DIRECTION OF ARRIVAL ESTIMATION USING SINGLE TRIPOLE RADIO ANTENNA

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ABSTRACT

We consider the problem of estimating the Direction of Arrival (DOA) of multiple waves incident on a single tripole sensor. Using the physical properties of the electric and magnetic fields, we show that we can disambiguate the DOA of multiple simultaneously incident waves using a set of time sampled 3D measurements from the single sensor. This is different from the traditional approach that uses arrays of antennas to estimate DOA. We use the Unitary Matrix Pencil method to estimate the frequencies of the waves, and use a least squares solver to estimate the amplitude and phase coefficients. We combine these to compute the DOA and evaluate the approach using simulations. We show that the method is very effective at estimating DOA for different numbers of incident waves, and different noise levels.

Index Terms— Direction of Arrival, Radio Astronomy, Matrix Pencil Method

1. INTRODUCTION

Radio astronomy has led to several key discoveries over the years, as waves penetrate much of the gas and dust in space as well as the clouds of planetary atmospheres and pass through the terrestrial atmosphere with little distortion. There is an overwhelming interest in the scientific community [1] to significantly expand (by several orders of magnitude) the exploration of the radio signal spectrum to image and understand the transient sky, to probe accretion onto black holes; to identify orphan gamma-ray burst afterglows; and finally to discover new and unknown transient phenomena from currently undiscovered celestial objects.

A key problem in this field involves identifying the direction of arrival of an electromagnetic signal incident on the radio antenna sensor - especially in the presence of multiple such signals, interference and noise. When multiple signals are incident on the antenna sensor, the result is a superposed combination, making it hard to identify the individual signals of interest. The problem of DOA estimation is related to Blind Source Separation, that has been studied in the astronomy and radar communities. Current approaches for DOA estimation D. S. Turaga, O. Verscheure and A. Biem

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use a spatially distributed array of multiple sensors (antennas) to disambiguate the multiple signals of interest¹. With the use a multiple sensors, various techniques are available for DOA estimation. Prior work includes work by [2] [3] and can be categorized into: Phase-based inteferometry methods, Eigen decomposition methods, and machine learning techniques. There is also a large body of related work on vector sensor [4], and 2D frequency estimation [5].

Phase interferometry (PI) based methods [6] use a measured phase differences across an array of sensors to estimate the DOA. These approaches have been successful for simpler radar signals, and have had limited appeal in radio astronomy applications. Decomposition techniques [7] exploit correlations inherent in time-dependent signals to estimate the components and directions of the incoming signals. These approaches include the Multiple Signal Classification (MU-SIC) algorithm [8], Maximum Likelihood Methods, and the ESPRIT algorithm for narrow-band planar signals [9]. These approaches offer asymptotically unbiased estimates of the direction of the irradiating sources, but are computationally expensive and not easily implemented in a real-time environment. The success of machine learning techniques is contingent on the availability of a sufficiently large training data set, especially for large-scale radio astronomy observations.

With the new interest in low frequency radio wave in astronomy and physic, as exemplified by the Low Frequency Array (LOFAR) with its centre in the Netherlands and the upcoming Square Kilometre Array (SKA), we can not expect a linear wave front through the array for all objects of interest. Unlike these prior approaches that rely on an antenna array, we focus on DOA estimation using a single tripole antenna. Our work is motivated by recent advances in antenna design that have led to the development of practical tripole antennas [10] that receive all 3 components (dimensions) of the electromagnetic signals in terms of the resulting electric field or generated current. It has been shown [11] that for a single incident wave, the DOA may be directly computed from a measurement of these three components. However, when

¹The number of sensors determines the accuracy of the estimate as well as the number of individual waves that may be disambiguated.

multiple waves with different frequencies are incident on the antenna, the resulting superposed wave exhibits a DOA that varies over time with a certain periodicity - even if the original individual waves have a fixed DOA. In this paper we present a method to disambiguate the DOA of multiple waves simultaneously incident on a single tripole antenna sensor. We use multiple temporal observations of the superposed waves, and use the Unitary Matrix Pencil (UMP) method [2] combined with Least Squares method for linear equations to identify the DOA for each incident wave. We present simulation results that demonstrate the technique on different number of observed waves, as well as different noise levels and highlight its accuracy and robustness to noise. This paper is organized as follows. We introduce our notation and background in Section 2. We then describe the details of our algorithm in Section 3. We present experimental results in Section 4 and conclude in Section 5.

2. BACKGROUND AND NOTATION

The electrical components of the electromagnetic field for a signal with a carrier frequency ν can be described by equation (1).

$$\vec{E}(t) = \left\{ A_x e^{i\theta_x}, A_y e^{i\theta_y}, A_z e^{i\theta_z} \right\} e^{i(\nu t)} \tag{1}$$

where A_x , A_y , and A_z represent the amplitude in the three dimensions ² and θ_x , θ_y and θ_z represent the corresponding phase.

From the Stokes parameter V [1] which describes the circular polarisation of the wave, we can define the 3D generalised vector \vec{V} as

$$\vec{V}(t) = -\frac{\epsilon_0}{2c} Im \left(\vec{E}(t) \times \vec{E}^{\star}(t) + c^2 \vec{B}(t) \times \vec{B}^{\star}(t) \right) \quad (2)$$

 $\vec{V}(t)$ represents the DOA of the incoming radio wave, where $Im(\cdot)$ represents an operator that extracts the imaginary part of a vector. $\vec{B}(t)$ is the magnetic field related the electric field \vec{E} . We assume transversal fields, where $\vec{E}(t) \times \vec{E}^{\star}(t) \simeq c^2 \vec{B}(t) \times \vec{B}^{\star}(t)$, which gives

$$\vec{V}(t) \simeq -Im\left(\vec{E}(t) \times \vec{E}^{\star}(t)\right)$$
 (3)

The equation may result in ambiguity in the estimate of $\pm \vec{V}(t)$ and non-conclusive DOA for linear waves – for which $\vec{V} = \vec{0}$ – which only occur in singular situations. When M such waves \vec{E}^m are simultaneously incident on a single antenna, the resulting superposed wave may be represented as:

$$\vec{E}_c(t) = \sum_{m=1}^{M} \vec{E}^m(t),$$
 (4)

or

$$\vec{E}_c(t) = \sum_{m=1}^M \left(A_x^m e^{i\theta_x^m}, A_y^m e^{i\theta_y^m}, A_z^m e^{i\theta_z^m} \right) e^{i\nu^m t}.$$
 (5)

We now focus on the X component $\left(\vec{E}_c\right)_{\tau}$:

$$\left(\vec{E}_c(t)\right)_x = \sum_{m=1}^M A_x^m e^{i\theta_x^m} e^{i\nu^m t}.$$
 (6)

Consider now that we observe N samples of this superposed wave, at time instances $t_0 \cdots t_{N-1}$. Without loss of generality we may assume these are evenly sampled with $t_n = t_0 + n\delta$, where $0 \le n \le N - 1$. We may rewrite eq. (6) in terms of a discrete signal X[n] as

$$X[n] = \sum_{m=1}^{M} A_x^m e^{i\theta_x^m} e^{i\nu^m (n\delta + t_0)} = \sum_{m=1}^{M} \left(A_x^m e^{i\theta_x^m} e^{i\nu^m t_0} \right) e^{i\nu^m n\delta}.$$
 (7)

In order to estimate the DOA for individual waves, we need to solve for their frequency, amplitude, as well as phase components for all three dimensions.

3. DOA ESTIMATION ALGORITHM

We use the UMP, a variant of the Matrix Pencil method [12] to solve for individual frequencies ν^m , while we solve a set of overspecified linear equations using the Least Squares Method to determine jointly the coefficients comprising phase θ_x^m and amplitudes A_x^m . We describe this in more detail in the following sub-sections.

3.1. Estimating Frequency

The UMP may be used to solve equations of the type

$$X[n] = \sum_{m=1}^{M} K_m \left(\alpha_m\right)^n \tag{8}$$

for the complex parameters α_m . It has been used to estimate DOA for phased antenna arrays, where the index n represents different antenna sensors. In this paper we use the method to solve for complex exponentials for time sampled signals from the same tripole antenna, as is clear by comparing equation (7) and (8). We only provide an outline of the UMP itself, more details can be obtained from [2]. The first step involves constructing an $(N - L) \times (L + 1)$ Hankel matrix whose columns are windowed versions of the original data, i.e.

$$\mathbf{Y} = \begin{bmatrix} X[0] & X[1] & \cdots & X[L] \\ X[1] & X[2] & \cdots & X[L+1] \\ \vdots & \vdots & \ddots & \vdots \\ X[N-L-1] & X[N-L] & \cdots & X[N-1] \end{bmatrix}.$$

²We assume these are not time varying - which is possible when the waves have angular rotation velocity.

The parameter L is labeled a pencil parameter and is selected for efficient noise filtering. The performance of the UMP method is controlled by parameters M, N and L. In order for the UMP to provide accurate estimates for the frequency parameters, we need to have the relationship $M \leq L \leq N - M$. This also implies that to detect M signals, we need to have at least 2M time samples. A typically recommended value for the pencil parameter is N/3 < L < N/2 [2]. The matrix Y may be shown to be centro-hermitian and we can determine a unitary transform matrix U that converts this into a purely real matrix, $\mathbf{X}_{R} = \mathbf{U}^{H} \mathbf{Y} \mathbf{U}$. A key advantage of this property is that \mathbf{X}_{R} may then be used for all computations, reducing complexity by a factor of 4. An SVD of \mathbf{X}_R is performed to compute \mathbf{A}_S , which contains the *M* largest singular vectors of \mathbf{X}_R . We then compute the M generalized eigenvalues $\gamma_1, \cdots, \gamma_M$ of the matrix $\left[Re\left(\mathbf{U}^{H}\mathbf{J}_{1}\mathbf{U}\right)\mathbf{A}_{S}\right]^{-1}Im\left(\mathbf{U}^{H}\mathbf{J}_{1}\mathbf{U}\right)\mathbf{A}_{S}$. The $(N-L-1) \times (N-L)$ matrix \mathbf{J}_1 is defined as

$$\mathbf{J}_{1} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

The frequencies ν^m may be directly computed from the resulting eigenvalues as $\nu^m = 2tan^{-1} (\gamma_m) / \delta$.

3.2. Estimating Coefficients Comprising Phase and Amplitude

Once the dominant frequencies are determined, we may construct a system of overspecified linear equations using (7) as:

$$\begin{bmatrix} X[0] \\ X[1] \\ \vdots \\ X[N-1] \end{bmatrix} = \mathbf{\Gamma} \begin{bmatrix} A_x^1 e^{i\nu^1 t_0} e^{i\theta_x^1} \\ A_x^2 e^{i\nu^2 t_0} e^{i\theta_x^2} \\ \vdots \\ A_x^M e^{i\nu^M t_0} e^{i\theta_x^M} \end{bmatrix}$$
(9)

with

$$\boldsymbol{\Gamma} = \begin{bmatrix} 1 & 1 & \dots & 1\\ \left(e^{i\nu^{1}\delta}\right) & \left(e^{i\nu^{2}\delta}\right) & \dots & \left(e^{i\nu^{M}\delta}\right)\\ \vdots & \vdots & \ddots & \vdots\\ \left(e^{i\nu^{1}\delta}\right)^{N-1} & \left(e^{i\nu^{2}\delta}\right)^{N-1} & \dots & \left(e^{i\nu^{M}\delta}\right)^{N-1} \end{bmatrix}$$

This system of linear equations may be solved using the Least Squares Method and SVD to get the resulting coefficients $A_x^m e^{i\nu^m t_0} e^{i\theta_x^m}$. While we cannot separate the phase from the amplitude, the computation of the DOA does not require that – as we can use this coefficient as is in the cross product.

3.3. Computing DOA or \vec{V}

Once we solve for the frequencies ν^m , and joint amplitudephase coefficients across all three dimensions, we have all the parameters needed to specify each individual signal \vec{E}^{m3} , and then may compute \vec{V}^m as in equation (3). For a staggered array of magnetic and electric tripole antennas we can use equation (2) for correct DOA under the assumption that the magnetic and the electric tripole antenna detect the same linear wavefront.

4. EXPERIMENTAL EVALUATION

We have implemented this algorithm for DOA, and an associated wave generator, in C++ using the Basic Linear Algebra Subprograms (BLAS) Library, the Gnu Statistical Library (GSL) and the Blitz package for object oriented scientific computing. We evaluate the performance of the algorithm under different settings with different values of M and different levels of noise. Firstly, under no noise, we have experimentally verified that setting $N \ge 2M + 2$ and $L \approx N/2$ we can perfectly disambiguate the DOA of each individual wave – when the waves have different frequencies.

We then present results under the presence of Additive White Gaussian Noise (AWGN) that is independent per dimension of the tripole antenna. In order to measure the accuracy of the DOA estimate, we compute the error in the resulting spherical coordinates, the Azimuth angle α and the Zenith angle ϕ . For a vector $\vec{V} = \{V_x, V_y, V_z\}$ these are defined as:

$$\alpha = \tan^{-1} \left(V_y / V_x \right), \phi = \cos^{-1} \left(V_z / \sqrt{V_x^2 + V_y^2 + V_z^2} \right)$$

Results were computed for three different cases, with M = 1, M = 2, and M = 3, each with two levels of noise SNR, 15dB and 30dB. Each experiment consisted of generating M circularly polarized waves with random frequency and phase, and performing 500 trials to understand the impact of noise. Quantitative results are computed by averaging across 10 experiments. Results for one experiment each for M = 1, M = 2, and M = 3 are presented in Figures 1 to Figure 3.

In the figures, results from one trial are represented as small dots, with different colors representing the estimate for a different component. Also shown are dark circles representing the true DOA angles for each component. Clearly, as the level of noise decreases, the spread of the estimates decreases, and they tend to cluster much closer to the true DOA. Interestingly, we observe a greater spread with increasing M. This is because we select the parameter N = 2M + 2, close to the minimum valid choice. We believe that increasing N and the pencil parameter L will lead to better estimates. We present

³We need to perform a matching step to make sure that the different coefficients across dimensions correspond to the same signal. This matching is performed using the frequency ν^m , as it remains the same across the three dimensions.



Fig. 1. DOA estimation for M = 1 at 15dB (left) and 30 dB (right)



Fig. 2. DOA estimation for M = 2 at 15dB (left) and 30 dB (right)

the average absolute error (averaged across the trials, experiments and M) in the estimates for the two angles in Table 1.

 Table 1. Averaged Absolute Error in DOA

e			
	M = 1	M = 2	M = 3
α 15dB	0.05 (6%)	0.10 (11%)	0.28 (44%)
ϕ 15dB	0.04 (2%)	0.08 (3%)	0.27 (16%)
α 30dB	0.01 (< 1%)	0.02 (2%)	0.03 (5%)
ϕ 30dB	0.01 (< 1%)	0.01 (< 1%)	0.05 (3%)

We also include the percentage error, normalized by the value of the true DOA. As can be seen, the resulting DOA estimates are accurate, especially at 30 dB.

5. CONCLUSION

We present a method for DOA estimation of multiple simultaneously incident waves on a single tripole antenna sensor. We use the Unitary Matrix Pencil Method combined with a least squares solver to estimate the frequency as well as amplitude and phase coefficients from a sampled 3D time series. Experimental evaluation demonstrates the effectiveness of the approach at accurately estimating the DOA for different numbers of incident waves M as well as different noise levels - with accuracies of within 1% for 30dB SNR. Directions for future research include improving the estimates at low SNR, evaluation of the algorithms with better noise and signal models, and validating the approach using a streaming system connected to a true tripole sensor.



Fig. 3. DOA estimation for M = 3 at 15dB (left) and 30 dB (right)

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