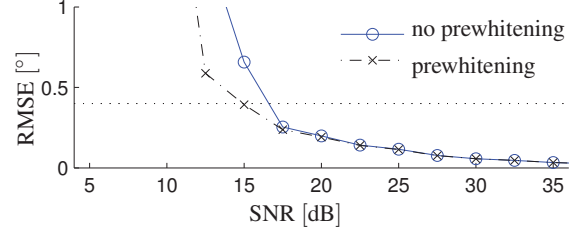


Fig. 3. Prewhitening after data-correction calibration

on DOA estimation (Fig. 3). Data-correction and FBSS decorrelation are applied to the array output of two coherent signals. This changes the spatially white sensor noise to a colored one. Without the prewhitening as described in section 5, the DOA estimation becomes worse for low SNR. With the prewhitening, the threshold region of SNR is shifted to the left considerably. For a RMSE of 0.4° as required in our automotive radar, there is a SNR improvement of 1.7dB. The improvement is even larger for larger model errors as shown in other simulations which we can not include due to limited space.

8. CONCLUSIONS

In this paper, we studied subspace based DOA estimation in the presence of both correlated signals and model errors. We described the phenomenon of coherent model error interferences. Decorrelation algorithms interpret model errors or residual calibration errors as coherent signals and degrade the DOA estimation if the signals have different power. We also proposed one prewhitening procedure which enables subspace based DOA estimation even in combination with data-correction calibration and decorrelation.

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