

CONSTRAINED OPTIMISATION OF 3D POLYGONAL MESH WATERMARKING BY QUADRATIC PROGRAMMING

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ABSTRACT

In this paper, we propose a blind and robust watermarking method for 3D polygonal meshes by minimising the mean square error between the original mesh and the watermarked mesh under several constraints. We have formulated the problem of assigning distortions to points in a 3D mesh to a quadratic programming problem, so it can be solved reliably and efficiently. Comparing with similar approaches in [1], experiments indicate the advantages of our method in resisting Gaussian noise.

Index Terms— 3D watermarking, Quadratic programming

1. INTRODUCTION

Digital watermarking has become a widespread approach to protect intellectual property rights since 1990's. Although many watermarking algorithms have been developed for images, audio and visual signals, the area of watermarking 3D objects is not immensely studied. With the rapid progress of 3D technology in computer aided design, video games and manufacturing industry, the area of 3D watermarking has received much attention in the past years, as can be seen in [2].

A 3D object is usually represented by a 3D mesh, with points and connections in it. Schemes for watermarking 3D meshes can be categorised into transform-domain and spatial-domain methods. The transform-domain methods treat 3D watermarking as a common signal processing problem. Regular signal processing conceptions, such as wavelet transform, are implemented into the watermarking approaches [3]. For the spatial-domain methods, the position of each point is changed individually based on some kind of criteria. For example, Benedens [4] proposed a watermarking scheme based on modifying the histogram of surface normals. Because statistics is applied to the watermarking scheme, it is sustainable to common watermarking attacks. Other local statistics, such as the distributions of vertex norms and

moments, are also used for watermarking 3D objects [1, 5]. Experiments have proved that histogram-based schemes can resist common watermarking attacks except cropping.

However, most histogram-based algorithms are based on heuristic approaches [1, 5, 6], which do not ensure global optimisation of the watermarking process. In this paper, we propose a histogram-based watermarking algorithm by globally optimising distortions between the watermarked mesh and the original mesh, with several constraints to be satisfied. We take similar approaches as in [1] to modify the distribution of vertex norms, but our modification is executed by incorporating quadratic programming (QP) into the watermarking scheme to minimise the mean square error (MSE) between the original mesh and the watermarked mesh. Experiments indicate that the proposed method is more sustainable to Gaussian noise compared to other two methods in [1].

2. THE PROPOSED METHOD

In this section, we take the same notations and terminologies as in [1] because our research work follows theirs.

Firstly, the Cartesian coordinates of a point (x_i, y_i, z_i) in the mesh are converted into spherical coordinates $(\rho_i, \theta_i, \phi_i)$:

$$\begin{aligned}\rho_i &= \sqrt{(x_i - x_g)^2 + (y_i - y_g)^2 + (z_i - z_g)^2} \\ \theta_i &= \tan^{-1} \frac{(y_i - y_g)}{(x_i - x_g)} \\ \phi_i &= \cos^{-1} \frac{(z_i - z_g)}{\rho_i}, \quad i \in \{0, 1, \dots, L-1\}\end{aligned}\quad (1)$$

where L is the number of points in the mesh, ρ_i is the i th vertex norm, and (x_g, y_g, z_g) is the mesh's centre of gravity, which can be calculated as:

$$x_g = \frac{1}{L} \sum_{i=0}^{L-1} x_i, \quad y_g = \frac{1}{L} \sum_{i=0}^{L-1} y_i, \quad z_g = \frac{1}{L} \sum_{i=0}^{L-1} z_i \quad (2)$$

Secondly, vertex norms are divided into N distinct bins according to their magnitude. Each bin is used to hide one bit of watermark, thus totally inserting N bits to the mesh. In

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this paper, we use ω_n to represent the watermarking bit to be embedded into the n th bin.

The maximal and minimal vertex norms, which are represented by ρ_{\min} and ρ_{\max} , are obtained in advance. Then the n th bin B_n is defined as follows:

$$\begin{aligned}\rho_{n,\min} &= \rho_{\min} + \frac{(\rho_{\max} - \rho_{\min})}{N} \cdot n \\ \rho_{n,\max} &= \rho_{\min} + \frac{(\rho_{\max} - \rho_{\min})}{N} \cdot (n + 1) \\ B_n &= \{\rho_{n,j} | \rho_{n,\min} < \rho_{n,j} < \rho_{n,\max}\}\end{aligned}\quad (3)$$

Here $\rho_{n,\min}$ and $\rho_{n,\max}$ are lower and upper boundaries of the n th bin, and $\rho_{n,j}$ is the j th vertex norm in the n th bin. In this paper, we also use $\theta_{n,j}$ and $\phi_{n,j}$ to represent the spherical angles of the j th vertex norm in the n th bin. In addition, we use M_n to represent the number of vertex norms belonging to the n th bin.

The third step is to map the vertex norms belonging to the n th bin to the normalised range $[0, 1]$:

$$\widetilde{\rho_{n,j}} = \frac{\rho_{n,j} - \rho_{n,\min}}{\rho_{n,\max} - \rho_{n,\min}} \quad (4)$$

where $\widetilde{\rho_{n,j}}$ is the normalised j th vertex norm in the n th bin. The aim of the watermarking process is to slightly change $\widetilde{\rho_{n,j}}$, so that the mean of the vertex norms is moved to a specific area according to the watermarking bit to be embedded. We introduce the normalised distortion for the j th vertex in the n th bin, which is represented by $\Delta\widetilde{\rho_{n,j}}$. Our aim is to calculate the value of each $\Delta\widetilde{\rho_{n,j}}$. After $\Delta\widetilde{\rho_{n,j}}$ is obtained, we can calculate the new vertex norm $\widetilde{\rho_{n,j}}'$ by adding the previous one with its distortion:

$$\widetilde{\rho_{n,j}}' = \widetilde{\rho_{n,j}} + \Delta\widetilde{\rho_{n,j}} \quad (5)$$

After distortions are added to all vertices, we need to transform the vertex norms to the original ones by equation (6), which is an inverse transformation of equation (4).

$$\rho_{n,j}' = \widetilde{\rho_{n,j}}' \cdot (\rho_{n,\max} - \rho_{n,\min}) + \rho_{n,\min} \quad (6)$$

The watermark embedding process is completed by converting the spherical coordinates to Cartesian coordinates. Let ρ_i' be the i th vertex norm. A watermarked mesh consisting of points (x_i', y_i', z_i') is obtained by

$$\begin{aligned}x_i' &= \rho_i' \cos \theta_i \sin \phi_i + x_g \\ y_i' &= \rho_i' \sin \theta_i \sin \phi_i + y_g \\ z_i' &= \rho_i' \cos \phi_i + z_g\end{aligned}\quad (7)$$

In [1], the distortion $\Delta\widetilde{\rho_{n,j}}$ is obtained by modifying the original vertex norm with a power function. However, there is no mathematical proof that this function is optimal in assigning distortions in the mesh. In this paper, we change the

problem of assigning distortions to an optimisation problem, with several constraints to be satisfied, thus providing a concrete mathematical background for norm-based watermarking algorithms.

Our aim is to minimise the sum of squares of $\Delta\widetilde{\rho_{n,j}}$, which is also the mean square error (MSE) between the original 3D mesh and the watermarked mesh.

$$\text{Minimise: } \sum_{n=0}^{N-1} \sum_{j=0}^{M_n-1} \Delta\widetilde{\rho_{n,j}}^2 \quad (8)$$

Three constraints are applied to ensure that the embedded watermarking bits are correctly decoded later. The first constraint is to limit the transformed vertex norm $\widetilde{\rho_{n,j}}'$ into the range of $[\Delta G, 1 - \Delta G]$. Here ΔG is a parameter to control the distance gap between adjacent bins. The points belonging to the n th bin still belong to that bin after the watermarking process, which is also implied in [1]. The constraint is given as follows:

Constraint 1: For every $n \in \{0, 1, \dots, N - 1\}$ and $j \in \{0, 1, \dots, M_n - 1\}$,

$$\Delta G - \widetilde{\rho_{n,j}} \leq \Delta\widetilde{\rho_{n,j}} \leq 1 - \widetilde{\rho_{n,j}} - \Delta G \quad (9)$$

We can see from equation (5) that $\widetilde{\rho_{n,j}}'$ will be in the range of $[\Delta G, 1 - \Delta G]$ if the above constraint is satisfied.

The second constraint is directly derived from [1], which ensures that the mean of the transformed vertex norms in the n th bin is greater (or smaller) than a reference value when the embedded watermarking bit $\omega_n = +1$ (or $\omega_n = -1$). This constraint must be satisfied to ensure that the embedded watermarking bits could be correctly extracted later. Our aim is to make the mean of the vertex norms in the n th bin

$$\widetilde{\mu}_n' = \frac{1}{M_n} \sum_{j=0}^{M_n-1} \widetilde{\rho_{n,j}}' \quad (10)$$

to be greater than $1/2 + \alpha$ (or smaller than $1/2 - \alpha$) when $\omega_n = +1$ (or $\omega_n = -1$). Here α is a strength factor to control the watermarking effect. The second constraint is given as follows:

Constraint 2: For every $n \in \{0, 1, \dots, N - 1\}$,

1. If $\omega_n = +1$, then

$$\sum_{j=0}^{M_n-1} \Delta\widetilde{\rho_{n,j}} > M_n \cdot (1/2 + \alpha) - \sum_{j=0}^{M_n-1} \widetilde{\rho_{n,j}} \quad (11)$$

2. If $\omega_n = -1$, then

$$\sum_{j=0}^{M_n-1} \Delta\widetilde{\rho_{n,j}} < M_n \cdot (1/2 - \alpha) - \sum_{j=0}^{M_n-1} \widetilde{\rho_{n,j}} \quad (12)$$

By equation (5) and (10), it can be deduced that when Constraint 2 is satisfied, the transformed vertex norm $\widetilde{\mu}_n$ is greater than $1/2 + \alpha$ (or smaller than $1/2 - \alpha$) when $\omega_n = +1$ (or $\omega_n = -1$).

We proposed another constraint to guarantee that the centre of gravity of the watermarked mesh is the same as the original one. If the centre of gravity (x_g, y_g, z_g) has been changed, by equation (1), the vertex norms ρ_i will also be changed. Thus, the watermarking decoding process will fail to extract the embedded watermarking bits. To avoid this, the following constraint has to be satisfied:

Constraint 3:

$$\begin{aligned} \sum_{n=0}^{N-1} \sum_{j=0}^{M_n-1} \Delta \widetilde{\rho}_{n,j} \cos \theta_{n,j} \sin \phi_{n,j} &= 0 \\ \sum_{n=0}^{N-1} \sum_{j=0}^{M_n-1} \Delta \widetilde{\rho}_{n,j} \sin \theta_{n,j} \sin \phi_{n,j} &= 0 \\ \sum_{n=0}^{N-1} \sum_{j=0}^{M_n-1} \Delta \widetilde{\rho}_{n,j} \cos \phi_{n,j} &= 0 \end{aligned} \quad (13)$$

Thus, we have changed the problem of assigning distortions to an optimisation problem, with a quadratic objective function and three constraints. This is exactly a quadratic programming problem. The theory of convex optimisation guarantees that such kind of problems can be solved reliably and efficiently, even in very large scales [7, Chapter 4].

The watermark decoding process is simple. Similar to the embedding process, the centre of gravity is firstly calculated by equation (2), then the coordinates are converted to spherical coordinates by equation (1). After obtaining the maximal and minimal vertex norms, the vertex norms are classified into N bins and mapped onto the range of $[0, 1]$ by equation (3) and (4). Then, the mean of the n th bin $\widetilde{\mu}_n$ is calculated by equation (10), and compared with the reference value $1/2$. The watermark hidden in the n th bin, represented by ω_n , is extracted by

$$\omega_n = \begin{cases} +1, & \text{if } \widetilde{\mu}_n > (1/2) \\ -1, & \text{if } \widetilde{\mu}_n < (1/2) \end{cases} \quad (14)$$

3. EXPERIMENTAL RESULTS

Simulations are carried out on the Stanford head model (Figure 1), which consists of 11703 vertices and 23402 triangles.

Our aim is to compare the noise resistance ability of the proposed method with other two methods described in [1]. The authors of [1] have proposed two watermarking schemes. One is based on modifying the mean of vertex norms, referred as ‘Method I’ in their paper, and the other is based on modifying the variance of the vertex norms, referred as ‘Method II’. We have implemented their methods as a baseline to compare with ours. For the proposed method, we set

the gap parameter $\Delta G = 0.1$, which is chosen by comparing the performance of different ΔG 's. The software to solve the QP problem is BPMPD [8], which is built for solving large-scale linear and quadratic programming problems.

Firstly, we randomly generate 55 watermarking bits ($N = 55$) and embed them into the original mesh. In order to obtain a fair comparison, we deliberately set α of method I to 0.05, then adjust the strength factors of method II and the proposed method so that they produce perceptually similar watermarked meshes. The perceptual similarities between the original mesh and the watermarked meshes are quantitatively measured by Metro [9], which is a software for calculating the Hausdorff distance (HD) between two meshes. Experiments indicate that when the strength factor $\alpha = 0.1212$ for method II, and $\alpha = 0.0814$ for the proposed method, the watermarked meshes produced by these three methods have similar Hausdorff distances to the original mesh, as shown in Figure 2.

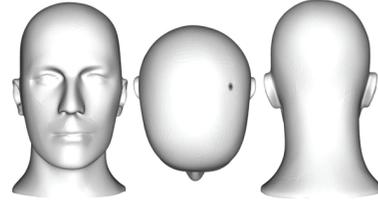


Fig. 1. The Stanford 3D head model, shown from three different angles.

Since these methods produce perceptually similar watermarked meshes, the next step is to test their robustness to noise. Gaussian noise is added to each of the points in the watermarked meshes. The mean of the Gaussian noise is zero, and its variance is proportional to the maximal vertex norm in the mesh. We define the ‘noise ratio’, or NR, as the ratio of the noise variance to the maximal vertex norm in the mesh. Then the noise-added mesh is decoded by the decoding process to obtain the watermarking bits. In order to filter out the randomness, we repeat this process for 100 times and obtain the correct bit rate (CBR), which is defined as the ratio of correctly decoded bits to all embedded bits.

Table 1 shows the CBR's of the three methods in different noise ratios. The greatest CBR in each noise ratio is printed in bold. We can see that the proposed method performs slightly worse than Method II when the noise magnitude is small ($NR = 0.001$), but it performs much better in greater noise ratios.

The same experiments are carried out on the bunny and beethoven models. It is shown that the proposed method consistently performs better in resisting Gaussian noise. However, space precludes further clarification of these experiments.

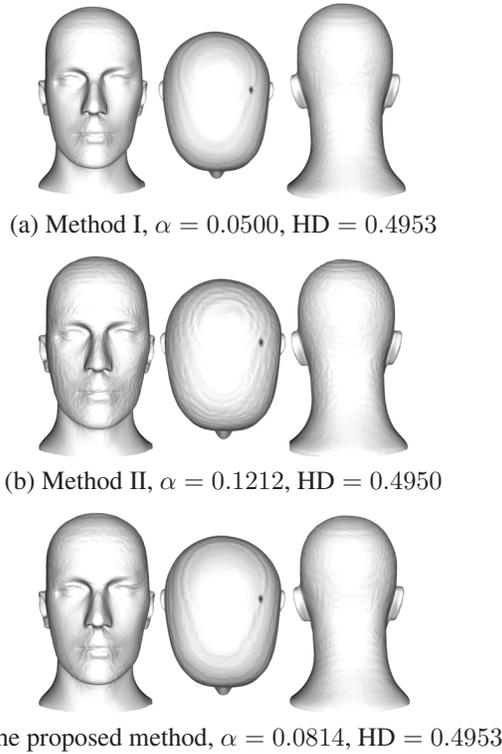


Fig. 2. The watermarking effects of the three methods. The strength factors are deliberately selected so that the watermarked meshes produced by these methods obtain similar Hausdorff distances to the original mesh.

4. CONCLUSIONS

This paper has proposed a histogram-based method for watermarking 3D polygonal meshes by using quadratic programming to minimise the mean square error between the original mesh and the watermarked mesh. Compared with the watermarking schemes in [1], this method performs better in resisting Gaussian noise, thus potentially is a better watermarking algorithm. However, this method has difficulties in dealing with large meshes because of the complexity limitations of computers and existing QP solvers. A segmentation-then-embedding scheme, as proposed in [6], can be used to solve this problem.

5. REFERENCES

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	Strength Factor (α)	Hausdorff Distance	NR	CBR (%)
Method I	0.0500	0.4953	0.001	92.85
			0.002	79.91
			0.003	67.76
			0.004	60.07
			0.005	56.45
Method II	0.1212	0.4950	0.001	97.82
			0.002	82.09
			0.003	63.33
			0.004	54.25
			0.005	51.24
Proposed	0.0814	0.4953	0.001	97.47
			0.002	84.89
			0.003	73.89
			0.004	65.31
			0.005	60.00

Table 1. The CBR's of the three methods when NR varies from 0.001 to 0.005. The strength factors of these methods are deliberately selected to produce similar Hausdorff distances. The greatest CBR in each NR is printed in bold.

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