TRANSMIT/RECEIVE BEAMFORMING FOR MIMO RADAR WITH COLOCATED ANTENNAS

Aboulnasr Hassanien and Sergiy A. Vorobyov

University of Alberta
Dept. of Electrical and Computer Engineering
Edmonton, AB, T6G 2V4 Canada
hassanie@ece.ualberta.ca
vorobyov@ece.ualberta.ca

ABSTRACT

We propose a new technique for multiple-input multiple-output (MIMO) radar with colocated antennas. The essence of the proposed technique is to partition the transmitting array into a number of subarrays that are allowed to overlap. Each subarray is used to coherently transmit a waveform which is orthogonal to the waveforms transmitted by other subarrays. Coherent processing gain can be achieved by designing a weight vector for each subarray to form a beam towards a certain direction in space. Moreover, the subarrays are combined jointly to form a MIMO radar resulting in higher resolution capabilities. Simulation results show the substantial improvements offered by the proposed technique as compared to previous techniques that validate its effectiveness.

***Index Terms*—** MIMO radar, phased-array radar, adaptive arrays, adaptive beamforming.

1. INTRODUCTION

Recently, the development of multiple-input multiple-output (MIMO) radar has been the focus of intensive research [1]–[7]. MIMO radars employ multiple antennas to emit several orthogonal waveforms and multiple antennas to receive the echoes reflected by the target. Based on the array configurations used, MIMO radars can be classified into two main types. The first type uses widely separated transmit/receive antennas to capture the spatial diversity of the target's radar cross section (RCS) (see [4], and references therein). The other type employs arrays of closely spaced transmit/receive antennas to cohere a beam towards a certain direction in space (see [5], and references therein).

Here, we focus on the latter type. As compared to phased-array radars, the use of MIMO radars with colocated antennas enables improving angular resolution, increasing the upper limit on the number of detectable targets, improving parameter identifiability, extending the array aperture by virtual sensors, and enhancing the flexibility for transmit/receive beampattern design [5]–[7]. However, the advantages offered by MIMO radars come at the price of loosing coherent processing gain offered by phased-array radars. Hence, MIMO radar systems with colocated arrays may suffer from beam-shape loss which leads to degradation in performance in the presence of target's RCS fading.

In this paper, we propose a new technique for MIMO radar with colocated antennas which combines the advantages of the phased-array and the MIMO radars. The main idea behind the proposed technique is partition the transmitting array into a number of subarrays that are allowed to overlap. Each subarray is used to coherently transmit a waveform which is orthogonal to the waveforms transmitted by other subarrays. Coherent processing gain can be achieved by designing the weight vector of each subarray to form a beam towards a certain direction in space. In the mean time, the subarrays are combined jointly to form a MIMO radar resulting in higher resolution capabilities. The new technique enables the use of existing beamforming techniaues at both the transmitting and the receiving ends. Simulation results are used to validate significant performance gains that can be achieved by the proposed algorithm as compared to the phased-array radar and perviously introduced MIMO radars.

2. MIMO RADAR: PRELIMINARIES

Consider a MIMO radar system of M_T colocated transmit and M_R colocated receive antennas. Both the transmitting and receiving arrays are assumed to be close to each other in space so that they see targets at same directions. The mth transmitting antenna emits the mth element of the waveforms vector $\boldsymbol{\phi}(t) \triangleq [\phi_1(t), \ldots, \phi_{M_T}(t)]^T$ which satisfies the orthogonality condition $\int_{T_0} \boldsymbol{\phi}(t)\boldsymbol{\phi}^H(t)dt = \mathbf{I}$, where T_0 is the radar pulse width, t is the time index within the radar pulse, $(\cdot)^T$ and $(\cdot)^H$ are the transpose and conjugate transpose re-

Supported in parts by the Natural Science and Engineering Research Council (NSERC) of Canada and the Alberta Ingenuity Foundation, Alberta, Canada.

spectively. The $M_R \times 1$ snapshot vector received by such a MIMO radar can be modeled as

$$\mathbf{x}(t) = \mathbf{x}_t(t) + \mathbf{x}_i(t) + \mathbf{n}(t) \quad (1)$$

where $\mathbf{x}_t(t)$, $\mathbf{x}_i(t)$, and $\mathbf{n}(t)$ are the independent components of the target signal, interference, and sensor noise, respectively. Under point target assumption, the target signal can be written as

$$\mathbf{x}_t(t) = \beta(\theta_t)\mathbf{a}_T^H(\theta_t)\boldsymbol{\phi}(t)\mathbf{a}_R(\theta_t) \quad (2)$$

where θ_t is the target direction, $\beta(\theta_t)$ is the complex-valued reflection coefficient of the focal point θ_t, and $\mathbf{a}_T(\theta_t)$ and $\mathbf{a}_R(\theta_t)$ are the actual transmit and the actual receive steering vectors associated with θ_t. The returns due to the mth transmitted waveform can be recovered by match filtering the received data to $\phi_m(t)$, i.e.,

$$\mathbf{x}_m \triangleq \int_{T_0} \mathbf{x}(t)\phi_m^*(t)dt. \quad (3)$$

where $(\cdot)^*$ is the conjugate operator. Then, the $M_T M_R \times 1$ virtual data vector can be written as

$$\mathbf{y} \triangleq [\mathbf{x}_1^T \cdots \mathbf{x}_{M_T}^T]^T = \beta(\theta_t)\mathbf{a}_T(\theta_t) \otimes \mathbf{a}_R(\theta_t) + \mathbf{y}_{i+n} \quad (4)$$

where $\otimes$ is the Kronker product and $\mathbf{y}_{i+n}$ accounts for the interference-plus-noise components. By using $\mathbf{y}$ for detection and estimation purposes, higher resolution and better performance can be achieved. Despite its advantages, the above MIMO radar formulation does not enable beamforming at the transmitter and, therefore, lack robustness against sensor noise and RCS fading.

3. PROPOSED MIMO RADAR FORMULATIONS

In this section, we propose new MIMO radar formulations which allow beamforming at the transmitter. The new formulations combine the advantages of the phased-array radar and the MIMO radar leading to substantial performance improvements. The key idea here is to partition the transmitting array into K subarrays ($1 \leq K \leq M_T$) which are allowed to overlap[1]. All elements of the kth subarray are used to coherently emit the signal $\phi_k(t)$ such that a beam is formed towards the target direction. Due to subarray overlap, each antenna transmits a linear combination of the waveforms $\{\phi_k(t)\}_{k=1}^K$ where the transmit amplifiers are assumed to be linear. Although $\{\phi_k(t)\}_{k=1}^K$ are required to satisfy the orthogonality condition, the signals transmitted by different antennas need not be orthogonal. The beamforming weight vector can be properly designed to maximize the coherent processing gain. The signal transmitted by the kth subarray can be modeled as

$$s_k(t) = \mathbf{w}_k^H \mathbf{a}_k(\theta_t)\phi_k(t) \quad (5)$$

[1] Without loss of generality, it is assumed that all subarrays have the same number of elements. If the subarrays are chosen to be fully-overlapped, then each subarray consists of $M_T - K + 1$ antennas.

where $\mathbf{a}_k(\theta_t)$ is the kth subarray's steering vector associated with θ_t, $\mathbf{w}_k$ is the $(M_T - K + 1) \times 1$ complex vector of beamforming weights.

At the receiver, the $M_R \times 1$ complex vector of array observations is given by (1). Then, the target signal component can be written as

$$\begin{aligned} \mathbf{x}_t(t) &= \sum_{k=1}^{K} \beta(\theta_t)s_k(t)\mathbf{a}_R(\theta_t) \\ &= \sum_{k=1}^{K} \beta(\theta_t)\phi_k(t)\mathbf{w}_k^H\mathbf{a}_k(\theta_t)\mathbf{a}_R(\theta_t) \end{aligned} \quad (6)$$

A bank of K filters is required to match filter the received data to the transmitted waveforms. The extracted components $\{\mathbf{x}_m\}_{m=1}^K$ are staked in one vector yielding the virtual data vector

$$\mathbf{y} \triangleq [\mathbf{x}_1^T \cdots \mathbf{x}_K^T]^T = \beta(\theta_t)\mathbf{v}(\theta_t) + \mathbf{y}_{i+n} \quad (7)$$

where $\mathbf{v}(\theta_t)$ is the $KM_R \times 1$ virtual steering vector associated with the target, i.e.,

$$\mathbf{v}(\theta_t) \triangleq \begin{bmatrix} \mathbf{w}_1^H\mathbf{a}_1(\theta_t) \\ \vdots \\ \mathbf{w}_K^H\mathbf{a}_K(\theta_t) \end{bmatrix} \otimes \mathbf{a}_R(\theta_t). \quad (8)$$

It is worth noting that if $K = 1$ is chosen, i.e., if the whole transmitting array is considered as one subarray, then the radar signal model (7) simplifies to

$$\mathbf{y} \triangleq \beta(\theta_t)\mathbf{w}_1^H\mathbf{a}_T(\theta_t)\mathbf{a}_R(\theta_t) + \mathbf{y}_{i+n} \quad (9)$$

which is the signal model for the conventional phased-array radar [8]. In (9), the data vector $\mathbf{y}$ is of dimension $M_R \times 1$ which explains the low resolution performance of phased-array radars. On the other hand, if $K = M_T$ is chosen, then the signal model (7) simplifies to (4) which is the signal model for MIMO radar without array partitioning. In this case, the $M_T M_R \times 1$ data vector $\mathbf{y}$ enables the highest possible resolution at the price of having no coherent processing gain at the transmitting side.

In contrast to the phased-array radar data model (9) and the MIMO radar data model (4), the proposed MIMO radar formulations (7) enjoy the following advantages:

- Transmit beamforming can be used at the transmitting side to maximize the coherent processing gain and to control the transmit power.
- The overall beampattern of the virtual array can be optimized by designing the transmit/receive beamforming weights jointly.
- Tradeoff between resolution and robustness against beamshape loss can be achieved by increasing/decreasing the number of subarrays used.
- Tradeoff between improvements in performance and the required computational complexity can be achieved.

4. TRANSMIT/RECEIVE BEAMFORMING

Adaptive processing techniques are applied to the proposed MIMO radar configurations (7) and compared to the MIMO radar without array partitioning (4) and the phased-array radar (9). At the transmitting side, existing uplink beamforming techniques lend themselves easily to design the weight vectors $\{\mathbf{w}_k\}_{k=1}^K$ for different subarrays such that certain beampattern and/or transmit power requirements are satisfied. However, for the sake of simplicity, in this paper we only consider using the conventional beamformer at the transmitting side while the use of other robust uplink beamforming techniques will be the focus of future work. Hence, the entries of the weight vector $\mathbf{w}_k$ are simply given by the elements of $\mathbf{a}_T(\theta_t)$ starting from the kth entry until the $(M_T - K + k)$th entry.

At the receiving end, we use the adaptive processing technique which aims at maximizing the signal-to-interference-plus-noise ratio (SINR). Hence, we resort to the famous minimum variance distortionless response (MVDR) beamformer [9]. The essence of the MVDR beamformer is to minimize the interference-plus-noise power while maintaining a distortionless response towards the direction of the target of interest. This can be expressed as the following optimization problem

$$\min_{\mathbf{w}_R} \ \mathbf{w}_R^H \mathbf{R}_{i+n} \mathbf{w}_R \quad \text{subject to} \ \ \mathbf{w}_R^H \mathbf{v}(\theta_t) = 1. \tag{10}$$

where $\mathbf{w}_R$ is the $KM_R \times 1$ receiving beamformer weight vector, $\mathbf{R}_{i+n} \triangleq \mathrm{E}\{\mathbf{y}_{i+n}\mathbf{y}_{i+n}^H\}$ is the interference-plus-noise covariance matrix, and $\mathrm{E}\{\cdot\}$ denotes the expectation operator. The solution to (10) is given by [9]

$$\mathbf{w}_R = \frac{\mathbf{R}_{i+n}^{-1}\mathbf{v}(\theta_t)}{\mathbf{v}^H(\theta_t)\mathbf{R}_{i+n}^{-1}\mathbf{v}(\theta_t)} \tag{11}$$

In practice, the matrix $\mathbf{R}_{i+n}$ is unavailable and, therefore, the sample covariance matrix $\hat{\mathbf{R}} \triangleq \sum_{n=1}^N \mathbf{y}_n \mathbf{y}_n^H$ is used, where $\{\mathbf{y}_n\}_{n=1}^N$ are data snapshots which can be collected from N different radar pulses within a coherent processing interval. It is worth noting that the target signal component is present in $\hat{\mathbf{R}}$. Alternatively, we obtain a target signal-free sample covariance matrix by collecting the data snapshots $\{\mathbf{y}_n\}_{n=1}^N$ for N different range bins [6], [10].

5. SIMULATION RESULTS

In our simulations, we assume a uniform linear array (ULA) of $M_T = 10$ omnidirectional sensors spaced half a wave length apart. The same ULA is used for both transmitting and receiving. The additive noise is modeled as a complex Gaussian zero-mean spatially and temporally white sequence that has identical variances in each array sensor. We assume two interfering targets located at directions $-30°$ and $-10°$, respectively. The target of interest is assumed to reflect a plane-wave that impinges on the array from direction $\theta_t = 10°$. The target reflection coefficient is assumed to be equal in magnitude to the reflection coefficient of the interference.

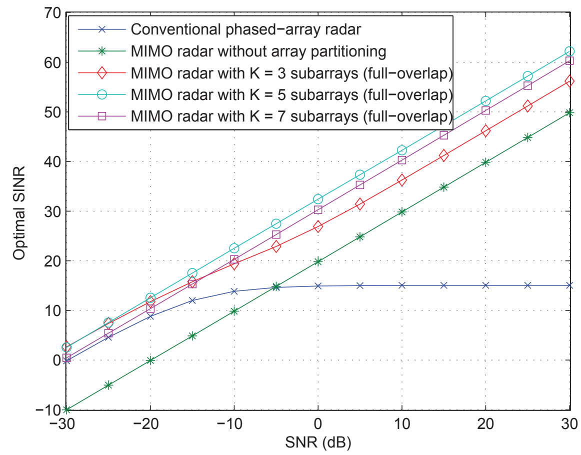

Fig. 1. Optimal output SINR versus SNR; first example.

In the first example, the comparison between the phased-array radar (9), the MIMO radar without array partinioning (4), and the proposed MIMO radar (7) is performed in terms of the optimal output SINR for three different cases $K = 3$, $K = 5$, and $K = 7$ subarrays, respectively. Fig. 1 shows the optimal SINR versus signal-to-noise ratio (SNR) for all methods tested. From this figure, we can see that the phased-array radar outperforms the MIMO radar without array partitioning at low SNR ranges. This can be attributed to robustness of the phased-array radar against sensor noise due to the use of uplink beamforming. On the other hand, MIMO radar without array partitioning outperforms the phased-array radar at moderate and high SNR ranges which can be attributed to the high resolution and high interference rejection capabilities offered by the virtual array of large aperture. Note that the performance of the phased-array radar saturates at high SNR because it is assumed in the simulations that the ratio of the reflected target power to the reflected interference power is fixed, i.e., $\mathrm{SNR}/\mathrm{INR} = const$ where INR stands for interference-to-noise ratio. It is important to note that the proposed MIMO radar offers a substantially better optimal SINR as compared to the phased-array radar and the MIMO radar without partitioning for all cases tested.

In the second example, we test all aforementioned methods using simulated data. The performance of all methods is compared in terms of the output SINR which is defined as $\mathrm{SINR} = \beta^2(\theta_t)|\mathbf{w}_R^H\mathbf{v}(\theta_t)|^2/\mathbf{w}_R^H\hat{\mathbf{R}}_{i+n}\mathbf{w}_R$, where $\mathbf{w}_R$ is computed using (11). At the transmitting side, the waveforms $\{\phi_k(t) = e^{j2\pi\frac{k}{T_0}t}\}_{k=1}^K$ are used. The sample covariance matrix is computed based on $N = 100$ data snapshots (i.e., 100 range bins) for all methods tested. Note that for the MIMO radar without partitioning, the sample covariance matrix is of size 100×100. To avoid the effect of low sample size, diagonal loading of $10\mathbf{I}$ is used when solving (11) for all methods tested. The output SINR versus SNR is plotted in Fig. 2 for all

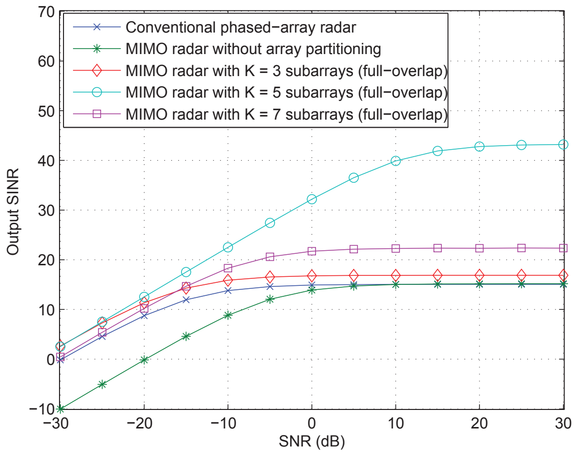

Fig. 2. Output SINR versus SNR, second example

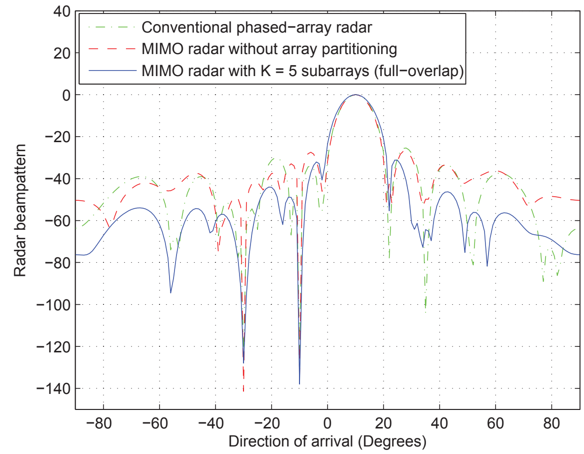

Fig. 3. MIMO radar beampattern.

methods. All results are calculated based on 100 independent simulation runs. It can be seen that the proposed MIMO radar (with $K = 3$, $K = 5$, and $K = 7$ subarrays) outperforms the phased-array radar and the MIMO radar without array partitioning for all tested SNR ranges. It is worth noting that as the number of subarrays increases, the benefit gained from waveform diversity also increases at the expense of decreased coherent processing gain. Hence, there is a tradeoff between these two desirable features. Figs. 1 and 2 show that the use of $K = 5$ subarrays is the best compromise.

Finally, radar beampatterns for the phased-array radar, the MIMO radar without partitioning, and the proposed MIMO radar with $K = 5$ subarrays are plotted in Fig. 3. The SNR is fixed at 20 dB. It can be observed that the beampattern of the proposed MIMO radar has substantially lower sidelobe levels as compared to the phased-array radar and the MIMO radar without array partitioning. At the same time, it has almost the same interference rejection capability as that of the MIMO radar without array partitioning. Hence, it is robust against both sensor noise and powerful interference.

6. CONCLUSIONS

A new technique for MIMO radar with colocated antennas has been proposed. This technique is based on partitioning the transmitting array into a number of subarrays which are allowed to overlap. Each subarray is used to coherently transmit a waveform which is orthogonal to the waveforms transmitted by other subarrays. Coherent processing gain is achieved by designing the weight vector of each subarray to form a beam towards a certain direction in space. The subarrays are combined jointly to form a MIMO radar resulting in higher resolution capabilities. The proposed technique combines the advantages of the phased-array radar and the MIMO radar and, therefore, it has a superior performance. Simulation results confirm our theoretical observations and demonstrate the effectiveness of the proposed MIMO radar technique.

7. REFERENCES

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