IMPROVED MAXIMUM LIKELIHOOD LOCATION ESTIMATION ACCURACY IN WIRELESS SENSOR NETWORKS USING THE CROSS-ENTROPY METHOD

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ABSTRACT

This paper considers the problem of target location estimation in a wireless sensor network based on IEEE 802.15.4 radio signals and proposes a novel implementation of the maximum likelihood (ML) location estimator based on the Cross-Entropy (CE) method. In the proposed CE method, the ML criterion is translated into a stochastic approximation problem which can be solved effectively. Simulation results comparing the performance of a ML target estimation scheme employing the conventional Newton method and the conjugate gradient method are presented. The simulation results show that the proposed CE method provides higher location estimation accuracy throughout the sensor field[1]

Index Terms—target location estimation, wireless sensor network, IEEE 802.15.4, maximum likelihood method, Cross-Entropy method

1. INTRODUCTION

The rapid development in the manufacturing of micro-electrical-machine systems and wireless communications has produced inexpensive tiny sensors with low cost and low power consumption. These sensors have detection capabilities in wireless communications systems and data processing. A network in which many wireless sensors are deployed and interconnected in a particular region is referred to as a wireless sensor network (WSN) [1]. One of the important tasks that WSNs need to perform is target location estimation because many important WSN applications require accurate target location and target tracking.

Generally speaking, the information used to estimate a target location is based on the measurement data of the time-of-arrival, time-difference-of-arrival, angle-of-arrival, and received signal strength (RSS) [2] gathered from a known station or sensor node. However, in this paper we focus on the location estimation method using RSS. We do so because the RSS can be measured using the IEEE 802.15.4 standard [3] that expects devices and other components to be developed in support of the realization of WSNs based on this specification. Consequently, location estimation with RSS is considered to be suitable for location estimation by WSNs. In addition, we also consider the probabilistic model which is based on the probability distribution function (pdf) that describes the wireless propagation characteristics in a real environment of observed RSSs. Once the pdfs of observed RSSs are available, the maximum likelihood (ML) method can be used for target location estimations.

Mathematically, the ML method used to solve the location estimation problem involves minimizing the negative of a highly complex multimodal log-likelihood function. Thus, traditional deterministic optimization algorithms are suitable for obtaining the ML solution. However, traditional deterministic optimization algorithms such as the Newton method [4] sometimes become stuck in the local minima because of their lack of a good initial guess, and no global convergence is guaranteed, in general. In this paper, we consider a WSN based on IEEE 802.15.4 radio signals and propose a novel implementation of the ML location estimator based on the Cross-Entropy (CE) method [5]. In the proposed CE method, the ML criterion is translated into a stochastic approximation problem which can be solved effectively. The simulation results show that the performance of the proposed CE method can provide higher location estimation accuracy throughout the sensor field comparing to the conventional methods.

2. SYSTEM MODEL AND PROBLEM DEFINITION

In this paper, we focus on the location estimation method based on the RSS of IEEE 802.15.4 radio signals. Since the RSS decreases according to the distance, denoted by r, between the transmitter and receiver, the distance decays of the average RSS, denoted by $\bar{P}$, is usually characterized as being inversely proportional to r^{n_p} and can be expressed as

$$\bar{P} = C \times r^{-n_p} , \qquad (1)$$

where n_p is the path-loss exponent that depends on the propagation environment, and C is a constant that depends on the transmission power, the antenna characteristics, the average channel attenuation, and so on. It is known that the effect of wireless propagation characteristics on the RSS can be modeled by means of a probability distribution. That is, the wireless propagation model can be represented as a mathematical model that predicts some properties of a radio signal at a given location. Based on the experimental results in [6], the pdf of the RSS against the distance for IEEE 802.15.4 radio signals in an indoor environment can be approximated well by an exponential distribution. Therefore, the conditional pdf $p(P|r)$ of the RSS P when r is given can be expressed by

$$p(P|r) = \frac{1}{\overline{P}}\exp\left(-\frac{P}{\overline{P}}\right), \qquad (2)$$

where $\overline{P}$ is the average RSS, expressed as (1).

After each sensor node measures the RSSs for a target node, all the information on the RSSs is sent to a data collection center. Given $\{P_i\}$ (i is the sensor node index) as the available RSS measurement data in the data collection center, the problem in hand now is how to accurately estimate a target's location (x,y) based on these measurement data. We assume that 1) the radio propagation channel among different target-sensor pairs are independent, 2) the pdf of the received power P_i with respect to the distance r_i from the transmitting node is modeled as an exponential distribution, and 3) there are N sensor nodes in the sensor field where the sensor nodes are supposed to exist. Hence, depending on the exact location of a target, the joint pdf for the RSS P_1, P_2, …, P_N can be expressed as

$$p(P_1,P_2,...,P_N|r_1,r_2,...,r_N) = \prod_{i=1}^{N} p(P_i|r_i) = \prod_{i=1}^{N} p(P_i|\boldsymbol{\theta}), \qquad (3)$$

where

$$r_i = \sqrt{(x-X_i)^2 + (y-Y_i)^2} \qquad (4)$$

is the distance between the unknown coordinate location of the target node, $\boldsymbol{\theta} = \begin{bmatrix} x & y \end{bmatrix}$, and the known coordinate location of the sensor node i, (X_i,Y_i), and $p(P_i|r_i)$ is given by (2), with P and r replaced by P_i and r_i, respectively. If we take the logarithm of (3), the log likelihood function of (3) is

$$L(\boldsymbol{\theta}) = \log \prod_{i=1}^{N} p(P_i|\boldsymbol{\theta}) = \sum_{i=1}^{N} \log p(P_i|\boldsymbol{\theta}). \qquad (5)$$

To derive a ML solution to (5), we differentiate (5) with respect to x and y and equate them to zero,

$$\frac{\partial L}{\partial x} = \frac{\partial}{\partial x}\sum_{i=1}^{N} \log p(P_i|\boldsymbol{\theta}) = 0; \quad \frac{\partial L}{\partial y} = \frac{\partial}{\partial y}\sum_{i=1}^{N} \log p(P_i|\boldsymbol{\theta}) = 0. \quad (6)$$

It is obvious that (6) is a system of non-linear equations. Instead of solving (6) directly, in the next section we propose a novel implementation for ML function in (5) based on the CE method.

3. THE CROSS-ENTROPY METHOD AND ITS APPLICATION TO ML LOCATION ESTIMATION

The CE method was first proposed by Rubinstein [5] to solve rare event estimation problems and was soon successfully applied to solving both combinatorial and continuous optimization problems. The CE method is a general algorithm for solving global optimization tasks of the form

$$\arg\max_{\boldsymbol{\theta} \in \Theta} S(\boldsymbol{\theta}) , \qquad (7)$$

that is, we wish to maximize the score function $S(\boldsymbol{\theta})$ over all $\boldsymbol{\theta}$ in set Θ. Instead of maintaining a simple solution candidate $\boldsymbol{\theta}_t$ in each time step for the conventional optimization algorithms, the main idea of the CE method is to maintain a distribution of possible solutions, and adaptively update this distribution according to the Kullback–Leibler distance, i.e., cross entropy, between the associated density and the optimal important sampling density. By doing so, one constructs a random sequence of solutions which converges (probabilistically) to the optimal or, at least, a reasonable solution. In short, the CE method involves the following two iterative phases: 1) Generate random samples in Θ according to a specified sampling distribution generated from the previous iteration. 2) Update the parameters on the basis of the best scoring samples in order to produce better scoring samples in the next iteration. For a concrete understanding of the CE method, the reader is referred to [5].

In the ML location estimation problem, we are interested in maximizing the log-likelihood function, expressed in (5), over the set Θ of all potential $\boldsymbol{\theta}$ such that

$$\arg\max_{\boldsymbol{\theta} \in \Theta} L(\boldsymbol{\theta}) = \arg\max_{\boldsymbol{\theta} \in \Theta} \sum_{i=1}^{N} \log p(P_i|\boldsymbol{\theta}). \qquad (8)$$

The CE method is an adaptive importance sampling method that transforms the deterministic optimization problem (8) into a family of stochastic sampling problems. Hence, the first step in using the CE method is to randomize our original deterministic problem (8) by including a set of sampling distribution over deterministic $\boldsymbol{\theta}$. In this paper, we take the sampling distribution to be a Gaussian distribution $N(a,b)$, where a and b are the mean and variance, respectively. It is important to emphasize that the sampling distribution can be quite arbitrary and does not need to be related to the function that is being optimized.

The reason we adopt the Gaussian distribution is that it gives formulas that are easy to update

At each iteration t of the algorithm, a collection of N random samples $\left\{\left\{\theta_i^{(t)}(n)\right\}_{i=1}^2\right\}_{n=1}^N$ is obtained from a Gaussian distribution, i.e., $\theta_i^{(t)}(n) \sim N\left(a_i^{(t)}, b_i^{(t)}\right)$, where $\theta_i^{(t)}(n)$ denotes the ith element of the sample n at iteration t, and $a_i^{(t)}$ and $b_i^{(t)}$ denote the mean and the variance of the ith element at iteration t, respectively. Next, with the N samples, we can derive a set of performance values $\left\{L\left(\boldsymbol{\theta}^{(t)}(n)\right)\right\}_{n=1}^N$ using the score function $L(\boldsymbol{\theta})$ expressed in (5). After sorting the performance values from the smallest to the largest, $L_{(1)} \leq \cdots \leq L_{(N)}$, then new values $a_i^{(t)}$ and $b_i^{(t)}$ are then updated based on the $N^{\text{elite}} = \lceil \rho N \rceil$ best performance samples, the elite samples, where ρ denotes the fraction of the best samples and $\lceil \rho N \rceil$ is the integer part of ρN. Let χ be the indices of the N^{elite} best performance samples. Then $a_i^{(t)}$ and $b_i^{(t)}$ can be updated via

$$\tilde{a}_i^{(t)} = \sum_{n \in \chi} \theta_i^{(t)}(n) \big/ N^{\text{elite}} \qquad (9)$$

and

$$\tilde{b}_i^{(t)} = \sum_{n \in \chi} \left(\theta_i^{(t)}(n) - a_i^{(t)}\right)^2 \big/ N^{\text{elite}}, \qquad (10)$$

respectively. That is, we update $a_i^{(t)}$ and $b_i^{(t)}$ as the mean and variance of the elite samples. In addition, instead of using $\tilde{a}_i^{(t)}$ and $\tilde{b}_i^{(t)}$ as the updated parameters, it is beneficial to add in a smoothing procedure for each iteration t as

$$a_i^{(t)} = \alpha \tilde{a}_i^{(t)} + (1-\alpha) a_i^{(t-1)} \qquad (11)$$

and

$$b_i^{(t)} = \alpha \tilde{b}_i^{(t)} + (1-\alpha) b_i^{(t-1)}, \qquad (12)$$

where α is the smoothing parameter, with $0 < \alpha \leq 1$. It is obvious that for $\alpha = 1$ we have the original updating rule. The main algorithm is summarized as follows:

Algorithm: CE Algorithm for the ML Location Estimation Problem

1.   *Initialize $a_i^{(0)}$ and $b_i^{(0)}$. Set t = 1.*

2.   *Use the density $N\left(a_i^{(t-1)}, b_i^{(t-1)}\right)$ to generate a random sample $\left\{\left\{\theta_i^{(t)}(n)\right\}_{i=1}^2\right\}_{n=1}^N$.*

3.   *Calculate the score function according to (5) to get*

a set of performance values $\left\{L\left(\boldsymbol{\theta}^{(t)}(n)\right)\right\}_{n=1}^N$.

4.   *Order the performance values from the smallest to the biggest, and select the best N^{elite} elite performance values according to predetermined quantile parameter ρ.*

5.   *Calculate the sample mean and sample variance of the elite samples according to (9) and (10), respectively.*

6.   *Update the mean and variance of the elite samples in a smooth way, as in (11) and (12), respectively.*

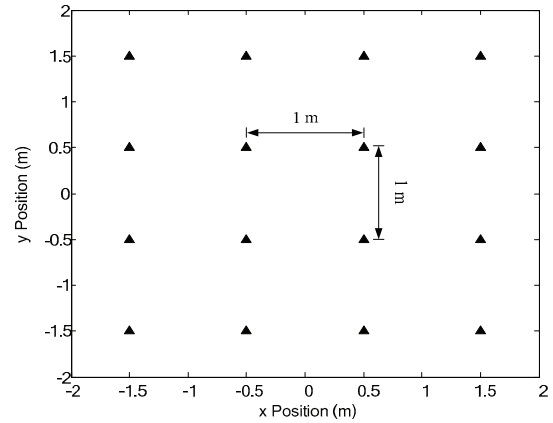7.   *Repeat step 2 to step 6 for t = t +1 until the stopping criterion is met.*

Fig. 1 Simulation scenario, where ▲ represents the location of a sensor node.

4. NUMERICAL RESULTS

Computer simulations are discussed in this section. For comparison, we also test four existing algorithms: 1) Newton's method (NT) [5] for solving (6); 2) the NT method with true target location (NTT) for solving (6); NTT is identical to the NT method except that it uses the true target location, not a random coordinate vector, as its initial guess; 3) the conjugate gradient method (CG) [5], which is an iterative optimization method used to solve the ML criterion in (5); and 4) the CG method with true target location (CGT), which is identical to the CG method, except that it uses the true target location, not a random coordinate vector, as its initial guess. The statistical model given by (1) contains two parameters used for advance estimation, that is, path-loss exponent n_p and constant C. In this paper, we assume that the path-loss exponent n_p and constant C are perfectly estimated. Here, $n_p = 3.51$ and $C = 0.0000141$ [7]. The simulation scenario is depicted in Fig. 1. Regarding the parameters used in the proposed CE algorithm, $a_i^{(0)}$ is randomly chosen over [0,100] m, $b_i^{(0)}$ is 10000, the smoothing parameter α is 0.8, $N^{\text{elite}} = 10$, $N = 100$, and

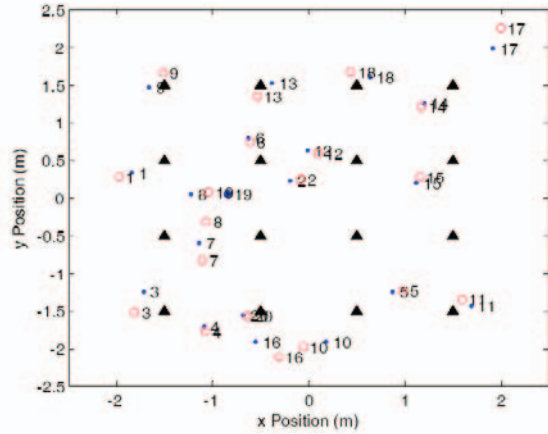the algorithm is stopped when the iteration number exceeds 100.

Fig. 2 Example of location estimation. ▲ represents the location of a sensor node, ○ represents the true location of the target, ● represents the estimated location of the target, respectively.

In our simulation, we randomly generate the x position and y position of the target node over [-2,2] (m) and compare the proposed method with the conventional methods with respect to the number of sensors. The sensor nodes that estimate a target location in this simulation are assumed to be those that receive the largest signal strength transmitted from the target node. Fig. 2 shows an example of location estimation result with 16 sensor nodes. Fig. 3 shows the root mean square error (RMSE) versus the number of sensor nodes for five algorithms with different sensor nodes. The location estimation is performed over 10,000 Monte Carlo trials. The maximum iteration number for the conventional NT method, the NTT method, the CG method, and the CGT method are 10,000, and these algorithms will be stopped when the iteration number exceeds 10,000. If the iteration number of above methods reaches the maximum iteration number, the algorithm is regarded as "divergence". From Fig. 3, we observe that 1) as expected, the RMSE decreases when the number of sensors increases; 2) a good initial guess definitely improves the performance of the conventional NT method and CG method, but it should be noted that the proposed CE method is independent of the initial guess; 3) the proposed CE method always outperform the other conventional methods, even though some of them use the true target location as their initial approximation to start their algorithm.

Finally, since the five compared algorithms are iterative algorithms, convergence is an important issue that needs to be discussed. The divergence probabilities for the NT method, the NTT method, the CG method, and the CGT method are 2.78%, 0.07%, 1.05%, and 1.01%, respectively. On the other hand, the proposed CE algorithm is stopped

when the iteration number exceeds 100. We can see from Fig. 3 that the performance of the proposed CE algorithm always converges to the optimal or, at least, a reasonable solution.
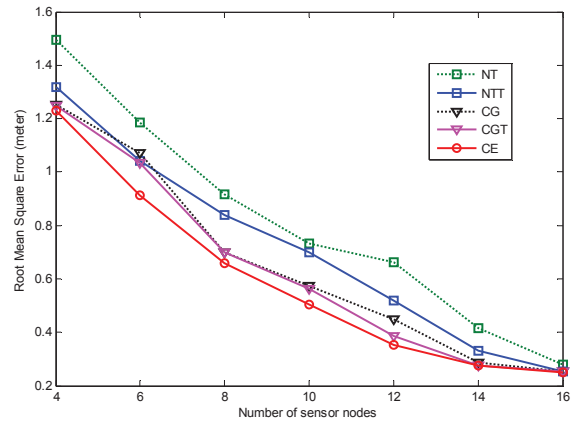

Fig. 3 Root mean square error (RMSE) versus number of sensor nodes.

5. CONCLUSION

This paper presented a CE-based method that performs ML location estimation based on IEEE 802.15.4 radio signals in WSNs. The performance of the CE algorithm was studied using computer simulations. Compared with the conventional Newton method and the conjugate gradient method, the proposed CE method provides higher location estimation accuracy throughout the sensor field.

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