FOCUSING-BASED APPROACH FOR WIDE-BAND SOURCE LOCALIZATION IN NEAR-FIELD

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ABSTRACT

A wide-band near-field source localization method is presented in this paper. Based on a pre-estimated source location, we form a focusing matrix which is able to compensate the wavefront distortion (with respect to far-field wavefront) due to near-field propagation and frequency dependent phase shift simultaneously. The focused covariance matrix has been proved to have a partially far-field narrow band structure, which allows us to estimate the bearings of the sources by the well studied far-field DOA estimators. The range estimation is then carried out via peak searching of the 1D MUSIC spectral with the estimated bearings. The performance of the algorithm is tested by simulations.

***Index Terms*—** Array signal processing, Direction of arrival estimation, Position measurement, Focusing

1. INTRODUCTION

The topic of source localization with an array of sensors has received a lot of attention because of its wide applications in military and civil services. Many methods have been proposed to solve this problem such as the MUSIC algorithm and its derivatives in [3, 4]. Most of these approaches make the assumption that the sources are located far from the array so that each signal wavefront can be characterized by a single DOA, however, when a source is located close to the array (i.e., in near-field), the wavefront must be characterized by both the azimuth and range. Far-field assumption-based approaches are no longer applicable to this situation. The near-field situation can occur, e.g., in sonar, electronic surveillance, and seismic exploration.

The estimation of near-field sources parameters has been discussed recently, e.g. the multi-dimensional MUSIC method proposed in [3] and the higher order ESPRIT algorithm addressed in [5]. Several methods have been presented in [1, 6] to avoid the high-cost multi-dimensional search and higher order statistics computation. The principle of these algorithms [1, 6] is to separate the estimation of DOA and range by using appropriate approximations. Based on this idea, a narrow-band focusing-based estimator has been proposed in [2], where the near-field wavefront distortion (with respect to far-field wavefront) is compensated via focusing in order to enable the application of far-field DOA algorithms for near-field bearing estimation.

In this paper we propose a new estimator for the localization of wide-band near-field sources. Based on the focusing technique proposed in [2], the wide-band near-field focusing matrix is able to compensate the wavefront distortion due to near-field propagation and the phase change caused by frequency shift within one time focusing calculation. The snapshots sampled in different frequencies are coherently summed and focused on a pre-estimated point. The focused covariance matrix is proved to have a partially far-field structure, which allows us to employ ESPRIT method for the bearing estimation. With the estimated source angles, the ranges of sources are estimated from 1D search-based MUSIC. The rest of the paper is organized as follows: section 2 addresses the received signal model; the focusing procedures are presented in section 3; in section 4, we discuss the estimation of bearings and ranges; the simulation results are shown in section 5 and section 6 concludes the whole paper.

2. SIGNAL MODEL

Consider a near-field scenario of K uncorrelated high frequency wide-band signals impinging to a M-element ULA with inter-element spacing d. Let the first element of the array be the phase reference point and the origin of the coordinates system. We assume that the array elements are on the axis $\theta = 0$. The received signal in time domain at the mth sensor can be modeled as

$$x_m(t) = \sum_{k=1}^{K} e^{j\tau_{mk}} s_k(t) + n_m(t) \tag{1}$$

where $s_k(t)$ is the kth source signal, $n_m(t)$ is the additive white Gaussian noise and τ_{mk} is the phase shift associated with propagation time delay between first sensor and mth sensor of the kth source signal, which is a function of source

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signal parameters, range r_k, angle θ_k and frequency f, given by

$$\tau_{mk} = \frac{2\pi f}{c}\left(y_{mk} - r_k\right) \tag{2}$$

with

$$y_{mk} = \sqrt{r_k^2 + (m-1)^2 d^2 - 2r_k(m-1)d\cos\theta_k} \tag{3}$$

being the distance between mth sensor and kth source. The received signal of the m-th sensor in frequency domain is the FFT of the time domain signal $x_m(t)$, which can be written as

$$X_m(f) = F\left(x_m(t)\right) = \sum_{k=1}^{K} e^{j\tau_{mk}(f)} S_k(f) + N_m(f) \tag{4}$$

where $S_k(f)$ and $N_m(f)$ are the FFT of the source signal and noise respectively.

The received signal of the array in frequency domain

$$\mathbf{X}(f) = \left[X_1(f), \ldots X_M(f)\right]^T \tag{5}$$

with the superscript T denoting matrix transpose, can be modeled as

$$\mathbf{X}(f) = \mathbf{A}(f)\mathbf{S}(f) + \mathbf{N}(f) \tag{6}$$

where $\mathbf{S}(f) = \left[S_1(f), \ldots S_K(f)\right]^T$ is the source signal vector in frequency domain, $\mathbf{N}(f) = \left[N_1(f), \ldots N_M(f)\right]^T$ is the noise vector, $\mathbf{A}(f) = \left[\mathbf{a}(r_1, \theta_1, f), \ldots, \mathbf{a}(r_K, \theta_K, f)\right]$ is the array manifold matrix in frequency f with its kth column $\mathbf{a}(r_k, \theta_k, f)$ being expressed as

$$\mathbf{a}(r_k, \theta_k, f) = \left[e^{j\tau_{1k}(f)}, \ldots, e^{j\tau_{Mk}(f)}\right]^T. \tag{7}$$

The phase shift τ_{mk} can be expressed below by using Taylor expansion:

$$\tau_{mk}(f) = \left(-\frac{2\pi f d}{c}\cos\theta_k\right)(m-1) + g\left(r_k, \theta_k, f, m\right) \tag{8}$$

where $g(r_k, \theta_k, f, m)$ contains the second and higher orders of the Taylor expansion, which is a function of the source location (r_k, θ_k), the frequency f and the sensor index m.

With the expression (8), we can rewrite the array manifold matrix with its kth column $\mathbf{a}(r_k, \theta_k, f)$ being

$$\mathbf{a}(r_k, \theta_k, f) = \begin{bmatrix} 1 \\ \vdots \\ e^{j\left(-\frac{2\pi f d}{c}\cos\theta_k\right)(M-1) + jg(r_k, \theta_k, f, M)} \end{bmatrix} \tag{9}$$

The observed covariance matrix of received signal in frequency f can then be written as

$$\mathbf{R}_X(f) = E\left[\mathbf{X}(f)\mathbf{X}^H(f)\right] = \mathbf{A}(f)\mathbf{R}_s(f)\mathbf{A}(f)^H + \sigma^2\mathbf{I} \tag{10}$$

where the superscript H denotes matrix transpose conjugate, σ^2 is the power of noise, and $\mathbf{R}_s(f) = E\left[\mathbf{S}(f)\mathbf{S}^H(f)\right]$ is the signal covariance matrix.

3. FOCUSING

The focusing-based estimator has been proposed in [2] for the position estimation of narrow-band sources, in which a focusing matrix is employed to compensate the distorted wavefront due to the near-field propagation. For high frequency wide-band sources formulated in the previous section, we propose here a frequency-dependent focusing matrix $\mathbf{B}(f) \in C^{M\times M}$. This new focusing matrix is able to compensate both the near-field wavefront distortion and the phase shift caused by the frequency shift within one time focusing calculation. $\mathbf{B}(f)$ can be written as below:

$$\mathbf{B}(f) = \mathrm{diag}\left(1, \ldots, e^{j\left(-\frac{2\pi f_0 d}{c}\cos\theta_e\right)(M-1) - j\tau_{Me}(f)}\right) \tag{11}$$

where f_0 is the known reference frequency, $\tau_{me}(f)$ is obtained from (2) and (3) with (r_e, θ_e) being a pre-estimated source location from the beamforming-based pre-estimator.

Since (r_e, θ_e) is obtained from a low-resolution pre-estimation, we can assume that L sources ($L \leq K$) are located in the area around (r_e, θ_e) with their positions being (r_l, θ_l) for $l = 1, 2, \ldots, L$.

Applying focusing technique on the array steering vector $\mathbf{a}(r_l, \theta_l, f)$ of the l-th source yields

$$\mathbf{c}_l = \mathbf{B}(f)\mathbf{a}(r_l, \theta_l, f) = \begin{bmatrix} 1 \\ \vdots \\ e^{j\left(-\frac{2\pi f_0 d}{c}\cos\theta_l\right)(M-1)} \end{bmatrix}, \tag{12}$$

where the approximations below are used

$$\begin{aligned} \tau_{ml}(f) - \tau_{me}(f) &\approx \tau_{ml}(f_0) - \tau_{me}(f_0) \\ &\approx \left(-\frac{2\pi f_0 d}{c}\left(\cos\theta_l - \cos\theta_e\right)\right)(m-1) \end{aligned} \tag{13}$$

We note that the first step of (13) is based on the assumption in classical focusing technique for wide-band signal. In the second step, we assume $g(r_e, \theta_e, f_0, m) \approx g(r_l, \theta_l, f_0, m)$ because the function g consists of the second and higher orders of the Taylor expansion of the phase shift function τ.

From (12), we find that $\mathbf{c}_l$ is independent of f and has a form similar to the directional vector in case of far-field narrow band sources. With this property, we can apply focusing technique to the covariance matrix observed in frequency f as follows

$$\begin{aligned}\mathbf{R}_f(f) &= \mathbf{B}(f)\mathbf{R}_x(f)\mathbf{B}^H(f)\\ &= \mathbf{C}(f)\mathbf{R}_s(f)\mathbf{C}^H(f)+\sigma^2\mathbf{I}\end{aligned} \quad (14)$$

with $\mathbf{C}(f)$ being expressed as

$$\mathbf{C}(f)=\mathbf{B}(f)\mathbf{A}(f)=[\mathbf{B}(f)\mathbf{a}(r_1,\theta_1,f)\ldots,\mathbf{c}_l\ldots] \quad (15)$$

To facilitate our presentation, we assume $[\mathbf{c}_1,\ldots,\mathbf{c}_L]$ to be the first L columns of $\mathbf{C}(f)$, i.e. $\mathbf{C}(f)=[\mathbf{G},\mathbf{D}(f)]$ with $\mathbf{G}=[\mathbf{c}_1,\ldots,\mathbf{c}_L]$ and $\mathbf{D}(f)\in C^{M\times(K-L)}$ denoting the rest $(K-L)$ columns of $\mathbf{C}(f)$. We note that $\mathbf{G}$ is independent of the frequency f, so $\mathbf{G}$ is a part of $\mathbf{C}(f)$ for any f. The averaging summation of $\mathbf{R}_f(f)$ in different frequencies yields

$$\mathbf{R}_y=\frac{1}{J}\sum_{i=1}^{J}\mathbf{R}_f(f_j)=\frac{1}{J}\sum_{j=1}^{J}\mathbf{C}(f_j)\mathbf{R}_s(f_j)\mathbf{C}^H(f_j)+\sigma^2\mathbf{I} \quad (16)$$

where J denotes the number of frequencies. With the property that $\mathbf{G}$ is a part of $\mathbf{C}(f)$ for any f, we can rewrite (16) as

$$\mathbf{R}_y=\mathbf{C}_f\mathbf{R}'_s\mathbf{C}_f^H+\sigma^2\mathbf{I} \quad (17)$$

where $\mathbf{C}_f=[\mathbf{G},\mathbf{D}_f]$, $\mathbf{D}_f\in C^{M\times(K-L)}$ and $\mathbf{R}'_s\in C^{K\times K}$ are full rank matrices which depend on $\mathbf{R}_s(f_j)$ and $\mathbf{C}(f_j)$ with $j=1,\ldots,J$. Obviously, $\mathbf{C}_f$ has a partially far-field narrow-band structure, which enables us to apply the far-field DOA algorithm in the next section.

In general, we suppose that we obtain Q $(Q\leq K)$ estimated source locations from the beamforming-based pre-estimator. We can divide the interesting area into Q subareas. Focusing technique is then applied to each subarea separately with the Q estimates. Far-field narrow band DOA estimators are consequently applicable to estimate the sources in the q-th subarea when it is focused.

4. SOURCE LOCALIZATION

4.1. DOA Estimation

The eigen-decomposition of $\mathbf{R}_y$ yields

$$\mathbf{R}_y=\mathbf{U}_s\mathbf{\Lambda}_s\mathbf{U}_s^H+\mathbf{U}_n\mathbf{\Lambda}_n\mathbf{U}_n^H \quad (18)$$

where $\mathbf{U}_s\in C^{M\times K}$ contains K eigenvectors spanning the signal subspace of $\mathbf{R}_y$, and the diagonal matrix $\mathbf{\Lambda}_s\in C^{K\times K}$ contains the corresponding eigenvalues. Similarly, $\mathbf{U}_n\in C^{M\times(M-K)}$ contains $M-K$ eigenvectors in the noise subspace of $\mathbf{R}_y$, and the diagonal matrix $\mathbf{\Lambda}_n\in C^{(M-K)\times(M-K)}$ contains the corresponding eigenvalues.

Then we decompose the array into two sub-arrays, so we have the two subarray manifold matrices $\mathbf{C}_1$ and $\mathbf{C}_2$ be expressing as

$$\mathbf{C}_f=\begin{bmatrix}\mathbf{C}_1\\ \text{last row}\end{bmatrix}=\begin{bmatrix}\text{first row}\\ \mathbf{C}_2\end{bmatrix} \quad (19)$$

With the property of $\mathbf{C}_f$ obtained in the last section, we find that $\mathbf{C}_1$ and $\mathbf{C}_2$ satisfy $\mathbf{C}_1=\mathbf{C}_2\mathbf{\Sigma}$ with $\mathbf{\Sigma}\in C^{K\times K}$ being written as

$$\mathbf{\Sigma}=\begin{bmatrix}\mathbf{\Phi} & \mathbf{0}\\ \mathbf{0} & \mathbf{\Xi}\end{bmatrix} \quad (20)$$

where Φ is a $L\times L$ diagonal matrix given by

$$\mathbf{\Phi}=\text{diag}\left[e^{j\frac{2\pi f_0 d}{c}\cos\theta_1},\ldots,e^{j\frac{2\pi f_0 d}{c}\cos\theta_L}\right] \quad (21)$$

and $\mathbf{\Xi}\in C^{(K-L)\times(K-L)}$ is a full rank matrix. Similarly, the signal subspace $\mathbf{U}_s$ can be partitioned as

$$\mathbf{U}_s=\begin{bmatrix}\mathbf{U}_{s1}\\ \text{last row}\end{bmatrix}=\begin{bmatrix}\text{first row}\\ \mathbf{U}_{s2}\end{bmatrix} \quad (22)$$

From the signal model in (6) and the matrix eigen decomposition in (18), it is obvious that there exists a $K\times K$ full-rank matrix $\mathbf{V}$ satisfying $\mathbf{U}_s=\mathbf{C}_f\mathbf{V}$. Thus, we have $\mathbf{U}_{s1}\mathbf{V}=\mathbf{C}_1$ and $\mathbf{U}_{s2}\mathbf{V}=\mathbf{C}_2$. With the relation between $\mathbf{C}_1$ and $\mathbf{C}_2$ presented previously, we can write the following equation

$$\mathbf{U}_{s1}=\mathbf{U}_{s2}\mathbf{V}\mathbf{\Sigma}\mathbf{V}^{-1} \quad (23)$$

where $\mathbf{V}^{-1}$ denotes the inverse matrix of $\mathbf{V}$.

By multiply the pseudo-inverse matrix of $\mathbf{U}_{s2}$ on the two sides of (23), we have

$$\mathbf{\Psi}=\left(\mathbf{U}_{s2}^H\mathbf{U}_{s2}\right)^{-1}\mathbf{U}_{s2}^H\mathbf{U}_{s1}=\mathbf{V}\mathbf{\Sigma}\mathbf{V}^{-1} \quad (24)$$

We note that $\mathbf{\Psi}$ and $\mathbf{\Sigma}$ have the same eigenvalues. In addition, from (20), we know thatthe diagonal elements of $\mathbf{\Phi}$ are L eigenvalues of $\mathbf{\Sigma}$.

We can then estimate the DOAs of the L sources around the pre-estimated location (r_{eq},θ_{eq}) by

$$\hat{\theta}_l=\beta\mid_{\beta\in\mathbf{p}(r_{eq},\theta_{eq})} \quad (25)$$

where $\mathbf{p}(r_{eq},\theta_{eq})$ denotes the subarea around (r_{eq},θ_{eq}) and

$$\beta=\arcsin\left(\frac{\lambda}{2\pi d}\arg\left(eigenvalues\,of\,\mathbf{\Psi}\right)\right). \quad (26)$$

4.2. Range Estimation

By substituting the estimated angle back into the steering vectors, the problem is reduced to finding the parameter r in $\mathbf{a}\left(r,\hat{\theta}_k,f_0\right)$ with the received signal. We employ here 1-D MUSIC method for the range estimation. The orthogonal projector $\mathbf{M}_n$ is obtained by

$$\mathbf{M}_n=\mathbf{B}^H(f_0)\mathbf{U}_n\mathbf{U}_n^H\mathbf{B}(f_0) \quad (27)$$

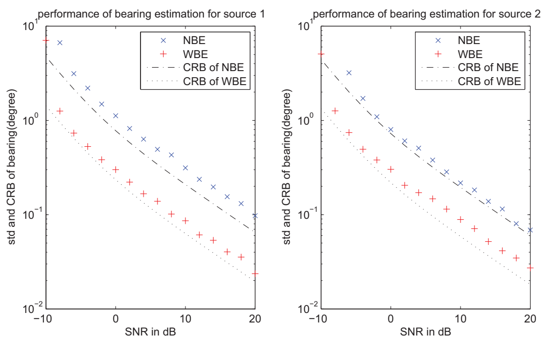

Fig. 1. STD of bearing and CRB versus SNR

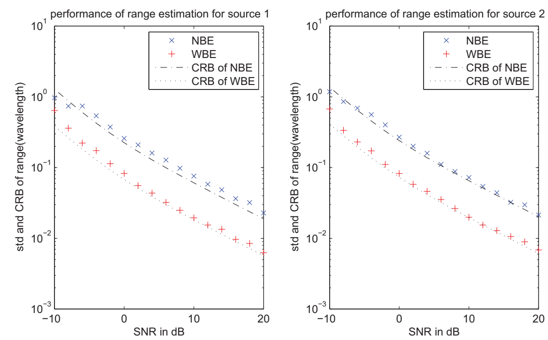

Fig. 2. STD of Range and CRB versus SNR

And the range estimates is obtained by maximizing the MUSIC spectrum

$$\hat{r}_l = \arg \max_{r \in \mathbf{r_{eq}}} \left[P_{MUSIC}^{(l)}(r) \right] \tag{28}$$

where $\mathbf{r}_{eq}$ is the region around r_{eq} and the MUSIC spectrum is obtained by,

$$P_{MUSIC}^{(l)}(r) = \frac{1}{\mathbf{a}^H \left(r, \hat{\theta}_l, f_0\right) \mathbf{M}_n \mathbf{a} \left(r, \hat{\theta}_l, f_0\right)} \tag{29}$$

In addition, we execute the 1-D MUSIC search for each estimated DOA to avoid parameter pairing problem.

5. SIMULATION RESULTS

To test the performance of the proposed, we simulate a simple case: a ULA with $M = 9$ and $d = c/5f_0$ is employed to localize two uncorrelated wide-band sources with their locations $(r_1, \theta_1) = (4.2\lambda, 55°)$and $(r_2, \theta_2) = (4.5\lambda, 65°)$. 11 frequencies varied from 1.9GHz to 2GHz are used with focusing frequency $f_0 = 2$GHz. 200 independent Monte Carlo trials have been carried out at different SNRs (from -10dB to 20dB) with 1000 snapshots. The classical beamforming method has been chosen for the pre-estimation.

The results from the proposed method are compared with the narrow band focusing-based algorithm presented in [2] and the corresponding Crammer Rao Bounds (CRBs) given in [1]. Figs. 1 and 2 illustrate the standard deviation of the estimates and the corresponding CRBs. versus SNR, respectively. From these 2 figures, we can see that the wide-band estimator (WBE) has a better efficiency than the narrow-band estimator (NBE). This is because the WBE uses coherently the snapshots obtained in different frequencies, which brings a better estimate of the covariance matrix. The CRB of WBE is lower than that of NBE thanks to the same reason.

6. CONCLUSION

We present a new estimator for the localization of near-field wide-band sources. The focusing procedure embedded in this estimator can compensate the wavefront distortion due to near-field propagation and the phase change caused by frequency shift within one time focusing calculation, which enables the application of far-field narrow-band algorithms to estimate the bearings. It addition this estimator has a high efficiency compared to the narrow-band focusing-based estimator.

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