

NOVEL SIMILARITY INVARIANT FOR SPACE CURVES USING TURNING ANGLES AND ITS APPLICATION TO OBJECT RECOGNITION

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ABSTRACT

We present a new similarity invariant signature for space curves. This signature is based on the information contained in the turning angles of both the tangent and the binormal vectors at each point on the curve. For an accurate comparison of these signatures, we define a Riemannian metric on the space of the invariant. We show through relevant examples that, unlike classical invariants, the one we define in this paper enjoys multiple important properties at the same time, namely, a high discrimination level, independence of any reference point, uniqueness property, as well as a good preservation of the correspondence between curves. Moreover, we illustrate how to match 3D objects by extracting and comparing the invariant signatures of their curved skeletons.

Index Terms— Space curve, similarity invariant, turning angle, curvature, torsion.

1. INTRODUCTION

Multiple 3D modeling methods use spatial curves for recognition and matching of objects. Spatial curves are exploited in different configurations. They may, for instance, be extracted as contours of landmark surfaces [1], as level curves of a Morse function [2], or also as elements of curved skeletons [3, 4]. All these techniques, despite their differences, agree in relying on curves' properties in solving computer vision problems. This common approach is motivated by the fact that curves in 3D are fairly well known geometric entities; moreover, under some conditions, they can accurately describe the overall geometry of an object in 3D space [1]. Translating the constraints of 3D shape representation techniques to curves reduces the level of difficulty associated with the representation problem and makes it more tractable.

Pose invariance of surfaces is a common requirement in object modeling. It is also a good illustration of the simplification from surfaces to curves. Indeed, an effective and economical solution for curves pose invariance may be provided through Euclidean/similarity invariants or invariant signatures [5]. Besides this pose invariance property, additional constraints are imposed on 3D curves as a direct result of the nature of 3D shape recognition applications. These constraints may be summarized as follows: invariance to a group of transforms, uniqueness, local characterization (local support), ability to determine shape properties such as symmetries and part correspondences. To the best of our knowledge, none of the available references seem to gather all these properties at once. The most

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complete work is the one of Mokhtarian and Bober [1], as they succeed in citing and addressing all the necessary properties; however, to provide an invariance to scaling transforms, the authors use a multi-resolutional procedure. In the present work, we provide the advantage of having an invariant that is, by definition, i.e., without additional steps, fully invariant to all similarity transforms. The key contribution of our work resides in using turning angles based on curvature and torsion instead of using curvature and torsion directly. We further ensure a natural registration of all the invariants on one curved space which leads to defining an accurate and computationally easy metric for curves comparison. Indeed, we show that while torsion and curvature are clearly variant with scaling, turning angles are not. The first inspiration of our work comes directly from [6], where an invariant for planar curves is defined. This invariant has the particularity to be an information theoretical measure of local geometric properties of curves. Moreover, this invariant comes as a proof for longtime psychological assumptions on mental shape perception. The operation of comparing invariants, although often overlooked, is crucial in assessing the properties of the invariant, and accurately achieving recognition operations. This is why, in the present work, we complete the proposed invariant signature by defining its Riemannian space equipped with an intrinsic measure.

We start this paper by reviewing, in Section 2, important geometric and information theoretic notions used throughout this work. We briefly cover the work of Feldman and Singh [6] in Section 2.3 as it was the precursor and the inspiration for the present effort. In Section 3, we introduce our new invariant signature for space curves and define a Riemannian metric for the comparison of the invariant signature curves. Finally, in Section 5, we support our theoretical claims through relevant experimental examples.

2. BACKGROUND AND FORMULATION

2.1. Turning angles

A space curve is uniquely determined, up to a Euclidean transform, by its curvature function $\kappa(t)$, and torsion function $\tau(t)$, both continuous functions of the parameter t ; hence, we naturally use these measurements to define an adequate invariant signature curve; however, since we target the group of similarity transforms, and knowing that curvature and torsion are not scale invariant, we use turning angles as the geometric features describing space curves [6]. In what follows, we show how it is possible to relate curvature and torsion as

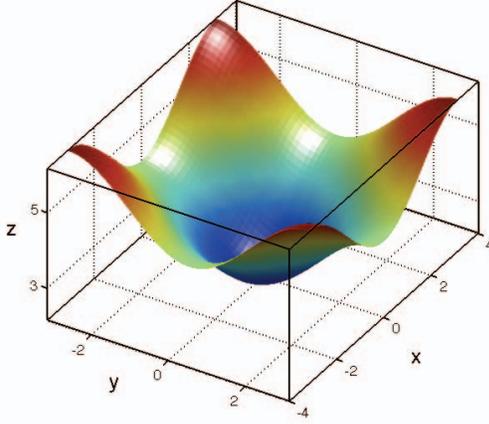


Fig. 1. Manifold defined by the invariant signature curve as defined in (9).

linear functions of turning angles. Using the Frenet-Serret formulae:

$$\frac{d\mathbf{T}}{dt} = \kappa\mathbf{N}, \quad (1)$$

$$\frac{d\mathbf{N}}{dt} = -\kappa\mathbf{T} + \tau\mathbf{B}, \quad (2)$$

$$\frac{d\mathbf{B}}{dt} = -\tau\mathbf{N}, \quad (3)$$

where \mathbf{T} , \mathbf{N} and \mathbf{B} are the tangent, the normal and the binormal vectors, respectively, we define two turning angles α_T and α_B . α_T is the change in the direction of \mathbf{T} , and α_B is the change in the direction of \mathbf{B} such that: $\alpha_T(t) \approx \kappa(t) \cdot dt$ and $\alpha_B(t) \approx \tau(t) \cdot dt$.

2.2. Shannon surprisal

We extract the information contained in the proposed turning angles, i.e., our chosen geometric measures, by using a notion in information theory known as ‘‘Shannon surprisal’’ [7, 6]. We assume $\alpha(t)$ to follow, without loss of generality, a *von Mises* distribution with a zero mean and a spread parameter b equal to 1. We then define the probability density function of $\alpha(t)$ as $f_\alpha(\alpha(t)) = A \exp(\cos(\alpha(t)))$, for all t , with $\alpha \in [-\pi, +\pi]$ and $A = \frac{1}{2\pi \cdot Bessel(0,b)}$. *Bessel*(0, b) being the Bessel distribution of mean 0 and variance b . The surprisal of $\alpha(t)$ is by definition:

$$\begin{aligned} \theta(t) &= -\ln(f_\alpha(\alpha(t))) \\ &= -\ln(A) - \cos(\alpha(t)) \quad \forall t. \end{aligned} \quad (4)$$

2.3. Invariant signature for planar curves

In [6], Feldman and Singh present an invariant signature for planar curves. This invariant is based on the turning angle $\alpha(t)$, the change in the direction of the tangent vector at the instant t . The actually considered invariant signature is the information gained when measuring $\alpha(t)$ at all instants t . This exactly corresponds to $\theta(t)$, the surprisal of $\alpha(t)$ as defined in (4). For a simple planar curve of length L sampled at N equal intervals of arc length Δt , $\alpha(t)$ is related to the curvature $\kappa(t)$ at a given point by the following approximation $\alpha(t) \approx \kappa(t) \cdot \Delta t$. We note that scaling the curve implies scaling

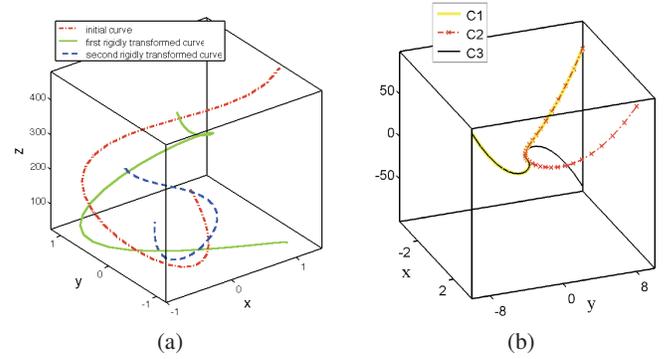


Fig. 2. Two different sets of synthetic space curves. In (a), T1 and T2 are two similarity transforms. In (b), $C1$, $C2$ and $C3$ are three similar curves except at one inflection point.

both $\kappa(t)$ and L while keeping the value of $\kappa(t) \frac{L}{N}$ invariant. It thus follows that $\alpha(t)$ is a measure equivalent to $\kappa(t)$ except that it is scale-invariant.

3. INVARIANT SIGNATURE FOR SPACE CURVES

In our case, we deal with space curves instead of planar curves. For this reason, we require two turning angles (α_T and α_B) versus one in the planar case; hence, we naturally use these two measurements to define an adequate invariant signature curve. Now, instead of separately considering the marginals of two random variables $\alpha_T(t)$ and $\alpha_B(t)$, we define a third invariant term that considers the random vector $[\alpha_T(t), \alpha_B(t)]^T$. Thus, the distribution f_α of interest becomes the binary *von Mises* distribution of the independent variables $\alpha_T(t)$ and $\alpha_B(t)$, such that:

$$f_\alpha(\alpha_T(t), \alpha_B(t)) = A^2 \exp(\cos(\alpha_T(t)) + \cos(\alpha_B(t))). \quad (5)$$

The corresponding surprisal function $\theta(\alpha_T(t), \alpha_B(t)) \equiv \theta(t)$ becomes $\theta(t) = -\ln(f_\alpha(\alpha_T, \alpha_B))$. That is $\theta(t) = -2 \ln(A) - \cos(\alpha_T(t)) - \cos(\alpha_B(t))$, with $(\alpha_T(t), \alpha_B(t)) \in ([-\pi, \pi])^2$. The range of the new function $\theta(\cdot)$ constitutes a curved space, as shown in Fig. 1. All the invariant signatures are constrained to live on this space. Moreover, defining this space provides a natural way to register all the signatures. The only case for which two non identical space curves are going to have identical signatures is when curves are periodic with the same period, but with different lengths. To compare two given curves γ_i and γ_j , we compare their invariants. The invariant we defined in (12) is a signature curve embedded in the curved space created by the two variables α_T and α_B (Fig. 1). All the invariant signature curves that we are to compare are thus constrained to live on the defined invariant space that we call \mathcal{T} . Defining a space \mathcal{T} that holds all the possible invariants is a natural way to register them. As a consequence, we may directly apply a distance measure to compare these invariant curves without worrying about ensuring a prior registration. We thus choose to compare two invariant curves γ_1 and γ_2 , corresponding to two space curves $C1$ and $C2$, by considering the oriented curve $\gamma_\Delta = \gamma_1 - \gamma_2$. We use tools from measure theory and choose to refer to their physical intuition in relating them to our problem [8]. We start by viewing the oriented version of the space \mathcal{T} as a vector field $\vec{\mathbf{F}}$ on the $(2\pi \times 2\pi)$ plane defined by the variables α_T and α_B . We directly relate $\vec{\mathbf{F}}$ to

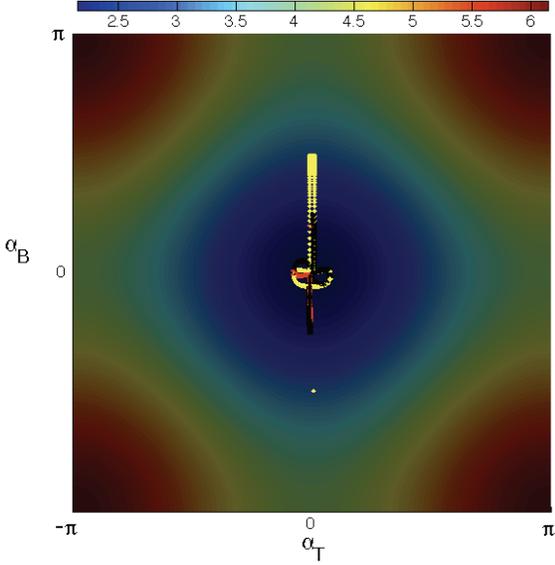


Fig. 4. Invariant signatures γ_1 , γ_2 and γ_3 for the curves $C1$, $C2$, and $C3$ of Figure 2 (b).

$\theta(t)$, and define it as follows:

$$\begin{aligned} \vec{\mathbf{F}} : ([-\pi, \pi])^2 &\rightarrow \mathbb{R}^2 \\ (\alpha_T, \alpha_B) &\mapsto -\ln(f_\alpha(\alpha_T)) \vec{i} - \ln(f_\alpha(\alpha_B)) \vec{j}. \end{aligned}$$

We also define γ_Δ^* ; the projection of γ_Δ on the $(2\pi)^2$ plane. γ_Δ^* is a 1-current in the space dual to the space of 1-forms $\mathcal{D}^1([- \pi, \pi])^2$. This means that if we consider $\vec{\mathbf{F}}(t) \equiv \vec{\mathbf{F}}(\alpha_T(t), \alpha_B(t))$ and for all ϕ from $\mathcal{D}^1([- \pi, \pi])^2$,

$$\gamma_\Delta^*(\phi) := \int_{\gamma_\Delta^*} \phi, \quad (6)$$

$$= \int_{\gamma_\Delta^*} \phi(\vec{\mathbf{F}}(t)) dt. \quad (7)$$

With these notions of measure theory, we naturally use the flat norm $\mathbb{F}(\gamma_\Delta^*)$ as the intrinsic distance between two curves $C1$ and $C2$ whose invariants are γ_1 and γ_2 , respectively. We thus may write,

$$\mathbb{D}(C1, C2) = \mathbb{F}(\gamma_1^* - \gamma_2^*), \quad (8)$$

$$:= \sup\{\gamma_\Delta^*(\phi) : \|d\phi\| \leq 1 \text{ for all } \|\phi\| \leq 1\} \quad (9)$$

where $\gamma_\Delta^* = (\gamma_1^* - \gamma_2^*)$.

4. APPLICATIONS AND EXPERIMENTAL RESULTS

In order to investigate and check the properties of the proposed space curve representation we use synthetic space curves and simulate different scenarios. In Fig. 2 (a), we test the first property of independence to pose (translations and rotations) and scaling. We note that all the curves, except $C2$, are similar to $C1$. We find two sets of invariants as shown in Fig. 3. Those corresponding to the family of

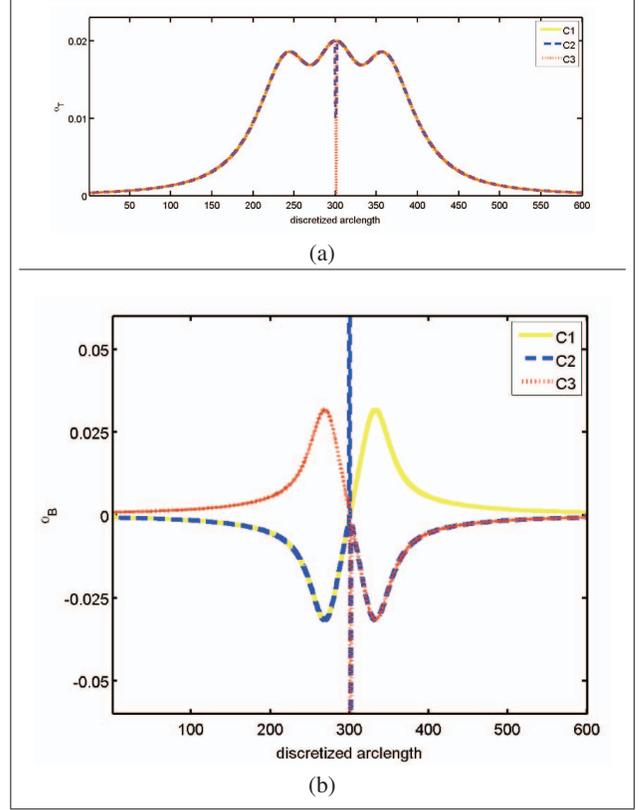


Fig. 5. Turning angles for the space curves of Figure 4 (b).

$C1$ are represented in red. Those in green are for the curve $C2$. This characterization into two groups confirms the invariance of the turning angle measures to the group of similarity transforms. We use the curves illustrated in Fig. 2 (b) to test a tricky case where local versus global representations are confronted. The objective of this experiment is to check whether the proposed signatures are able to locate the inflection points causing the dissimilarities between the proposed curves. In Fig. 5 (a) and (b), we observe overlaps and symmetries between some parts of the turning angles. This observation exactly translates what is happening at the curves level because of the effect of the inflection points. The actual signature curves sitting on the space \mathcal{T} are shown in Fig. 4. We further apply these signature invariant curves to compare 3D shapes through their curved skeletons that we defined in [4] and illustrated in Fig. 6. The particularity of these skeletons is that they have spatial curves replacing their edges. These characteristic curves are new means to compactly carry the

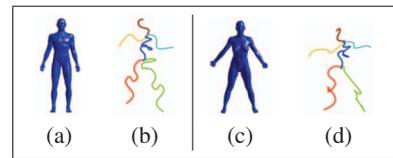


Fig. 6. Comparison of the 3D objects in (a) and (c) using their curved skeletons in (b) and (d), respectively.

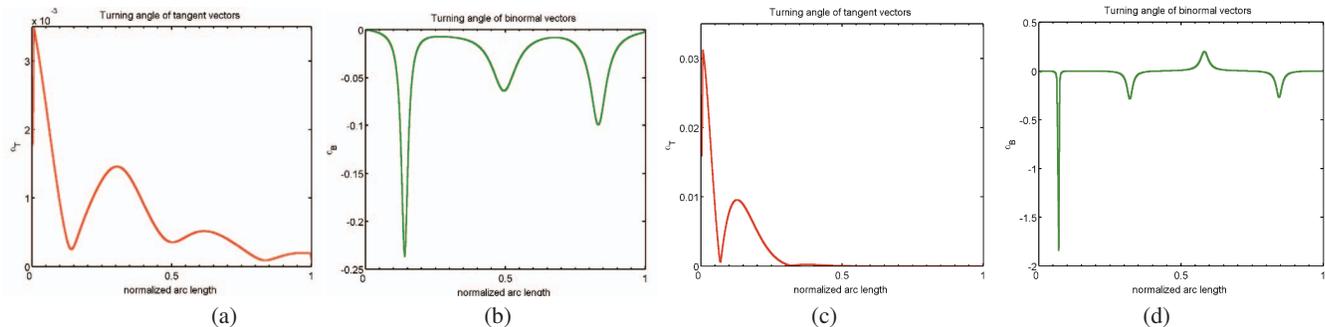


Fig. 3. Invariants for the space curves of Figure 4 (a). (a) and (b) correspond to turning angles of the binormal vectors. (c) and (d) correspond to turning angles of the tangent vectors. In red are the signature curves for $C1$'s family. In green are the ones for $C2$.

geometric information of 3D shapes. In Fig. 6 we illustrate a typical example for which the practical importance of the proposed invariant signature becomes obvious. The 3D shape comparison technique simplifies to comparing the signatures of the edge curves with the same colors in Fig. 6. For these skeletons, we define a new global metric based on Eq. 8. So we consider that $C1$ and $C2$ are now two sets of curves in 3D such that each set contains N curves C_1^i and C_2^i , $i = 1, \dots, N$, respectively representing the geometrical shapes of the 3-dimensional parts S_1^i and S_2^i , $i = 1, \dots, N$, that constitute each 3D object. We define in (10) the new distance between the two sets $C1$ and $C2$, which is also the distance between the corresponding objects $S1$ and $S2$. We show in Fig. 7 the results of using this distance in comparing six different subjects.

$$\mathbb{D}(C1, C2) = \frac{1}{2} \sum_{i=1}^N \left(\frac{\text{area}(S_1^i)}{\text{area}(S1)} + \frac{\text{area}(S_2^i)}{\text{area}(S2)} \right) \cdot \mathbb{D}(C_1^i, C_2^i). \quad (10)$$

5. CONCLUSION

In this paper, we presented a new similarity invariant signature for space curves. This invariant, since based on the tangent and the binormal turning angles, has the advantage of being local, unique and fully invariant to similarity transforms. The proposed invariant proves to be very practical to use in 3D shape modeling/comparison problems.

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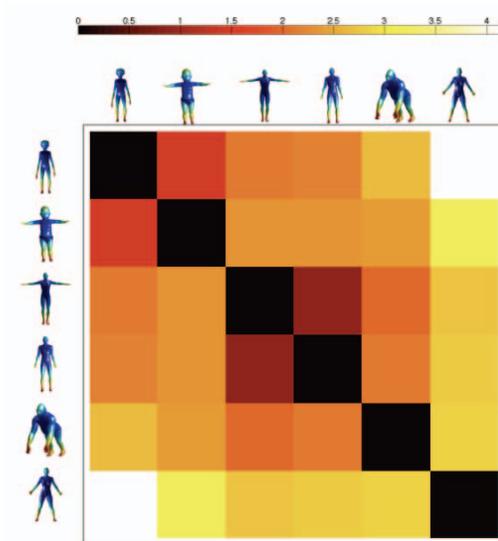


Fig. 7. Confusion matrix for 6 different subjects using the distance \mathbb{D} between curved skeletons.

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