# OPTIMAL AND ROBUST WAVEFORM DESIGN FOR MIMO RADARS

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**ABSTRACT**

Waveform design for Target identification and classification in MIMO radar systems has been studied in several recent works. While the previous works considered signal independent noise, here we extend the results to the case where signal-dependent noise, clutter, is also present and then we find the optimum waveform for several estimators differing in the assumptions on the given statistics. Computing the optimal waveforms for MMSE estimator leads to the Semi-definite programming (SDP) problem. Finding the optimal transmit signals for CSLS estimator results in a minimax eigenvalue problem. Finally it is shown that equal power waveforms are the best transmit signals for the SLS estimator.

***Index Terms***— Estimators, Optimization, Waveform

## 1. INTRODUCTION

Waveform design in the presence of signal dependent noise, i.e., clutter, is one of the most intensively investigated problems in radar systems. Recently, Kay in [1] has shown that for optimal detection with Nyman Pearson criterion, transmit signals must whiten the clutter. Most of the existing literatures deal with the point target model. However, as the resolution of radars increases, a better model becomes that of an extended target which is spread in range, azimuth, and Doppler.

Recently a great interest has emerged in MIMO radars employing multiple antennas at both the transmitter and receiver and performing space-time processing on both. An extended target model for MIMO radars has been developed in [2]. Since a MIMO radar system can transmit multiple probing signals via its antennas, better optimization through waveform selection is possible. For example, in [3], use of multiple signals with arbitrary cross-correlation matrix has been proposed and shown that the cross-correlation matrix can be chosen to achieve a desired spatial transmit beampattern. Waveform optimization problem for parameter estimation for the general case of multiple targets in the presence of spatially colored noise has been investigated in [4]. An information theoretic approach has been considered for waveform optimization problem by [5]. Also some works are done on MIMO waveform design in the presence of clutter. In [6] a procedure is developed to design the optimal waveform which maximizes the signal-to-interference plus-noise ratio (SINR) at the output of the detector. It leads to a nonlinear optimization problem and the suggested adaptive algorithm is neither guaranteed to converge nor, if converged, to produce the global maximum.

In this work we consider the MIMO radar system and attempt to find the appropriate waveform for optimal parameter estimation and target identification in the presence of clutter and noise. This work is organized as follows. First the modelling assumption is presented in section 2, and then transmit waveforms are found such that the error of MMSE estimator in estimating the target impulse response is minimized. This problem leads to a semidefinite programming (SDP) problem which is discussed in section 3. Then the optimal signal design for covariance shaping least square estimator (CSLS), which requires less information than MMSE estimator, is investigated in section 4. Since it assumes the worst possible target conditions, it guarantees the fixed level of performance. So it will be robust in any realization of target. Finally section 5 relates to the numerical results and a comparison between the performances of different estimators.

Notation: throughout this paper, we use bold upper case letters to denote matrices, and bold lower case letters to signify column vectors. Superscripts $\{.\}^H$ and $\{.\}^\dagger$ will be used to denote the complex conjugate transpose and the pseudoinverse of the corresponding matrix respectively. We use $tr\{.\}$ for the trace of a matrix, and $E\{.\}$ for expectation with respect to all the random variables within the brackets. Given a matrix $\boldsymbol{G}$, the symbol $\|\boldsymbol{G}\|_{\mathrm{f}}^2$ denotes the Frobenius norm of $\boldsymbol{G}$ and it is defined as $\|\boldsymbol{G}\|_{\mathrm{f}}^2 = trace\{\boldsymbol{G}^H\boldsymbol{G}\}$. We let $\boldsymbol{I}$ denote the identity matrix. The notation $\boldsymbol{X} \succcurlyeq 0$ means that $\boldsymbol{X}$ is Hermitian and nonnegative definite, and $\boldsymbol{X} \succcurlyeq \boldsymbol{Y}$ means that $\boldsymbol{X} - \boldsymbol{Y} \succcurlyeq 0$. Finally, given a Hermitian matrix $\boldsymbol{A}$, we denote by $(\boldsymbol{A})^{\frac{1}{2}}$ the Hermitian square root and by $\lambda_{max}(\boldsymbol{A})$ and $\lambda_{min}(\boldsymbol{A})$ the largest and smallest eigenvalues, respectively.

## 2. MODEL

Consider MIMO radar equipped with $t$ transmitter and $r$ receiver antennas. The $r \times 1$ received signal can be expressed as [2] with the difference that we consider this model in discrete-time for simplicity

$$\boldsymbol{z}_n = \boldsymbol{H}\boldsymbol{s}_n + \boldsymbol{w}_n \,,$$

where $r \times t$ matrix $\boldsymbol{H} = [\boldsymbol{h}_1, \boldsymbol{h}_2, \dots, \boldsymbol{h}_r]^H$ is the target scattering matrix similar to the channel matrix in [2], signal vector $\boldsymbol{s_n} = [\, s_1(n), \dots, s_t(n)\, ]^H$ is transmitted by transmitters at time n, $\boldsymbol{z_n} = [z_1(n), \dots z_r(n)\, ]^H$ is the collection of the received signals at the various receiving elements at time n and $\boldsymbol{w}_n$ is the additive white noise. In order to estimate $\boldsymbol{H}$, let finite length signal $s_k[n]$ by length $N \geq t$ be transmitted from each element. Due to the $t \times N$ transmitted matrix $\boldsymbol{S} = [\boldsymbol{s}_1, \boldsymbol{s}_2, \dots\dots \boldsymbol{s}_N]$, the $r \times N$ received matrix $\boldsymbol{Z}$ can be expressed as:

$$\boldsymbol{Z} = \boldsymbol{HS} + \boldsymbol{W}\,,$$

where $\boldsymbol{W}$ is the noise matrix defined by $\boldsymbol{W} = [\boldsymbol{w}_1, \boldsymbol{w}_2, \dots\dots \boldsymbol{w}_N]$. The reflection from a target is almost always accompanied by reflections from the surrounding environment (ground, ocean), referred to as clutter. By denoting the target related quantities by subscript, t and clutter by c, the complete signal model for received radar signal $\boldsymbol{Z}$ is then given by:

$$\boldsymbol{Z} = \boldsymbol{H_t S} + \boldsymbol{H_c S} + \boldsymbol{W}\,, \tag{1}$$

where $\boldsymbol{H_t}$ and $\boldsymbol{H_c}$ are assumed to be Gaussian distributed matrices with zero mean and covariance $\boldsymbol{R}_t$ and $\boldsymbol{R}_c$ respectively. These covariance matrices are defined by $\boldsymbol{R}_t = E\{\boldsymbol{H}_t{}^H \boldsymbol{H}_t\}$ and $\boldsymbol{R}_c = E\{\boldsymbol{H}_c{}^H \boldsymbol{H}_c\}$. Now our problem is to find the optimal transmit matrix $\boldsymbol{S}$ such that target estimation error is minimized.

## 3. OPTIMAL WAVEFORM DESIGN FOR MMSE ESTIMATOR

The task of a target estimation algorithm is to recover the target scattering matrix $\boldsymbol{H}_t$. In this section we obtain a linear estimator that minimizes the estimation MSE of $\boldsymbol{H_t}$ and choosing $\boldsymbol{S}$ such that the MMSE estimation error is minimized. The linear MMSE estimator can be expressed as:

$$\widehat{\boldsymbol{H}}_{MMSE} = \boldsymbol{Z}\boldsymbol{G}_{op}$$

where $\boldsymbol{G}_{op}$ has to be obtained so that the MSE is minimized.

$$\boldsymbol{G}_{op} = \arg\min_{\boldsymbol{G}} E\{\|\boldsymbol{H}_t - \boldsymbol{ZG}\|_F^2\}$$

The estimation error can be expressed as:

$$\begin{aligned}\varepsilon &= E\{\|\boldsymbol{H}_t - (\boldsymbol{H}_t\boldsymbol{S} + \boldsymbol{H}_c\boldsymbol{S} + \boldsymbol{W}).\boldsymbol{G}\|_F^2\} \\ &= tr\{\boldsymbol{R}_t\} - tr\{\boldsymbol{R}_t\boldsymbol{SG}\} - tr\{\boldsymbol{G}^H\boldsymbol{S}^H\boldsymbol{R}_t\} \\ &\quad + tr\{\boldsymbol{G}^H[\boldsymbol{S}^H(\boldsymbol{R}_t + \boldsymbol{R}_c)\boldsymbol{S}\sigma^2.r\boldsymbol{I}]\boldsymbol{G}\}\end{aligned} \tag{2}$$

The optimal estimator $\boldsymbol{G}_{op}$ can be found from $\frac{\partial\varepsilon}{\partial\boldsymbol{G}} = 0$ and is given by:

$$\boldsymbol{G}_{op} = [\boldsymbol{S^H}(\boldsymbol{R_t} + \boldsymbol{R_c})\boldsymbol{S} + \sigma^2.r\boldsymbol{I}]^{-1}\boldsymbol{S^H R_t}$$

Hence the linear MMSE estimation of matrix $\boldsymbol{H_t}$ can be written as:

$$\widehat{\boldsymbol{H}}_{MMSE} = \boldsymbol{ZG} = \boldsymbol{Z}[\boldsymbol{S^H}(\boldsymbol{R_t} + \boldsymbol{R_c})\boldsymbol{S} + \sigma^2.r\boldsymbol{I}]^{-1}\boldsymbol{S^H R_t}$$

The performance of this estimator is characterized by the error matrix $\boldsymbol{E} = \boldsymbol{H_t} - \widehat{\boldsymbol{H}}_{\boldsymbol{MMSE}}$ with zero mean and covariance

$$\boldsymbol{R_E} = E\{(\boldsymbol{H_t} - \widehat{\boldsymbol{H}}_{\boldsymbol{MMSE}})^H.(\boldsymbol{H_t} - \widehat{\boldsymbol{H}}_{\boldsymbol{MMSE}})\}.$$

Therefore the MMSE estimation error can be computed as:

$$\begin{aligned}\varepsilon_{MMSE} &= tr\{\boldsymbol{R_E}\} \\ &= tr\{\boldsymbol{R}_t - \boldsymbol{R}_t\boldsymbol{S}(\boldsymbol{S}^H(\boldsymbol{R}_c + \boldsymbol{R}_t)\boldsymbol{S} + \sigma^2.r\boldsymbol{I})^{-1}\boldsymbol{S}^H\boldsymbol{R}_t\}\end{aligned} \tag{3}$$

Our goal is to find those transmitted waveforms $\boldsymbol{S}$ that minimize estimation error $\varepsilon_{MMSE}$ under the power constraint $tr\{\boldsymbol{SS^H}\} = P$. Therefore we can express the problem of waveform design as:

$$\begin{aligned}&Min_{\boldsymbol{S}} \quad \varepsilon_{MMSE} \\ &\text{s.t} \quad tr\{\boldsymbol{SS^H}\} = P\end{aligned}$$

Now by setting

$$\boldsymbol{R}_{tc} = \boldsymbol{R}_t + \boldsymbol{R}_c = \boldsymbol{V}_{tc}\boldsymbol{V}_{tc}^H, \tag{4}$$

, using matrix inversion Lemma and some matrix manipulations, not shown due to the lack of space, $\varepsilon_{MMSE}$ can be rewritten as:

$$\begin{aligned}\varepsilon_{MMSE} = {} & tr\{\boldsymbol{R_t} - \boldsymbol{R_t}.\boldsymbol{R_{tc}^{-1}}.\boldsymbol{R_t}\} \\ & + \sigma^2.r\,tr\{\boldsymbol{R_t V_{tc}^{-H}}(\boldsymbol{V_{tc}^H SS^H V_{tc}} + \sigma^2.r\boldsymbol{I})^{-1}\boldsymbol{V_{tc}^{-1} R_t}\}\end{aligned} \tag{5}$$

For the reason that the first expression, $tr\{\boldsymbol{R_t} - \boldsymbol{R_t R_{tc}^{-1} R_t}\}$, is independent of $\boldsymbol{S},$ it's enough to minimize the second expression.

$$\begin{aligned}& Min_{\boldsymbol{S}}\, tr\left\{\boldsymbol{R}_t\boldsymbol{V}_{tc}^{-H}\left(\boldsymbol{V}_{tc}^H\,\frac{1}{\sigma^2.r}\boldsymbol{SS}^H\boldsymbol{V}_{tc} + \boldsymbol{I}\right)^{-1}\boldsymbol{V}_{tc}^{-1}\boldsymbol{R}_t\right\} \\ &= Min_{\boldsymbol{S}}\ tr\left\{\left(\tfrac{1}{\sigma^2.r}\boldsymbol{R_1}\boldsymbol{\mathcal{S}}\,\boldsymbol{R_1}^H + \boldsymbol{R_2}\right)^{-1}\right\}\end{aligned} \tag{6}$$

where

$$\boldsymbol{\mathcal{S}} = \boldsymbol{SS}^H \quad , \quad \boldsymbol{R}_1 = \boldsymbol{R}_t^{-1}\boldsymbol{R}_{tc}\ ,\ \boldsymbol{R}_2 = \boldsymbol{R}_t^{-1}\boldsymbol{R}_{tc}\boldsymbol{R}_t^{-1}\,. \tag{7}$$

This optimization problem can be formulated as an SDP convex optimization problem. The following Lemma is useful for developing equations specifying optimal waveforms.

Lemma1. (Schur's complement): let $\boldsymbol{Z} = \begin{bmatrix}\boldsymbol{P} & \boldsymbol{S^H} \\ \boldsymbol{S} & \boldsymbol{Q}\end{bmatrix}$ be an Hermitian matrix. Then with $\boldsymbol{Q} \succ 0$ , $\boldsymbol{Z} \succcurlyeq 0$ if and only if $\boldsymbol{P} - \boldsymbol{S^H Q^{-1} S} \succcurlyeq 0$, [7].

Theorem 1: The optimal waveforms for MMSE estimator can be found as the solution to the SDP:

$$Min_{\boldsymbol{\mathcal{S}},\boldsymbol{X}}\ tr\{\boldsymbol{X}\}$$

Subject to

$$\begin{bmatrix}\boldsymbol{X} & \boldsymbol{I} \\ \boldsymbol{I} & \frac{1}{\sigma^2.r}\boldsymbol{R_1}\boldsymbol{\mathcal{S}}\,\boldsymbol{R_1}^H + \boldsymbol{R_2}\end{bmatrix} \succcurlyeq 0 \tag{8}$$

$$\begin{aligned}&\boldsymbol{\mathcal{S}} \succcurlyeq 0 \\ &tr\{\boldsymbol{\mathcal{S}}\} = P\end{aligned} \tag{9}$$

*Proof:* employing auxiliary variable$\boldsymbol{X}$, (6) can be rewritten as:

$$\begin{aligned}&Min_{\boldsymbol{\mathcal{S}},\boldsymbol{X}}\ tr\{\boldsymbol{X}\} \\ &\boldsymbol{X} \succcurlyeq \left(\tfrac{1}{\sigma^2.r}\boldsymbol{R_1}\boldsymbol{\mathcal{S}}\,\boldsymbol{R_1}^H + \boldsymbol{R_2}\right)^{-1} \\ &\boldsymbol{\mathcal{S}} \succcurlyeq 0 \\ &tr\{\boldsymbol{\mathcal{S}}\} = P\end{aligned} \tag{10}$$

now by using Lemma 1, (10) is equivalent to the Linear Matrix Inequality (LMI) in (8). The mathematical optimization problems formulated in [5] and [8] can be seen to be the special cases of that treated in Theorem 1.

## 4. ROBUST WAVEFORM DESIGN

Consider the case where we know the clutter statistics but not that of the target. The clutter matrix information can be obtained through field measurements in areas assumed to be target free. Since we have no knowledge of the second order statistics of the target matrix, in this section we characterize $\boldsymbol{H}_t$ as an unknown, deterministic matrix in the linear model

$$\boldsymbol{Z} = \boldsymbol{H_t S} + \boldsymbol{H_c S} + \boldsymbol{W}.$$

A linear estimator $\boldsymbol{G}$ is chosen to minimize MSE of $\widehat{\boldsymbol{Z}} = \widehat{\boldsymbol{H_t}}\boldsymbol{S}$ subject to the constraint that the covariance of the error in the estimate $\widehat{\boldsymbol{H_t}}$ defined by $\widehat{\boldsymbol{H_t}} = \boldsymbol{ZG}$, is proportional to a given covariance matrix $\boldsymbol{R}$. Therefore we can control the dynamic range and spectral shape of the covariance of the estimation error. This is the CSLS estimator the theory of which can be found in [9]. This can be formulated as [9]:

$$Min_{\boldsymbol{G}}\ E\left\{(\boldsymbol{Z} - \boldsymbol{ZGS})^{\boldsymbol{H}} \boldsymbol{C}_{\boldsymbol{W}}^{-1} (\boldsymbol{Z} - \boldsymbol{ZGS})\right\} \tag{11}$$

$$\text{s.t.} \quad \boldsymbol{G}^H \boldsymbol{C}_W^{-1} \boldsymbol{G} = c^2 \boldsymbol{R} \tag{12}$$

where $\boldsymbol{R}$ is a given covariance matrix, c >0 is a constant and, $\boldsymbol{C}_W = (\boldsymbol{S}^H \boldsymbol{R}_c \boldsymbol{S} + \sigma^2.r\boldsymbol{I})$ is the covariance of the estimation error. Note that if (12) is replaced by constraint $\boldsymbol{GS} = \boldsymbol{I}$, we obtain the LS estimator that estimates $\widehat{\boldsymbol{H_t}} = \boldsymbol{H_t} + \boldsymbol{H_c}$ and cannot discriminate between $\boldsymbol{H_t}$ and $\boldsymbol{H}_c$. It happens due to the well known fact that unbiased estimators such as the LS estimator do not necessarily lead to the minimum MSE [9]. Without any knowledge about target statistics, we can only control the spectral shape of the estimation error. That is what the CSLS estimator exactly does. CSLS estimator is the biased estimator. Eldar and Oppenheim in [9] have shown that it improves the performance at low to moderate SNRs. Without loss of generality, we choose $c^2\boldsymbol{R} = \boldsymbol{I}$ in order to whiten the estimation error in (12). $\boldsymbol{G}$ can be obtained by solving (11) subject to (12) as [9]:

$$\boldsymbol{G} = \boldsymbol{C}_W^{-1} \boldsymbol{S}^H (\boldsymbol{S} \boldsymbol{C}_W^{-1} \boldsymbol{S}^H)^{-\frac{1}{2}}$$

So the estimation error can be expressed as:

$$\varepsilon_{CSLS} = E\left\{\left\|(\boldsymbol{H}_t - \widehat{\boldsymbol{H}_t})\right\|_{\mathrm{f}}^2\right\}$$

$$= tr\left\{\boldsymbol{H}_t \left(\boldsymbol{I} - (\boldsymbol{S}\boldsymbol{C}_W^{-1}\boldsymbol{S}^H)^{\frac{1}{2}}\right)^2 {\boldsymbol{H}_t}^H\right\} + tr\{\boldsymbol{I}\} \tag{13}$$

In designing optimal waveform, one possible approach is to minimize the MSE. However, as it is seen in (13) in the case of deterministic target matrices, the MSE depends explicitly on $\boldsymbol{H}_t$ and therefore cannot be minimized.We therefore proposed seeking waveform that minimizes a worst-case function of the MSE over all possible values of $\boldsymbol{H}_t$ that satisfy a Frobenuis norm constraint of the form $tr({\boldsymbol{H}_t}^{\boldsymbol{H}}.\boldsymbol{H}_t) < U^2$ for some constant *U*. This is a robust waveform design procedure and it can guarantee a fixed level of performance for any realization of the target channels.

$$MinMax\ tr\left\{\boldsymbol{H}_t \left(\boldsymbol{I} - (\boldsymbol{S}\boldsymbol{C}_W^{-1}\boldsymbol{S}^H)^{\frac{1}{2}}\right)^{\boldsymbol{2}} {\boldsymbol{H}_t}^{\boldsymbol{H}}\right\}$$

$$= Min_{\boldsymbol{S}} Max_{\boldsymbol{H}_t} \textstyle\sum_{i=1}^{r} {\boldsymbol{h}_i}^H \boldsymbol{T}\, \boldsymbol{h}_i \tag{14}$$

$$\text{s.t.} \quad tr({\boldsymbol{H}_t}^H \boldsymbol{H}_t) < U^2$$

where $\boldsymbol{T} = \left(\boldsymbol{I} - (\boldsymbol{S}\boldsymbol{C}_W^{-1}\boldsymbol{S}^H)^{\frac{1}{2}}\right)^2$.The worst–case will happen when all rows of $\boldsymbol{H}_t$, ${\boldsymbol{h}_i}^H$, are in the direction of eigenvector corresponding to largest eigenvalue of matrix $\boldsymbol{T}$. Therefore (14) can be rewritten as:

$$Min_{\boldsymbol{S}}\ \lambda_{max}(\boldsymbol{T}) \tag{15}$$

By setting

$$\boldsymbol{D} = (\boldsymbol{S}\boldsymbol{C}_{\boldsymbol{W}}^{-\boldsymbol{1}}\boldsymbol{S}^{\boldsymbol{H}})^{-1},$$

eigenvalues of $\boldsymbol{T}$ are:

$$\lambda(\boldsymbol{T}) = (1 - \lambda^{-\frac{1}{2}}(\boldsymbol{D}))^2 \tag{16}$$

If all of the eigenvalues of $\boldsymbol{D}$ are larger than 1 then it can be seen from (16) that $\lambda(\boldsymbol{T})$ is monotonically increasing in $\lambda(\boldsymbol{D})$ and therefore $\lambda_{max}(\boldsymbol{T})$ will correspond to $\lambda_{max}(\boldsymbol{D})$, but if $\boldsymbol{D}$ has some eigenvalues smaller than 1, $\lambda_{max}(\boldsymbol{T})$ may correspond to $\lambda_{min}(\boldsymbol{D})$. So (15) can be rewritten as the nonlinear optimization:

$$Min_{\boldsymbol{S}} \max\left\{\left(1 - {\lambda_{max}}^{-\frac{1}{2}}(\boldsymbol{D})\right)^2, \left(1 - {\lambda_{min}}^{-\frac{1}{2}}(\boldsymbol{D})\right)^2\right\}. \tag{17}$$

This is a nonlinear optimization problem and in general we must resort to numerical optimization techniques to solve for the optimal transmit waveform **S**. As discussed earlier, CSLS estimator is used in low to moderate SNR. Hence due to the fact that

$$\boldsymbol{D} = (\boldsymbol{S}\boldsymbol{C}_W^{-1}\boldsymbol{S}^H)^{-1} = \boldsymbol{R}_c + \sigma^2.r\boldsymbol{\mathcal{S}}^{-1} \tag{18}$$

eigenvalues of $\boldsymbol{D}$ are generally more than 1. Therefore we can relax the nonlinear optimization problem (17) by assuming that all the eigenvalues of $\boldsymbol{D}$ are larger than 1:

$$\max\left\{\left(1 - {\lambda_{max}}^{-\frac{1}{2}}(\boldsymbol{D})\right)^2, \left(1 - {\lambda_{min}}^{-\frac{1}{2}}(\boldsymbol{D})\right)^2\right\}$$

$$= \left(1 - {\lambda_{max}}^{-\frac{1}{2}}(\boldsymbol{D})\right)^2 \tag{19}$$

Therefore (17) becomes:

$$Min_{\boldsymbol{S}} \left(1 - {\lambda_{max}}^{-\frac{1}{2}}(\boldsymbol{D})\right)^2$$

that is equivalent to $Min_{\boldsymbol{S}}\ \lambda_{max}(\boldsymbol{D})$. Furthermore $\lambda_{max}(\boldsymbol{D})$ can be expressed as the solution to the SDP problem:

$$Min_{\lambda}\ \lambda \ \ subject\ to\text{:}\ \ \lambda\boldsymbol{I} \succcurlyeq \boldsymbol{D} \tag{20}$$

Therefore the optimal transmit signals for CSLS estimator can be found as the solution to the following SDP problem:

$$Min_{\boldsymbol{\mathcal{S}},\lambda}\ \lambda$$

Subject to

$$\begin{bmatrix} \lambda\boldsymbol{I} - \boldsymbol{R}_c & \boldsymbol{I} \\ \boldsymbol{I} & \frac{1}{\sigma^2.r}\boldsymbol{\mathcal{S}} \end{bmatrix} \succcurlyeq 0 \tag{21}$$

$$tr\{\boldsymbol{\mathcal{S}}\} = P$$

$$\boldsymbol{\mathcal{S}} \succ 0$$

We used Lemma 1 to express $\lambda\boldsymbol{I} \succcurlyeq \mathbf{D} = \boldsymbol{R}_c + \sigma^2.r\boldsymbol{\mathcal{S}}^{-1}$as an LMI in (21).

## 5. NUMERICAL RESULTS

In this section, we present numerical examples in order to illustrate the waveform design solutions derived in this paper. Here we consider a simple MIMO radar case with $t = 4$ and $r = 3$. We assume the additive white Gaussian noise. In all the simulations we fix target power, $tr(\boldsymbol{R_t})$, at 30 and select eigenvalues of $\boldsymbol{R_t}$ such that $\lambda_i(\boldsymbol{R_t}) \in [1,15]\ for\ i = 1,..,4$. The SINR at the input of the estimator is defined by $\frac{tr(\boldsymbol{R_t})}{tr(\boldsymbol{R_c})+\frac{\sigma^2.r.t}{P}}$. It can be seen that SINR is a function of both $tr(\boldsymbol{R_c})$ and $\frac{\sigma^2.r.t}{P}$.

Fig. 1 illustrates the MSE of MMSE estimator under both the optimal and equal power allocation schemes ($\boldsymbol{SS^H} = \frac{P}{t}\boldsymbol{I}$) versus SINR values for fixed $tr(\boldsymbol{R_c})$ of 50. It shows that equal power waveforms are asymptotically optimum in the sense of minimizing the MSE. Fig 1 also compares MMSE and CSLS performances. As we expected CSLS is very conservative but it will guarantee the fix value of error in all cases.

When there is no knowledge about target statistics, a class of estimators is employed which assume deterministic signal model and only needs the knowledge of noise statistics in order to obtain $\widehat{\boldsymbol{H_t}}$. ML estimator is the most popular estimator of this class and by following similar procedure suggested in this paper, it is easy to see that the ML optimal waveforms are orthogonal equal power schemes ($\boldsymbol{SS^H} = \frac{P}{t}\boldsymbol{I}$). In Fig. 2 we plot the MSE in estimating $\boldsymbol{H_t}$ by using CSLS and ML estimators for different values of SINR for fixed $\frac{\sigma^2.r.t}{P}$ equal to 1. Since ML estimator cannot discriminate between $\boldsymbol{H_t}$ and $\boldsymbol{H_c}$, for high clutter power ML estimator leads to the large MSE values. Instead of ML estimator, in this work we proposed CSLS estimator that exhibits a much better performance than ML estimator in high clutter power.

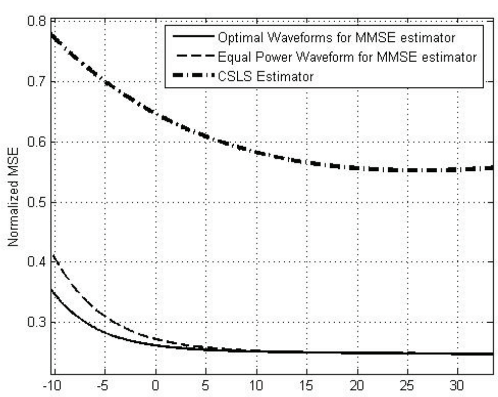

Fig. 1 CSLS estimator compared by MMSE estimator under both optimal and equal-power waveforms.

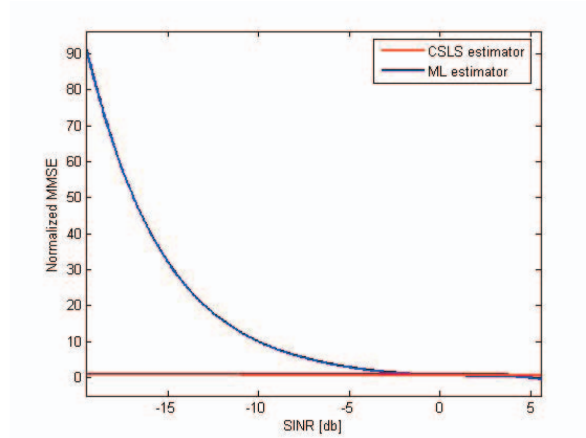

Fig. 2 CSLS estimator compared by ML estimator, both under their optimal transmitted signals

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