BLIND ADAPTIVE BEAMFORMING FOR WIDEBAND CIRCULAR ARRAYS

Wei Liu

Communications Research Group
Department of Electronic and Electrical Engineering
University of Sheffield, Sheffield, UK
w.liu@sheffield.ac.uk

Abstract. An approach for blind adaptive wideband beamforming is proposed based on a uniform circular array. The received array signals are first transformed into different phase modes and each phase mode output is then processed by a filter to achieve a frequency independent response. As a result, a set of instantaneous mixtures of the original source signals is obtained and the original wideband beamforming problem can be readily solved using the standard instantaneous BSS algorithms and the original source signals can be recovered one by one or simultaneously, depending on the specific requirement.

Keywords. blind beamforming, circular arrays, wideband, complexity

1. INTRODUCTION

Beamforming has found many applications in various areas ranging from sonar and radar to wireless communications [1]. For wideband signals, it is usually achieved by the use of tapped delay-lines (TDLs) or FIR/IIR filters in its discrete form, which can form a frequency dependent response for each of the received wideband sensor signals to compensate the phase difference for different frequency components. To perform adaptive beamforming with high interference rejection and resolution, we need to employ a large number of sensors and long TDLs or FIR/IIR filters, which leads to a large number of adaptive coefficients and unavoidably increases the computational complexity of its adaptive part and slows down the convergence of the system. To alleviate this problem, many methods have been proposed, such as the subband adaptive and beamspace adaptive beamformers [2, 3].

In this paper, we will focus on the class of uniform circular arrays (UCAs) and propose a novel approach for wideband adaptive beamforming with a significantly reduced number of adaptive coefficients. Compared to uniform linear arrays, uniform circular arrays can achieve uniform resolution at the azimuth angle over the full range of 360° [4, 5, 6, 7, 8]. In the proposed scheme, the received array signals are first transformed into different phase modes and each phase mode output is then processed by a filter to compensate the frequency dependent terms. As a result, a set of instantaneous mixtures of the original source signals is obtained and the original wideband beamforming problem is transformed into an instantaneous blind source separation (BSS) problem [9, 10, 11]. This BSS problem can be readily solved using the standard instantaneous BSS algorithms and the original source signals can be recovered one by one or simultaneously, depending on the specific requirement. This is done without estimating the direction of arrival (DOA) angle of the signal of interest. Therefore it can be considered as a blind wideband beamforming scheme to differentiate it from the traditional adaptive beamforming approaches.

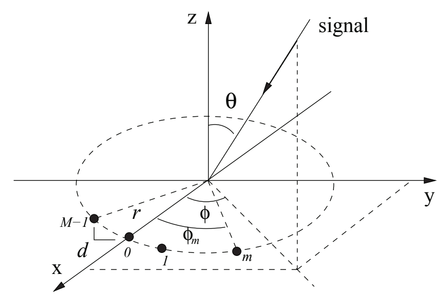

Figure 1: A uniform circular array with M sensors and a circumferential sensor spacing d and a radius r, where a signal impinges from the direction (θ, ϕ).

This paper is organised as follows. The proposed beamforming scheme is introduced in Section 2, followed by simulations in Section 3; conclusions are drawn in Section 4.

2. PROPOSED WIDEBAND ADAPTIVE BEAMFORMING SCHEME FOR CIRCULAR ARRAYS

Consider the uniform circular array with M omnidirectional sensors and a circumferential sensor spacing of d, as shown in Figure 1, with its radius $r = \frac{Md}{2\pi}$. The position of the $m-th$ sensor is given by $(r\cos\phi_m,\, r\sin\phi_m,\, 0)$, $m = 0, 1, \ldots, M-1$, where $\phi_m = m\frac{2\pi}{M}$ is the angle measured from the x axis to the $m-th$ sensor. The sensor spacing d is $\alpha/2$ times the wavelength $\lambda_{\min}$ of the highest frequency component of the impinging signal, i.e. $d = \alpha\frac{\lambda_{\min}}{2}$, then we have

$$r = \alpha\frac{\lambda_{\min}}{2}\frac{M}{2\pi} = \alpha\beta\frac{\lambda_{\min}}{2} \;, \tag{1}$$

where $\beta = \frac{M}{2\pi}$.

For a signal with an angular frequency ω, the phase difference between the center of the circular array and the $m-th$ sensor is given by

$$\Phi = \alpha\beta\frac{\omega\lambda_{\min}}{2c}\sin\theta\cos(\phi - \phi_m) \;, \tag{2}$$

where c is propagation speed of the signal.

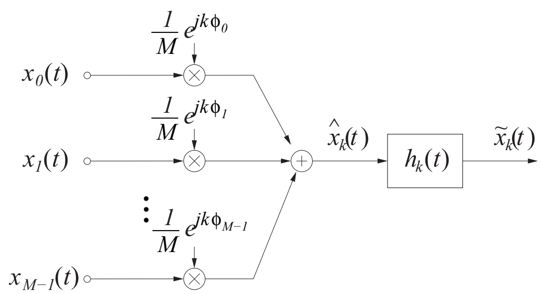

Figure 2: Processing of the received array signals for the $k-th$ phase mode by the weight vector $\mathbf{w}_k$ and the following filter $h_k(t)$, with $\hat{x}_k(t)$ and $\tilde{x}_k(t)$ as the respective outputs.

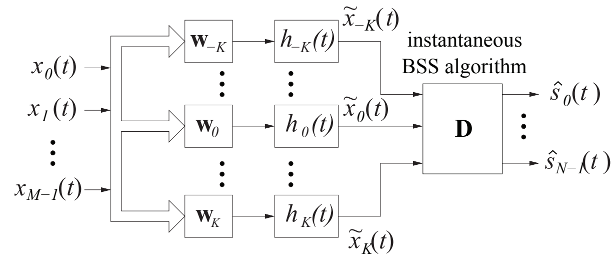

Figure 3: The $2K+1$ processing blocks with outputs $\tilde{x}_k(t)$, $k=-K,\ldots,-1,0,1,\ldots,K$, followed by an instantaneous BSS algorithm to recover the source signals.

In the discrete form, if we sample the signal at the Nyquest rate, i.e. the sampling period $T_s = \frac{\lambda_{\min}}{2c}$, then we have $\frac{\omega\lambda_{\min}}{2c} = \Omega$, where $\Omega = \omega T_s$ is the normalised angular frequency of the signal. Without loss of generality, we assume the signals arrive from the (x,y) plane, i.e. $\theta = \frac{\pi}{2}$. Then the $m-th$ element of the steering vector $\mathbf{d}(\theta,\phi,\Omega)$ of this uniform circular array is given by

$$d_m = e^{j\alpha\beta\Omega\cos(\phi-\phi_m)} , \tag{3}$$

which can be expanded to the following form [12]

$$d_m = \sum_{n=-\infty}^{+\infty} j^n J_n(\alpha\beta\Omega) e^{jn(\phi-\phi_m)} , \tag{4}$$

where J_n is the Bessel function of the first kind.

Applying a normalised weight vector $\mathbf{w}_k$ to the received array signals $x_m(t)$, $m=0,1,\ldots,M-1$, as shown in Figure 2, we obtain the output $\hat{x}_k(t)$, given by

$$\hat{x}_k(t) = \mathbf{w}_k^H \mathbf{x}(t) , \tag{5}$$

where

$$\begin{aligned} \mathbf{w}_k &= \frac{1}{M}[e^{jk\phi_0}\ e^{jk\phi_1}\ \cdots\ e^{jk\phi_{M-1}}]^H \\ \mathbf{x}(t) &= [x_0(t)\ x_1(t)\ \cdots\ x_{M-1}(t)]^T . \end{aligned} \tag{6}$$

The response of this beamformer with the weight vector $\mathbf{w}_k$ is given by [4, 6]

$$\begin{aligned} P_k(\phi,\Omega) &= \mathbf{w}_k^H \mathbf{d}(\phi,\Omega) \\ &= j^k J_k(\alpha\beta\Omega)e^{jk\phi} + \sum_{n=k+lM} j^n J_n(\alpha\beta\Omega)e^{jn\phi} , \end{aligned} \tag{7}$$

where l is a non-zero integer ($l \in \mathbf{Z}$ but $l \neq 0$).

The higher-order components in Equation (7) can be ignored if the absolute value of k does not exceed some threshold K. In this case, $P_k(\phi,\Omega)$ can be approximated by

$$P_k(\phi,\Omega) \approx j^k J_k(\alpha\beta\Omega)e^{jk\phi} \tag{8}$$

for $|k| \leq K$. According to [4, 6], for each frequency Ω, we can choose

$$K(\Omega) \approx \alpha\beta\Omega \tag{9}$$

and

$$K(\Omega) < \frac{M}{2} , \tag{10}$$

where $K(\Omega)$ means the value of K is dependent on Ω.

To meet both conditions, the limit is to choose the circumferential sensor spacing d to be half the wavelength of the corresponding frequency Ω. For wideband signals with a frequency range $\Omega \in [\Omega_{\min};\ \Omega_{\max}]$, we will choose $d = \frac{\lambda_{\min}}{2}$, i.e. $\alpha = 1$ and $K \approx \beta\Omega_{\min}$.

Note the beam pattern in Equation (8) is frequency dependent. In order to achieve a frequency independent response, we further process the output $\hat{x}_k(t)$ of the $k-th$ phase mode by a filter $h_k(t)$, which has a frequency response $H_k(\Omega) = \frac{1}{j^k J_k(\alpha\beta\Omega)}$ and its output is given by

$$\tilde{x}_k(t) = h_k(t) * \hat{x}_k(t) = h_k(t) * (\mathbf{w}_k^H \mathbf{x}(t)) , \tag{11}$$

where $*$ denotes the convolution operation. The beam response with the weight vector $\mathbf{w}_k$ and the filter $h_k(t)$ can be expressed as

$$\hat{P}_k(\phi) = e^{jk\phi} \tag{12}$$

Suppose we have L impinging plane-wave signals $s_l(t)$ from the azimuth angle $\hat{\phi}_l$, $l=0,\ldots,L-1$, respectively. All of them are bandlimited to the range $\Omega \in [\Omega_{\min};\ \Omega_{\max}]$. Then after the processing shown in Figure 2, the output $\tilde{x}_k(t)$ can be expressed as

$$\tilde{x}_k(t) = \sum_{l=0}^{L-1} e^{jk\hat{\phi}_l} s_l(t) = \mathbf{a}_k^T \mathbf{s}(t) , \tag{13}$$

where

$$\begin{aligned} \mathbf{a}_k &= \left[e^{jk\hat{\phi}_0}\ e^{jk\hat{\phi}_1}\ ,\cdots, e^{jk\hat{\phi}_{L-1}}\right]^T \\ \mathbf{s}(t) &= [s_0(t)\ s_1(t)\ \cdots\ s_{L-1}(t)]^T . \end{aligned} \tag{14}$$

Clearly, after the processing of $\mathbf{w}_k$ and $h_k(t)$, its output $\tilde{x}_k(t)$ becomes a simple weighted sum of the L wideband source signals.

If we set up $2K+1$ such processing blocks with a weight vector $\mathbf{w}_k$ and the corresponding filter $h_k(t)$, $k=-K,\ldots,-1,0,1,\ldots,K$, as shown in Figure 3, their outputs $\tilde{x}_k(t)$, $k=-K,\ldots,-1,0,1,\ldots,K$, can be expressed in a vector form as follows

$$\tilde{\mathbf{x}}(t) = \mathbf{A}^T \mathbf{s}(t) , \tag{15}$$

where

$$\begin{aligned} \tilde{\mathbf{x}}(t) &= [\tilde{x}_{-K}(t)\ \cdots\ \tilde{x}_{-1}(t)\ \tilde{x}_0(t)\ \tilde{x}_1(t)\ \cdots\ \tilde{x}_K(t)]^T \\ \mathbf{A} &= [\mathbf{a}_{-K}\ \cdots\ \mathbf{a}_{-1}\ \mathbf{a}_0\ \mathbf{a}_1\ \cdots \mathbf{a}_K] . \end{aligned} \tag{16}$$

Since the matrix $\mathbf{A}$ is independent of signal frequency and its entries are of a constant value for fixed signal arrival angles $\hat{\phi}_l$, $l = 0, \ldots, L-1$, Equation (15) now represents a classical instantaneous mixing problem in the well-known blind source separation area [10], and we can recover the L source signals blindly from the mixtures $\tilde{\mathbf{x}}(t)$ without estimating their arrival angles or knowing the mixing matrix $\mathbf{A}$. This can be achieved by employing the standard instantaneous BSS algorithms, such as those employing second-order statistics or higher-order statistics [10], as shown in Figure 3, where $\mathbf{D}$ is the demixing matrix and should be an inverse of the mixing matrix $\mathbf{A}$ up to some scaling and permutation for $2K+1 \geq L$ after the source signals are successfully recovered/separated. Since the proposed scheme does not need the DOA (direction of arrival) information of the source signals, it can be considered as a blind wideband beamforming approach [11].

Many instantaneous BSS algorithms can be applied to the mixtures $\tilde{\mathbf{x}}(t)$ to recover the source signals and here we only consider a simple example to illustrate the proposed blind beamforming scheme. Suppose the source signals are independent of each other and at most one of them is Gaussian. Then a higher-order statistics based instantaneous BSS algorithm can be applied here. In the simulations part, we will use a normalised kurtosis based blind source extraction (BSE) algorithm to extract one of source signals [10], instead of recovering all of the L source signals simultaneously in one single step. The update equation for this algorithm is given by

$$\mathbf{d}_l[n+1] = \mathbf{d}_l[n] + \mu\phi(\hat{s}_l[n])\tilde{\mathbf{x}}[n] \; , \tag{17}$$

where $\mathbf{d}_l[n]$ is the extraction vector applied to the mixtures $\tilde{\mathbf{x}}$, $\hat{s}_l[n] = \mathbf{d}_l^T[n]\tilde{\mathbf{x}}[n]$ is the extracted source signal, and

$$\phi(\hat{s}_l[n]) = \beta\frac{m_4(\hat{s}_l)}{m_2(\hat{s}_l)^3}\left[\frac{m_2(\hat{s}_l)}{m_4(\hat{s}_l)}\hat{s}_l^3[n] - \hat{s}_l[n]\right] \tag{18}$$

with

$$m_q(\hat{s}_l)[n] = (1-\lambda)m_q(\hat{s}_l)[n-1] + \lambda|\hat{s}_l[n]|^q, \quad q = 2, 4 \; . \tag{19}$$

λ is the forgetting factor and $\beta = 1$ for the extraction of source signals with positive kurtosis and -1 for sources with negative kurtosis.

The proposed approach has an advantage that it simplifies the adaptive beamforming process for wideband signals greatly, since there is no need to estimate the DOA angle of the signal of interest and the length $2K+1$ of the adaptive demixing vector $\mathbf{d}_i$ is much shorter than the length of the adaptive coefficients in the traditional TDL-based direct beamforming approaches.

3. SIMULATIONS

The simulations are based on a UCA with 10 sensors and the circumferential adjacent sensor spacing d is chosen to be half wavelength corresponding to the highest frequency $\alpha = 1$. There are three source signals, two of which are non-Gaussian and the other one is Gaussian. Their normalised kurtosis values are -1.38, -0.32, and -0.04, respectively. They arrive at the array from three different directions $\hat{\phi} = 0^\circ$, -80° and 110°, and have been pre-filtered and bandlimited to the range $\Omega \in [0.5\pi; \pi]$. According to [10], when we apply the algorithm ($\beta = -1$) in (17) to the processed signals $\tilde{\mathbf{x}}(t)$, the one with the minimum kurtosis value -1.38 (the first one with the DOA angle 0°) will be extracted. We can assume that the first signal is the signal of interest and the other two are interfering signals. The signal to interference ratio (SIR) is about 0 dB and the signal to noise ratio (SNR) is about 20 dB.

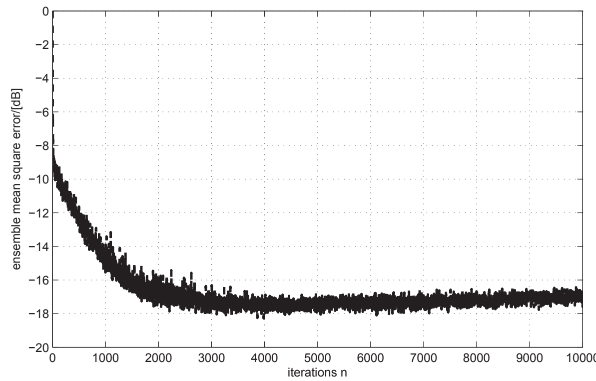

Figure 4: Learning curve of the proposed method using the algorithm in (17).

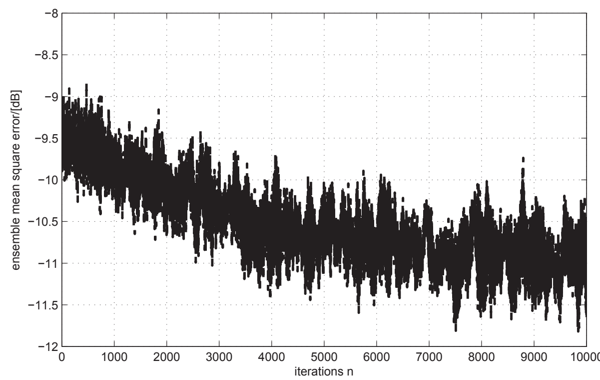

Figure 5: Learning curve of the generalised sidelobe canceller based on the same uniform linear array.

With $\beta = \frac{M}{2\pi} = \frac{5}{\pi}$, we have

$$K \approx \beta\Omega_{\min} = \frac{5}{\pi} \times 0.5\pi = 2.5 \approx 2 \; . \tag{20}$$

Therefore in total we can have $2 \times 2 + 1 = 5$ phase mode outputs. However, we will only choose the three phase modes $k = -2, 0, 2$ in our simulation. Each of the filter $h_k(t)$ is designed according to the requirement $H_k(\Omega) = \frac{1}{j^k J_k(\alpha\beta\Omega)}$ and realised by a 128-tap FIR filter. The reason for ignoring the modes $k = -1, 1$ is that for these two modes, there is a point with zero response value for $J_k(\alpha\beta\Omega)$ in the frequency range $\Omega \in [0.5\pi; \pi]$, which will lead to an unstable filter after the inversion. This is a limitation for the proposed approach and one solution is to employ multiple-ring circular arrays [7, 8].

The learning curve of the BSE algorithm in Equation (17) is shown in Fig. 4, with $\mu = 0.001$ and $\lambda = 0.03$. The ensemble mean square error is the one between the extracted signal and the first source signal averaged over 1000 runs. As a reference, we also show the learning curve of a generalised sidelobe canceller

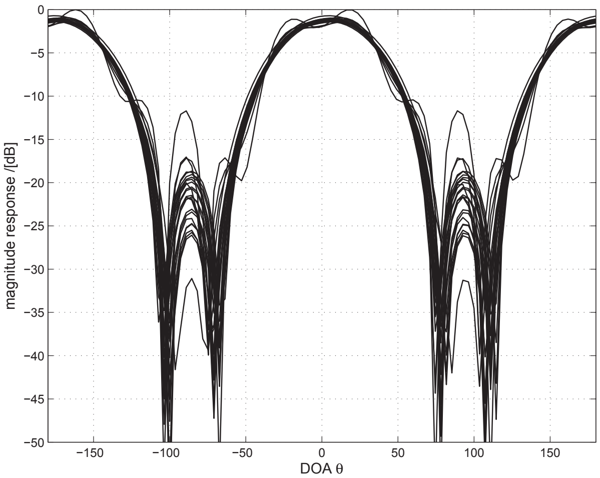

Figure 6: The resultant beam pattern of the proposed method for the frequency range $\Omega \in [0.5\pi; \pi]$.

(GSC) [13, 14] in Fig. 5 based on the same structure, using a normalised LMS adaptive algorithm with a step size of 0.03 [15]. In the GSC, the TDL length is 128 and total number of adaptive coefficients is $(10-1)*128 = 1152$. Note that the GSC needs the DOA information of the signal of interest to design the linear constraints, while the proposed method works without any such information. So these two curves are not directly comparable. However, at least we can see that the proposed method can achieve a very fast convergence speed due to the very small number of adaptive coefficients involved, which is only 3 in this case since we only have three phase mode outputs and much smaller than 1152 of the GSC. Moreover, the steady state error of the GSC is higher than the proposed approach. To reduce the steady state error, we can have a smaller step size for the normalised LMS algorithm, but this means that the convergence speed of the GSC will be even slower compared to the proposed method. The resultant beam pattern for the proposed method is shown in Fig. 6, where the null at the interference direction 110° is clearly visible. For the direction -80°, although the null is not as clear as the one at 110°, the attenuation is about 20 dB.

4. CONCLUSIONS

A novel approach has been proposed for adaptive wideband beamforming with a significantly reduced number of adaptive coefficients. This approach is based on a uniform circular array and achieved by first transforming the received array signals into different phase modes and then processed by a filter to compensate the frequency dependence of the response. As a result, a set of instantaneous mixtures of the original source signals is obtained and the original wideband beamforming problem is transformed into an instantaneous BSS problem, which can be readily solved using the standard instantaneous BSS algorithms. Since no DOA information is required for the proposed scheme, it can be considered as a blind wideband beamforming approach. Simulation results have shown that it can successfully extract the signal of interest.

5. REFERENCES

[1] H. L. Van Trees, *Optimum Array Processing, Part IV of Detection, Estimation, and Modulation Theory*, John Wiley & Sons, Inc., New York, U.S.A., 2002.

[2] W. Liu, S. Weiss, and L. Hanzo, "A subband-selective broadband GSC with cosine-modulated blocking matrix," *IEEE Transactions on Antennas and Propagation*, vol. 52, pp. 813–820, March 2004.

[3] W. Liu, R. Wu, and R. Langley, "Design and analysis of broadband beamspace adaptive arrays," *IEEE Transactions on Antennas and Propagation*, vol. 55, no. 12, pp. 3413–3420, December 2007.

[4] D. E. N. Davies, "Circular arrays," in *The Handbook of Antenna Design*, A. W. Rudge, K. Milne, A. D. Olver, and P. Knight, Eds., vol. 2, chapter 12. Peregrinus, London, 1983.

[5] R. Eiges and H. D. Griffiths, "Mode-space spatial spectral estimation for circular arrays," *IEE Proceedings – Radar, Sonar and Navigation*, vol. 141, no. 6, pp. 300–306, December 1994.

[6] C. P. Mathews and M. D. Zoltowski, "Eigenstructure techniques for 2-D angle estimation with uniform circular arrays," *IEEE Transactions on Signal Processing*, vol. 42, no. 9, pp. 2395–2407, September 1994.

[7] E. D. Di Claudio, "Asymptotically perfect wideband focusing of multiring circular arrays," *IEEE Transactions on Signal Processing*, vol. 53, pp. 3661–3673, October 2005.

[8] S. C. Chan and H. H. Chen, "Uniform concentric circular arrays with frequency-invariant characteristics–theory, design, adaptive beamforming and DOA estimation," *IEEE Transactions on Signal Processing*, vol. 55, pp. 165–177, January 2007.

[9] A. Hyvarinen, J. Karhunen, and E. Oja, *Independent Component Analysis*, John Wiley & Sons, Inc., New York, 2001.

[10] A. Cichocki and S. Amari, *Adaptive Blind Signal and Image Processing*, John Wiley & Sons, Inc., New York, 2003.

[11] W. Liu and D. P. Mandic, "Semi-blind source separation for convolutive mixtures based on frequency invariant transformation," in *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing*, Philadelphia, USA, March 2005, vol. 5, pp. 285–288.

[12] M. Abramowitz and I. A. Stegun, Eds., *Handbook of Mathematical Functions*, Dover Publications, New York, 1970.

[13] L. J. Griffiths and C. W. Jim, "An alternative approach to linearly constrained adaptive beamforming," *IEEE Transactions on Antennas and Propagation*, vol. 30, no. 1, pp. 27–34, January 1982.

[14] K. M. Buckley and L. J. Griffith, "An adaptive generalized sidelobe canceller with derivative constraints," *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 3, pp. 311–319, March 1986.

[15] S. Haykin, *Adaptive Filter Theory*, Prentice Hall, Englewood Cliffs, 3rd edition, 1996.