ADAPTIVE RECONSTRUCTION METHOD OF MISSING TEXTURES BASED ON KERNEL CANONICAL CORRELATION ANALYSIS

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ABSTRACT

This paper presents an adaptive reconstruction method of missing textures based on kernel canonical correlation analysis (CCA). The proposed method calculates the correlation between two areas, which respectively correspond to a missing area and its neighbor area, from known parts within the target image and realizes the estimation of the missing textures. In order to obtain this correlation, the kernel CCA is applied to each set containing the same kind of textures, and the optimal result is selected for the target missing area. Specifically, a new approach monitoring errors caused in the above estimation process enables the selection of the optimal result. This approach provides a solution to the problem in traditional methods of not being able to perform adaptive reconstruction of the target textures due to the missing intensities. Experimental results show subjective and quantitative improvement of the proposed reconstruction technique over previously reported reconstruction techniques.

Index Terms— Image restoration, image texture analysis, interpolation, nonlinear estimation.

1. INTRODUCTION

In the field of image restoration, reconstruction of missing areas in digital images is a very important issue since it has a number of fundamental applications. For example, it is applied to removal of unnecessary objects, restoration of corrupted old films, and error concealment for video communications. In order to realize these applications, many methods, whose goals are successful reconstruction of important visual features such as texture features, have been proposed. In this paper, we focus on the reconstruction approach of the texture features.

In recent work, the study of missing texture reconstruction has developed rapidly and its achievements have become a center of attraction [1]–[3]. Most algorithms reported in the literature reconstruct missing areas by utilizing known textures within the target image as training patterns. However, such approaches assume that arbitrary local textures within the target image are quite similar to each other, that is, the target image consists of only one type of texture. Thus, if the target image contains various textures, the missing textures should be adaptively reconstructed from only the same kinds of textures. Unfortunately, such textures cannot be selected by traditional methods since the distance between the target missing textures and the other ones cannot be calculated.

In this paper, an adaptive texture reconstruction method based on kernel canonical correlation analysis (CCA) [4] is proposed. Since the kernel CCA is a useful method for finding relationships underlying between two different data sets, we utilize this method for finding the correlation of intensities between missing areas and the other known areas. Specifically, the proposed method calculates the correlation between two areas, which respectively correspond to a missing area and its neighbor area, within known parts of the target image, and estimates the missing intensities. In this procedure, the kernel CCA is applied to each set containing the same kind of textures, and the optimal result is selected for the target missing area based on errors caused in the above estimation scheme. Then, since each missing texture can be adaptively reconstructed by the optimal correlation obtained from only the same kind of texture, the successful restoration of the missing areas is expected.

This paper is organized as follows. In order to realize the adaptive reconstruction method, we have to first perform the clustering of known local textures within the target image. Thus, the texture clustering based on the kernel CCA is explained in Section 2. Next, the kernel CCA-based texture reconstruction method is presented in Section 3. Furthermore, experimental results that verify the performance of the proposed method are shown in Section 4. Finally, conclusion remarks are shown in Section 5.

2. KERNEL CCA-BASED TEXTURE CLUSTERING

In the the proposed method, a local image f ($w \times h$ pixels) including missing areas is clipped from the target image, and the missing intensities are estimated based on the kernel CCA. For the following explanation, we respectively denote the two areas, whose intensities are unknown and known within the target local image f, as Ω and $\bar{\Omega}$. Note that in the target image, there are many known local images whose textures are quite different from that of the target local image f. Such local images should not affect the reconstruction of the target local image f. Therefore, the proposed method applies the kernel CCA to each set of local images containing the same kind of texture, and the optimal result is adaptively utilized for the reconstruction of the target local image f. In order to realize this scheme, clustering of the known local images within the target image must first be performed before the reconstruction process. Thus, in this section, we show a kernel CCA-based clustering method of the known local images. Furthermore, the overview of the proposed method is shown in Fig. 1 for the following explanation.

First, we clip known local images f_i ($i = 1, 2, \cdots, N$) whose size is $w \times h$ pixels from the target image in the same interval. Next, for each local image f_i, two vectors $\mathbf{x}_i$ ($\in \mathbf{R}^{wh-N_\Omega}$) and $\mathbf{y}_i$ ($\in \mathbf{R}^{N_\Omega}$), whose elements are respectively raster scanned intensities in the corresponding areas of $\bar{\Omega}$ and Ω, are defined, where N_Ω represents the number of pixels in Ω. Furthermore, the proposed method maps $\mathbf{x}_i$ into the feature space via the nonlinear map $\phi_\mathbf{x}$:$\mathbf{R}^{wh-N_\Omega} \rightarrow \mathbf{F}$ [4].

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In this paper, we use the nonlinear map whose kernel function is a Gaussian kernel. Note that exact pre-image, which is the inverse mapping from the feature space back to the input space, typically does not exist [6]. Therefore, the estimated pre-image includes some errors. Since the final results estimated in the proposed method are the missing intensities, we do not utilize the nonlinear map for $\mathbf{y}_i$.

From the obtained results $\phi_{\mathbf{x}}(\mathbf{x}_i)$ and $\mathbf{y}_i$ ($i = 1, 2, \cdots, N$), the proposed method performs their clustering that minimizes the following criterion:

$$C = \sum_{k=1}^{K} \sum_{j=1}^{N^k} \left\| \mathbf{B}^{k\prime} \left(\mathbf{y}_j^k - \overline{\mathbf{y}}^k \right) - \mathbf{\Lambda}^k \mathbf{A}^{k\prime} \left(\phi_{\mathbf{x}}(\mathbf{x}_j^k) - \overline{\phi}_{\mathbf{x}}^k \right) \right\|^2 / D^k, \quad (1)$$

where K is the number of the clusters. The vectors $\mathbf{x}_j^k$ and $\mathbf{y}_j^k$ ($j = 1, 2, \cdots, N^k$) respectively represents $\mathbf{x}_i$ and $\mathbf{y}_i$ ($i = 1, 2, \cdots, N$) assigned to cluster k. Furthermore, $\overline{\phi}_{\mathbf{x}}^k$ and $\overline{\mathbf{y}}^k$ are respectively the mean vectors of $\phi_{\mathbf{x}}(\mathbf{x}_j^k)$ and $\mathbf{y}_j^k$ ($j = 1, 2, \cdots, N^k$) and defined below.

$$\overline{\phi}_{\mathbf{x}}^k = \frac{1}{N^k} \mathbf{\Xi}_{\mathbf{x}}^k \mathbf{1}^k, \quad (2)$$

$$\overline{\mathbf{y}}^k = \frac{1}{N^k} \mathbf{Y}^k \mathbf{1}^k, \quad (3)$$

where $\mathbf{\Xi}_{\mathbf{x}}^k = [\phi_{\mathbf{x}}(\mathbf{x}_1^k), \phi_{\mathbf{x}}(\mathbf{x}_2^k), \cdots, \phi_{\mathbf{x}}(\mathbf{x}_{N^k}^k)]$, $\mathbf{Y}^k = [\mathbf{y}_1^k, \mathbf{y}_2^k, \cdots, \mathbf{y}_{N^k}^k]$, and $\mathbf{1}^k = [1, 1, \cdots, 1]'$ is an $N^k \times 1$ vector. Given $\{(\phi_{\mathbf{x}}(\mathbf{x}_j^k), \mathbf{y}_j^k) | j = 1, 2, \cdots, N^k\}$, $\mathbf{A}^k$ and $\mathbf{B}^k$ in Eq. (1) are matrices maximizing the correlation between the following two D^k-dimensional vectors:

$$\mathbf{s}_j^k = \mathbf{A}^{k\prime} \left(\phi_{\mathbf{x}}(\mathbf{x}_j^k) - \overline{\phi}_{\mathbf{x}}^k \right), \quad (4)$$

$$\mathbf{t}_j^k = \mathbf{B}^{k\prime} \left(\mathbf{y}_j^k - \overline{\mathbf{y}}^k \right), \quad (5)$$

and $\mathbf{\Lambda}^k$ is a $D^k \times D^k$ matrix whose diagonal elements represent the correlation coefficients. The dimension D^k is set to N^{Ω} in our method.

In order to obtain $\mathbf{A}^k$, $\mathbf{B}^k$, and $\mathbf{\Lambda}^k$, we utilize the regularized kernel CCA [5]. Note that the optimal matrix $\mathbf{A}^k$ is given by

$$\mathbf{A}^k = \mathbf{\Xi}_{\mathbf{x}}^k \mathbf{E}^k, \quad (6)$$

where $\mathbf{E}^k$ is an $N^k \times N^k$ matrix. Then, the matrices $\mathbf{E}^k$ and $\mathbf{B}^k$ can be obtained as the solutions of the following eigenvalue problems:

$$\mathbf{M}^{k^{-1}} \mathbf{L}^k \mathbf{P}^{k^{-1}} \mathbf{L}^{k\prime} \mathbf{E}^k = \mathbf{\Lambda}^{k^2} \mathbf{E}^k, \quad (7)$$

$$\mathbf{P}^{k^{-1}} \mathbf{L}^{k\prime} \mathbf{M}^{k^{-1}} \mathbf{L}^k \mathbf{B}^k = \mathbf{\Lambda}^{k^2} \mathbf{B}^k, \quad (8)$$

where $\mathbf{\Lambda}^k$ is obtained as an eigenvalue matrix, and $\mathbf{L}^k$, $\mathbf{M}^k$, and $\mathbf{P}^k$ are calculated as follows:

$$\mathbf{L}^k = \frac{1}{N^k} \mathbf{H}^k \mathbf{K}_{\mathbf{x}}^k \mathbf{H}^k \mathbf{H}^k \mathbf{Y}^{k\prime}, \quad (9)$$

$$\mathbf{M}^k = \frac{1}{N^k} \mathbf{H}^k \mathbf{K}_{\mathbf{x}}^k \mathbf{H}^k \mathbf{H}^k \mathbf{K}_{\mathbf{x}}^k \mathbf{H}^k - \eta_1 \mathbf{H}^k \mathbf{K}_{\mathbf{x}}^k \mathbf{H}^k, \quad (10)$$

$$\mathbf{P}^k = \frac{1}{N^k} \mathbf{Y}^k \mathbf{H}^k \mathbf{H}^k \mathbf{Y}^{k\prime} - \eta_2 \mathbf{I}^k. \quad (11)$$

In the above equations,

$$\mathbf{H}^k = \mathbf{I}^k - \frac{1}{N^k} \mathbf{1}^k \mathbf{1}^{k\prime} \quad (12)$$

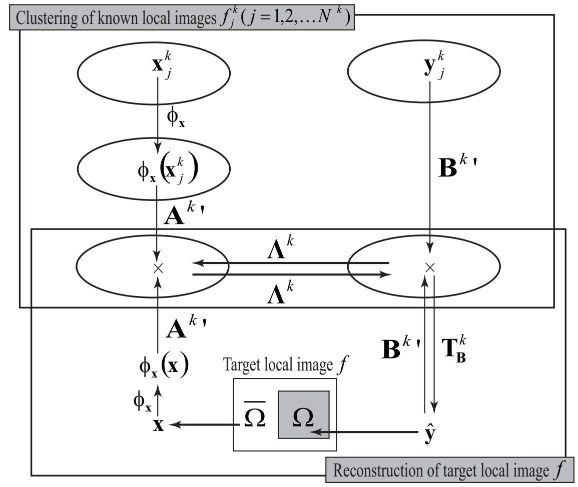

Fig. 1. Outline of the texture clustering and reconstruction algorithms based on kernel CCA.

is a centering matrix, where $\mathbf{I}^k$ is the $N^k \times N^k$ identity matrix. Furthermore, $\mathbf{K}_{\mathbf{x}}^k$ ($= \mathbf{\Xi}_{\mathbf{x}}^{k\prime} \mathbf{\Xi}_{\mathbf{x}}^k$) is the matrix whose (p, q)-th ($p = 1, 2, \cdots, N^k$, $q = 1, 2, \cdots, N^k$) element is defined as $\kappa_{\mathbf{x}}(\mathbf{x}_p^k, \mathbf{x}_q^k)$ by using the Gaussian kernel function $\kappa_{\mathbf{x}}(\cdot, \cdot)$. The values η_1 and η_2 in Eqs. (10) and (11) are regularization parameters.

From the obtained matrices $\mathbf{B}^k$, $\mathbf{E}^k$, $\mathbf{\Lambda}^k$, and Eqs.(2) and (3), Eq. (1) can be rewritten as follows:

$$C = \sum_{k=1}^{K} \sum_{j=1}^{N^k} \left\| \mathbf{B}^{k\prime} \left(\mathbf{y}_j^k - \frac{1}{N^k} \mathbf{Y}^k \mathbf{1}^k \right) - \mathbf{\Lambda}^k \mathbf{E}^{k\prime} \left(\kappa_j^k - \frac{1}{N^k} \mathbf{K}_{\mathbf{x}}^k \mathbf{1}^k \right) \right\|^2 / D^k, \quad (13)$$

where κ_j^k is an $N^k \times 1$ vector whose p-th element is $\kappa_{\mathbf{x}}(\mathbf{x}_j^k, \mathbf{x}_p^k)$. From the above equation, the mapped result

$$\begin{aligned} \hat{\mathbf{t}}_j^k &= \mathbf{\Lambda}^k \mathbf{E}^{k\prime} \left(\kappa_j^k - \frac{1}{N^k} \mathbf{K}_{\mathbf{x}}^k \mathbf{1}^k \right) \\ &= \mathbf{\Lambda}^k \mathbf{s}_j^k \end{aligned} \quad (14)$$

is the optimal approximation result of $\mathbf{t}_j^k$ in Eq. (5) as shown in Fig. 1. Therefore, $\|\mathbf{t}_j^k - \hat{\mathbf{t}}_j^k\|^2$ corresponds to the minimum distance between the new variate $\mathbf{t}_j^k$ of $\mathbf{y}_j^k$ and $\hat{\mathbf{t}}_j^k$ obtained from the new variate $\mathbf{s}_j^k$ of $\phi_{\mathbf{x}}(\mathbf{x}_j^k)$. Then, by using criterion C, we can effectively perform the clustering of the local images f_i ($i = 1, 2, \cdots, N$).

3. KERNEL CCA-BASED TEXTURE RECONSTRUCTION

In this section, we present a reconstruction method of the missing texture in the target local image f from the clustering results obtained by the previous section. First, we respectively denote the vectors of the raster scanned intensities in $\overline{\Omega}$ and Ω as $\mathbf{x}$ and $\mathbf{y}$. As shown in Fig. 1, the estimation result $\hat{\mathbf{y}}^k$ of the unknown vector $\mathbf{y}$ by cluster k is obtained as follows:

$$\hat{\mathbf{y}}^k = \mathbf{T}_{\mathbf{B}}^k \mathbf{\Lambda}^k \mathbf{A}^{k\prime} \left(\phi_{\mathbf{x}}(\mathbf{x}) - \overline{\phi}_{\mathbf{x}}^k \right) + \overline{\mathbf{y}}^k. \quad (15)$$

In the above equation, the matrix $\mathbf{T}_{\mathbf{B}}^k$ satisfies

$$\mathbf{T}_{\mathbf{B}}^k \mathbf{B}^{k\prime} \mathbf{Y}^k \mathbf{H}^k = \mathbf{Y}^k \mathbf{H}^k, \quad (16)$$

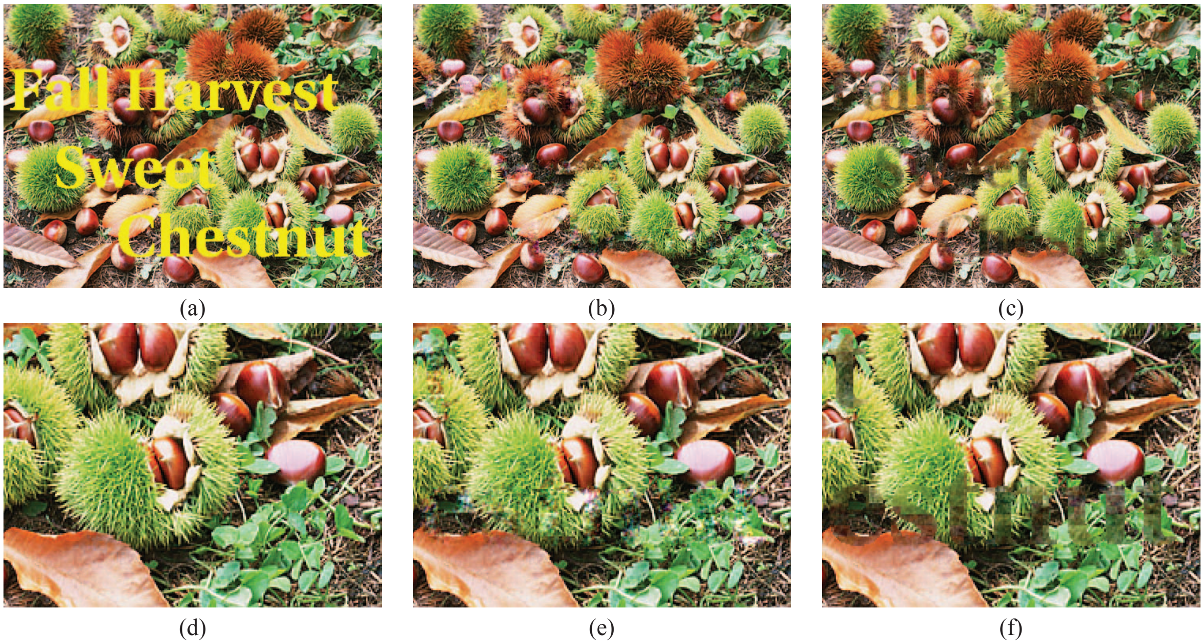

Fig. 2. (a) Corrupted image including text regions (11.3 % loss), (b) Reconstructed image by the proposed method (21.59 dB), (c) Reconstructed image by the traditional method (21.04 dB), (d) Zoomed portion of the original image, (e) Zoomed portion of (b), (f) Zoomed portion of (c).

and it is obtained by calculating the pseudo-inverse matrix of $\mathbf{B}^{k\prime}\mathbf{Y}^k\mathbf{H}^k$ as follows:

$$\mathbf{T}^k_{\mathbf{B}} = \mathbf{Y}^k\mathbf{H}^k\mathbf{H}^k\mathbf{Y}^{k\prime}\mathbf{B}^k\left(\mathbf{B}^{k\prime}\mathbf{Y}^k\mathbf{H}^k\mathbf{H}^k\mathbf{Y}^{k\prime}\mathbf{B}^k\right)^{-1}. \tag{17}$$

Furthermore, by utilizing the calculation scheme of Eq. (13), Eq.(15) is rewritten as follows:

$$\hat{\mathbf{y}}^k = \mathbf{T}^k_{\mathbf{B}}\boldsymbol{\Lambda}^k\mathbf{E}^{k\prime}\left(\kappa^k - \frac{1}{N^k}\mathbf{K}^k_{\mathbf{x}}\mathbf{1}^k\right) + \frac{1}{N^k}\mathbf{Y}^k\mathbf{1}^k, \tag{18}$$

where κ^k is an $N^k \times 1$ vector whose p-th element is $\kappa_{\mathbf{x}}(\mathbf{x}, \mathbf{x}^k_p)$.

By calculating $\hat{\mathbf{y}}^k$ in Eq. (18), the missing intensities in Ω can be estimated from cluster k. In the proposed method, the matrices $\mathbf{A}^k$ and $\mathbf{B}^k$, which maximize the correlation between the new variates in Eqs. (4) and (5), are calculated from $\phi_{\mathbf{x}}(\mathbf{x}^k_j)$ and $\mathbf{y}^k_j$ $(j = 1, 2, \cdots, N^k)$ by the kernel CCA. Then, from the obtained matrices $\mathbf{A}^k$, $\mathbf{B}^k$, and $\boldsymbol{\Lambda}^k$, $\mathbf{t}^k_j$ in Eq. (5) of cluster k can be optimally approximated by $\hat{\mathbf{t}}^k_j$ in Eq. (14) as shown in Fig. 1. Therefore, if we can classify the target image f into the optimal cluster k^{opt}, the proposed method accurately estimates the unknown vector $\mathbf{y}$ from the known vector $\phi_{\mathbf{x}}(\mathbf{x})$ in Eq. (18). However, since the target image f contains the missing area Ω, it cannot be classified by criterion C in Eq. (13). Thus, in order to achieve the classification of f, the proposed method utilizes the following novel criterion as a substitute for Eq. (13):

$$\tilde{C}^k = \left\|\mathbf{A}^{k\prime}\left(\phi_{\mathbf{x}}(\mathbf{x}) - \bar{\phi}^k_{\mathbf{x}}\right) - \boldsymbol{\Lambda}^k\mathbf{B}^{k\prime}\left(\hat{\mathbf{y}}^k - \bar{\mathbf{y}}^k\right)\right\|^2 / D^k. \tag{19}$$

By utilizing the calculation scheme of Eqs. (13) and (18), the above equation is rewritten as follows:

$$\tilde{C}^k = \left\|\mathbf{E}^{k\prime}\left(\kappa^k - \frac{1}{N^k}\mathbf{K}^k_{\mathbf{x}}\mathbf{1}^k\right) - \boldsymbol{\Lambda}^k\mathbf{B}^{k\prime}\left(\hat{\mathbf{y}}^k - \frac{1}{N^k}\mathbf{Y}^k\mathbf{1}^k\right)\right\|^2 / D^k. \tag{20}$$

As shown in the previous section, the mapped result

$$\begin{aligned} \hat{\mathbf{s}}^k &= \boldsymbol{\Lambda}^k\mathbf{B}^{k\prime}\left(\hat{\mathbf{y}}^k - \frac{1}{N^k}\mathbf{Y}^k\mathbf{1}^k\right) \\ &= \boldsymbol{\Lambda}^k\hat{\mathbf{t}}^k \end{aligned} \tag{21}$$

becomes the optimal approximation result of the known vector

$$\mathbf{s}^k = \mathbf{E}^{k\prime}\left(\kappa^k - \frac{1}{N^k}\mathbf{K}^k_{\mathbf{x}}\mathbf{1}^k\right), \tag{22}$$

when the target local image f belongs to cluster k. This means that the criterion $\tilde{C}^k$ corresponds to the minimum distance between the new variate $\mathbf{s}^k$ of the known vector $\phi_{\mathbf{x}}(\mathbf{x})$ and $\hat{\mathbf{s}}^k$ obtained from the new variate $\hat{\mathbf{t}}^k$ of the estimation result $\hat{\mathbf{y}}^k$. Therefore, this criterion is applicable for the classification of the target local image f. Then, the selection of the optimal cluster for the target local image f becomes possible. Furthermore, the proposed method regards the result $\hat{\mathbf{y}}^{k^{\mathrm{opt}}}$ obtained by the optimal cluster k^{opt} as the output. Consequently, by performing the non-conventional approach, which adaptively selects the optimal cluster for the missing texture, we can reconstruct all of the missing textures in the target image accurately.

As described above, we can reconstruct the missing texture in the target local image. The proposed method clips local images ($w \times h$ pixels) including missing textures in a raster scanning order and reconstructs them by using the above approach. Note that each restored pixel has multiple estimation results if the clipping interval is smaller than the size of the local images. In this case, the proposed method regards the result minimizing Eq. (20) as the final one.

4. EXPERIMENTAL RESULTS

The performance of the proposed method is shown in this section. Figure 2(a) is a test texture image (480 × 360 pixels, 24-bit color

(a) (b) (c)

Fig. 3. (a) Target image including text (8.9 % loss), (b) Reconstructed image by the proposed method (29.85 dB), (c) Reconstructed image by the traditional method (27.79 dB).

(a) (b) (c)

Fig. 4. (a) Target image including text (11.9 % loss), (b) Reconstructed image by the proposed method (26.46 dB), (c) Reconstructed image by the traditional method (25.82 dB).

levels) that includes the text regions "Fall Harvest Sweet Chestnut". Figure 2(b) shows the results of reconstruction by the proposed method. For comparison, Fig. 2(c) shows the results obtained by the traditional eigenspace method using projection of the nonlinear subspace obtained by the kernel PCA in [7]. For better subjective evaluation, the enlarged portions around the lower right of the images are shown in Figs. 2 (d)–(f). It can be seen that the use of the proposed method has achieved noticeable improvements. In the conventional method, different kinds of textures affect the reconstruction of the target missing textures. On the other hand, the proposed method can adaptively reconstruct the missing textures from only the reliable ones by selecting the optimal cluster including the same kinds of textures. Therefore, the proposed method has higher performance than that of the conventional method.

Different experimental results are shown in Figs. 3 and 4. Compared to the results obtained by the conventional method, it can be seen that various kinds of textures can be accurately restored by using the proposed method. Furthermore, in order to quantitatively evaluate the performance of the proposed method, we show the PSNR[1] of the reconstruction results in the captions of Figs. 2–4. It can be seen that our method has achieved an improvement of 0.55–2.06 dB over the conventional method. Therefore, high performance of the proposed method was verified by the experiments.

5. CONCLUSIONS

In this paper, we have proposed a kernel CCA-based texture reconstruction method. The proposed method applies the kernel CCA to each set containing the same kind of textures and adaptively estimates the missing intensities from the optimal correlation. In order to select the optimal one, the errors caused in the estimation scheme is introduced as a new criterion. Consequently, since the reliable textures can be utilized for the reconstruction of the target textures, impressive improvements in both objective and subjective measures have been achieved.

[1] PSNR $= 10\log_{10}\frac{\mathrm{MAX}^2}{\mathrm{MSE}}$, where MAX denotes the maximum value of intensities and MSE is the mean square error between the original image and the reconstructed image.

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