DEGRADED IMAGE ANALYSIS USING ZERNIKE MOMENT INVARIANTS

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ABSTRACT

In real imaging system, the observed image is usually corrupted by blurring, spatial degradations. The classical recognition methods in degraded image analysis are to obtain blur invariants based on geometric moments or complex moments. In this paper, we introduce blur invariants based on Zernike moments which are orthogonal over a unit circle. Both the expression of Zernike moments of blurred image and the set of blur invariants based on Zernike moments are presented and proved mathematically. Compared with the pattern classification results of complex moments, the experimental results of Zernike moment demonstrate that the proposed method performs well in object and pattern recognition.

***Index Terms*—** Zernike moments, radial moments, blur invariants, pattern recognition, classification.

1. INTRODUCTION

Since real imaging systems are usually imperfect, the observed image presents only a degraded version of the original scene, which makes the processing and recognition of images challenging. Applying a set of moment invariants with respect to blurring will be useful to solve this problem. Traditional methods for obtaining blur invariants are commonly based on geometric central moments or complex moments. Jan Flusser and Tomas Suk first introduced the blur, and combined blur and affine moment invariants based on geometric central moments [1-2]. In [3], a set of rotation, scale, translation and blur invariants based on complex moments were introduced. However, the kernel functions of central moments and complex moments are not orthogonal. It is proven that these moments suffer from high redundancy and overlap between moments of different orders [4]. In [5], Teague suggested the Zernike moments based on orthogonal Zernike polynomials to overcome these problems. Zernike moments have a simple rotation property due to its separable nature of angular dependence [4], and its translation invariants have been reported in [6]. However, its blur invariants have not been introduced until now.

In this paper, a novel blur invariants set based on the Zernike moments has been proposed. We give a detailed derivation for obtaining this set. The experimental results demonstrate that the proposed method performs well in pattern classification tasks.

2. MATHEMATICAL BACKGROUND

2.1. Definition of image blurring

Let $f(x, y)$ be an original image function and $h(x, y)$ be the point spread function (PSF). The blurred image can be described by the convolution as

$$g(x,y) = f(x,y) \otimes h(x,y) \tag{1}$$

Here, $\iint_{R^2} h(x,y)dxdy = 1$

The equation of a Gaussian PSF in two dimensions is

$$h(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \tag{2}$$

2.2. Zernike moment

The two-dimensional Zernike moments of an image intensity function $f(r,\theta)$ are defined as follows:

$$Z_{pq} = \frac{p+1}{\pi} \int_0^{2\pi} \int_0^1 V_{pq}^*(r,\theta) f(r,\theta) r dr d\theta \qquad |r| \le 1 \tag{3}$$

where Zernike polynomials of order p with repetition q ($q=0, \pm1, \pm2, \ldots$), and $V_{pq}(r,\theta)$ is defined as

$$V_{pq}(r,\theta) = R_{pq}(r) e^{jq\theta} \tag{4}$$

radial polynomial is given as

$$R_{pq}(r) = \sum_{k=0}^{(p-|q|)/2} \frac{(-1)^k (p-k)! r^{p-2k}}{k!((p+|q|)/2-k)!((p-|q|)/2-k)!} \tag{5}$$

where $0 \le |q| \le p$ and $p-|q| = even$.

Due to the symmetry property of Zernike polynomials, $Z_{p,-q}$ has same property as Z_{pq} . For simplification, only $q>0$ will be considered in this study. The relationship between Zernike moments Z_{pq} and radial moments D_{pq} are given as follows [7]

$$Z_{pq} = \frac{p+1}{\pi} \sum_{k=q}^{p} B_{pqk} D_{kq} \tag{6}$$

where

$$B_{pqk} = \frac{(-1)^{(p-k)/2} ((p+k)/2)!}{((p-k)/2)!((k+q)/2)!((k-q)/2)!} \tag{7}$$

The radial central moment D_{pq} is defined as follows

$$D_{pq} = \int_x \int_y [(x-x_0) - j(y-y_0)]^{(p+q)/2} \times [(x-x_0)+j(y-y_0)]^{(p-q)/2} f(x,y)dxdy \qquad (8)$$

where *p-q=even*.

3. BLUR INVARIANTS OF ZERNIKE MOMENTS

In this section, the blur property of Zernike moments is investigated. Theorem 1 gives the Zernike moments of degraded image.

Theorem 1. The blurred image *g(x, y)* is derived from the convolution image *f(x, y)* with PSF. The expression of Z^g_{pq} is

$$Z^g_{pq} = \frac{p+1}{\pi} \sum_{k=q}^{p} \sum_{n=0}^{(k-q)/2} \sum_{m=0}^{(k+q)/2} \sum_{s=0}^{k-q-2n} \sum_{t=0}^{2n} \binom{(k+q)/2}{m} \binom{(k-q)/2}{n} \times B_{pqk} \gamma_{(q-m+n+s)} \gamma_{(m-n+t)} \delta_{(k-m-n)(q-m+n+s)} \delta_{(m+n)(m-n+t)} \times Z^h_{(q-m+n+s)(q-m+n)} Z^f_{(m-n+t)(m-n)} \qquad (9)$$

The proof is deferred to Appendix A. On the assumption that PSF *h(x, y)* is Gaussian PSF, we derive the set of blur invariants of Zernike moments by virtue of Eq.(9) and present it as Theorem 2.

Theorem 2. Zernike moment Z_{pq} (the subscript $p = q$) are not affected by Gaussian blurring. (For proof of Theorem 2, see Appendix B).

Here, a set of blur invariants from zero order to fourth order is listed below:

$\phi_1 = Z^f_{00}$, $\phi_2 = Z^f_{11}$, $\phi_3 = Z^f_{22}$, $\phi_4 = Z^f_{33}$, $\phi_5 = Z^f_{44}$, $\phi_6 = Z^f_{55}$

From the above invariants we can see that Zernike moment easily becomes invariant under blurring.

4. EXPERIMENT RESULT

In this section, the experiment is carried out to evaluate the proposed Zernike blur invariants. The five original images (size 100×100) used in the experiment are from Columbia object image database [8]. Each image is blurred by Gaussian blur with a 5×5 vector under the condition of different standard deviation σ. The original images are shown in Fig.1 and the images blurred by Gaussian blur with σ =5.0 are shown in Fig.2 as examples. In Table 1 we list the invariants $\phi_1, \phi_2 ... \phi_6$ of the original image Fig.1 (a), Fig.1 (b) and their blurred images, respectively. From the data in Table 1, one can see that $\phi_1, \phi_2 ... \phi_6$ remain invariant with respect to different standard deviation σ.

The second experiment provides the experimental study on the classification accuracy of Zernike moments and complex moments under blurring. In our recognition task we use the following feature vector

$$V = [\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6]$$

$$V' = [\mathrm{Re}(C^2_{30}C^3_{02}), \mathrm{Im}(C^2_{30}C^3_{02}), \mathrm{Re}(C_{04}C^4_{12}), \mathrm{Im}(C_{04}C^4_{12}), \mathrm{Re}\left(C_{40}C^2_{02}\right), C_{40}C_{04}]$$

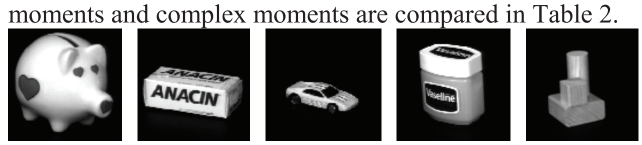
where the elements in vector *V'* are the blur invariant with respect to rotation and blurring based on complex moments [3]. Euclidean distance *d* is used as the classification measure and is defined by

$$d(V_s, V_t^{(k)}) = \sum_{j=1}^{T} (v_{sj} - v_{tj})^2 \qquad (10)$$

where V_s is the T-dimensional feature vector of unknown sample, and $V_t^{(k)}$ is the training vector of class k. And the classification accuracy η is defined as

$$\eta = \frac{\text{Number of correctly classified images}}{\text{The total number of images used in the test}} \times 100\% \qquad (11)$$

The test set comprises of 240 images, which are generated by Gaussian blur under $\sigma \in \{0.5, 1, 5, 8, 10\}$ and rotation with $\varphi \in \{\pm\frac{\pi}{6}, \pm\frac{\pi}{3}, \pm\frac{\pi}{2}, \pm\pi\}$. Fig.3 shows some of the testing images. The recognition accuracy of Zernike moments and complex moments are compared in Table 2.



(a) (b) (c) (d) (e)

Fig. 1. The original images

(a) (b) (c) (d) (e)

Fig. 2. Blurred images by Gaussian blur with σ =5.0

Table 1. Moment invariants of the blurred images of Fig.1 (a) and Fig.2 (b) under Gaussian blur with different σ.

		Original	σ =0.5	σ =1	σ =5
Fig.1(a)	ϕ_1	65.872	65.959	65.963	66.022
	ϕ_2	-5.5585	-5.6102	-5.6119	-5.639
	ϕ_3	-1.8174	-1.8584	-1.8605	-1.8822
	ϕ_4	9.3883	9.4688	9.4699	9.5058
	ϕ_5	1.9688	1.9702	1.9704	1.9748
	ϕ_6	-3.0092	-2.9526	-2.9502	-2.9196
Fig.1(b)	ϕ_1	84.538	84.511	84.46	84.37
	ϕ_2	86.307	86.275	86.182	85.999
	ϕ_3	99.291	99.255	99.129	98.815
	ϕ_4	79.318	79.301	79.377	79.597
	ϕ_5	103.09	103.04	102.83	102.28
	ϕ_6	96.818	96.76	96.451	95.627

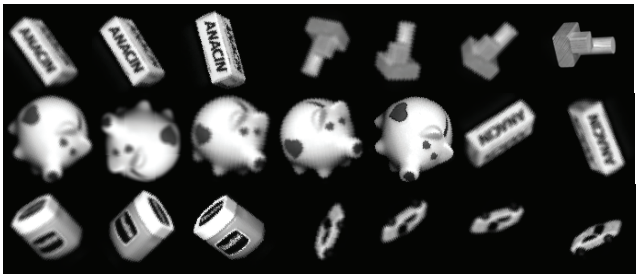


Fig. 3 part of the test images in the test set.

Table 2. Classification results of the blurred and rotated images.

	standard deviation σ				
	σ =0.5	σ =1	σ =2	σ =5	σ =10
Complex	95.3%	94.67%	94.33%	93.75%	93.32%
Zernike	98.17%	97.77%	96.92%	96.58%	95.41%

From Table 2, one can see that both the invariants based on Zernike moments and complex moments remain stable with the increase of standard deviation σ . However, the novel method performs better than the invariants based on complex moments in recognition of rotated and blurred images.

5. CONCLUSION

In this paper, we have introduced the blur invariants of Zernike moments for recognizing and classifying blur objects. This method eliminates the requirement of deblurring and normalization of recognition tasks. The results of simulation have illustrated that the invariant capability of the proposed invariants has advantages over the other similar ones. In the future work, the application of the method will be investigated with the images of complicated backgrounds and the comparison with Zernike moments and other orthogonal moments in degraded image analysis will be made.

APPENDIX A. PROOF OF THEORM 1

The derivation of Zernike moments of the blurred image

$$\frac{\pi}{p+1}Z_{pq}^{g}$$

$$=\sum_{k=q}^{p}B_{pqk}D_{kq}^{g}$$

$$=\sum_{k=q}^{p}B_{pqk}\iint[(x-x_0)-j(y-y_0)]^{(k+q)/2}[(x-x_0)$$

$$+j(y-y_0)]^{(k-q)/2}g(x,y)dxdy$$

$$=\sum_{k=q}^{p}B_{pqk}\iint[(x-x_0+a)-j(y-y_0+b)]^{(k-q)/2}[(x-x_0+a)$$

$$+j(y-y_0+b)]^{(k-q)/2}\iint h(a,b)f(x-a,y-b)dadbdxdy$$

$$=\sum_{k=q}^{p}B_{pqk}\iint\sum_{m=0}^{(k+q)/2}\binom{(k+q)/2}{m}[(x-x_0)-j(y-y_0)]^{m}$$

$$\times(a-jb)^{(k+q)/2-m}\sum_{n=0}^{(k-q)/2}\binom{(k-q)/2}{n}[(x-x_0)+j(y-y_0)]^{n}$$

$$\times(a+jb)^{(k-q)/2-n}\iint h(a,b)f(x,y)dxdydadb$$

$$=\sum_{k=q}^{p}\sum_{n=0}^{(k-q)/2}\sum_{m=0}^{(k+q)/2}\binom{(k+q)/2}{m}\binom{(k-q)/2}{n}B_{pqk}\iint[(a-0)$$

$$-j(b-0)]^{(k+q)/2-m}\iint[(a-0)+j(b-0)]^{(k-q)/2-n}h(a,b)dadb$$

$$\times\iint[(x-x_0)-j(y-y_0)]^{m}[(x-x_0)+j(y-y_0)]^{n}f(x,y)dxdy$$

$$=\sum_{k=q}^{p}\sum_{n=0}^{(k-q)/2}\sum_{m=0}^{(k+q)/2}\binom{(k+q)/2}{m}\binom{(k-q)/2}{n}$$

$$\times B_{pqk}D_{k-m-n,q-m+n}^{h}D_{m+n,m-n}^{f} \qquad \text{(A.1)}$$

here,

$$D_{pq}=\sum_{s=0}^{a}\gamma_{q+s}\delta_{p(q+s)}Z_{(q+s)q} \qquad \text{(A.2)}$$

where

$$\gamma_{q+s}=\frac{1}{\lambda_{q+s}B_{(q+s)q(q+s)}}$$

$$\delta_{pp}=1,\delta_{p(q+s)}=-C_{p(q+s)}+\sum_{b=2}^{a-s-2}-C_{(p-b)(q+s)}\delta_{p(p-b)}$$

for $p\neq(q+s)$, and a=p-q, a-s= even.

where
$$C_{p(q+s)}=\frac{B_{pq(q+s)}}{B_{pqp}}$$

where
$$B_{pqk}=\frac{(-1)^{(p-k)/2}((p+k)/2)!}{((p-k)/2)!((k+q)/2)!((k-q)/2)!}$$

subtitute Eq. (A.2) into Eq. (A.1), we get

$$Z_{pq}^{g}=\frac{p+1}{\pi}\sum_{k=q}^{p}\sum_{n=0}^{(k-q)/2}\sum_{m=0}^{(k+q)/2}\sum_{s=0}^{k-q-2n}\sum_{t=0}^{2n}\binom{(k+q)/2}{m}\binom{(k-q)/2}{n}$$

$$\times B_{pqk}\gamma_{(q-m+n+s)}\gamma_{(m-n+t)}\delta_{(k-m-n)(q-m+n+s)}\delta_{(m+n)(m-n+t)}$$

$$\times Z_{(q-m+n+s)(q-m+n)}^{h}Z_{(m-n+t)(m-n)}^{f} \qquad \text{(A.3)}$$

APPENDIX B. PROOF OF THEORM 2

The expression Z_{pq}^{g} of image *f(x, y)* after convolution with PSF is given in (A.3). If p=q, from (A.3), we get

$$Z_{pp}^{g}=\frac{p+1}{\pi}\sum_{m=0}^{p}\binom{p}{m}B_{ppp}\gamma_{(p-m)}\gamma_{(m)}\delta_{(p-m)(p-m)}\delta_{mm}Z_{(p-m)(p-m)}^{h}Z_{mm}^{f}$$

(B.1)

where

$$Z^h_{(p-m)(p-m)} = \frac{p-m+1}{\pi} B_{(p-m)(p-m)(p-m)} D^h_{(p-m)(p-m)}$$

where

$$D^h_{p-m,p-m} = \iint h(x,y)(x-iy)^{p-m}\,dxdy$$
$$= \iint h(x,y) \sum_{t=0}^{p-m} \binom{p-m}{t} (-i)^t x^{p-m-t} y^t\,dxdy$$
$$= \sum_{t=0}^{p-m} \binom{p-m}{t} (-i)^t \iint \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} x^{p-m-t} y^t\,dxdy$$
$$= \sum_{t=0}^{p-m} \binom{p-m}{t} (-i)^t g_{p-m-t} g_t$$

where g_i is defined as follows:

$$g_i g_j = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (x-0)^i (y-0)^j \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}\,dxdy$$

and $g_i = \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{+\infty} x^i e^{-\frac{x^2}{2\sigma^2}}\,dx$

$$= \begin{cases} 0 & \text{if } i \text{ is an odd number} \\ 1\cdot 3\cdots(i-1)\sigma^i & \text{if } i \text{ is an even number} \end{cases}$$

then

if *p-m* is odd, whatever *t* is odd or even $D^h_{p-m,p-m} = 0$

If *p-m* is even and *t* is odd $D^h_{p-m,p-m} = 0$

If *p-m* is even and *t* is even, let *t=2k*, then

$$D^h_{p-m,p-m} = \sum_{k=0}^{(p-m)/2} \binom{p-m}{2k} (-i)^{2k} g_{p-m-2k} g_{2k}$$

$$= \sum_{k=0}^{(p-m)/2} \binom{p-m}{2k} (-1)^k g_{p-m-2k} g_{2k} \qquad \text{(B.2)}$$

where

$$g_{2k} = 1\cdot 3\cdot 5\cdots(2k-1)\sigma^{2k} = \frac{(2k)!}{2^k\cdot(k)!}\sigma^{2k} \qquad \text{(B.3)}$$

$$g_{p-m-2k} = \frac{(p-m-2k)!}{2^{\frac{p-m}{2}-k}(\frac{p-m}{2}-k)!}\sigma^{p-m-2k} \qquad \text{(B.4)}$$

substitute (B.3) and (B.4)into (B.2), we get

$$D^h_{p-m,p-m}$$
$$= \sum_{k=0}^{(p-m)/2} \frac{(-1)^k (p-m)!}{(2k)!(p-m-2k)!} \frac{(2k)!}{2^k (k!)}$$
$$\times \frac{(p-m-2k)!}{2^{\frac{p-m}{2}-k}(\frac{p-m}{2}-k)!}\sigma^{2k}\sigma^{p-m-2k}$$

$$= \sigma^{p-m} \frac{(p-m)!}{2^{\frac{p-m}{2}}(\frac{p-m}{2})!} \sum_{k=0}^{(p-m)/2} \binom{\frac{(p-m)}{2}}{k} (-1)^k (1)^{(\frac{p-m}{2}-k)}$$

$$= \sigma^{p-m} \frac{(p-m)!}{2^{\frac{p-m}{2}}(\frac{p-m}{2})!} (-1+1)^{\frac{p-m}{2}} = 0 \qquad \text{(B.5)}$$

And, $Z^h_{00} = \frac{1}{\pi} B_{000} D^h_{00} = \frac{1}{\pi}$.

Therefore, when *p=q, $p \neq m$*, $Z^h_{(p-m)(p-m)} = 0$.

Substituting $Z^h_{(p-m)(p-m)} = 0$ into (B.1), we get

$$Z^g_{pp} = \frac{p+1}{\pi} B_{ppp}\gamma_0\gamma_p\delta_{00}\delta_{pp} Z^h_{00} Z^f_{pp}$$
$$= \frac{p+1}{\pi}\cdot 1\cdot\pi\cdot\frac{\pi}{p+1}\cdot 1\cdot 1\cdot\frac{1}{\pi}\cdot Z^f_{pp} = Z^f_{pp}$$

Therefore, when $p = q$, $Z^h_{pq} = 0$, and $Z^g_{pp} = Z^f_{pp}$.

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