SEPARATION OF LAYERS FROM IMAGES CONTAINING MULTIPLE REFLECTIONS AND TRANSPARENCY USING CYCLIC PERMUTATION

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ABSTRACT

In the paper, we propose a new method for blind separation of an arbitrary number of images from a set of their linear mixtures with unknown coefficients. This approach is as follows. We first introduce a novel multiple correlation between one image and a set of multiple images. Then this multiple correlation leads us to provide a set of simultaneous linear equations for updating each mixture of images. Finally, source images are recovered by iterating between solving the sets of equations and cyclically permuting the mixtures of images. The technique can be applied for extracting multiple layers from images containing multiple reflections and transparency.

***Index Terms*—** Image processing, Image reconstruction

1. INTRODUCTION

Blind source separation (BSS), which aims at recovering unknown source signals from their linear mixtures without knowing the mixing coefficients, has recently received considerable attention in the image processing community [4]. The most of current blind source separation techniques are based straightforwardly on the independent component analysis (ICA) [1, 6, 7]. ICA-based blind source separation methods are able to handle an arbitrary number of sources. However, as far as blind image separation is concerned, these methods often suffer from poor separation results [8, 9].

Recently, excellent image separation results have been obtained with a variety of approaches without explicitly using ICA [5, 8, 9, 10]. The most of these approaches are, however, limited to separation of mixtures of two images, and thus they cannot be used for separating layers from images containing multiple reflections and transparency (see Fig. 2(a)).

Our work is inspired by the recent work of Sarel and Irani [8], who successfully separated mixtures of two independent images by a correlation-based information exchange between mixtures of images. In this paper, their method will be extended to the case of mixtures of an arbitrary number of images. This approach is as follows. We first introduce a multiple correlation between one image and a set of multiple images. Then this multiple correlation leads us to provide a set of simultaneous linear equations for updating each mixture of images. Finally, source images are estimated by iterating between solving the sets of equations and cyclically permuting the mixtures of images. We show its effectiveness through experiments with synthetic and real images.

2. BLIND SEPARATION OF MIXTURES OF MULTIPLE IMAGES

We propose extension of the two-layer separation method in [8], adapted to the multi-layer separation. Let $\{\boldsymbol{\alpha}_k\}_{k=1}^K = \{(\alpha_{k,1}, \alpha_{k,2}, \cdots, \alpha_{k,K})^T\}_{k=1}^K$ be a set of K different values for a K-dimensional coefficient vector. Let $\{I_k\}_{k=1}^K$ be the corresponding set of the K images obtained by linearly combining K independent images $\{L_k\}_{k=1}^K$ as,

$$\begin{aligned} I_1(i) &= \alpha_{1,1}L_1(i) + \alpha_{1,2}L_2(i) + \cdots + \alpha_{1,K}L_K(i), \\ I_2(i) &= \alpha_{2,1}L_1(i) + \alpha_{2,2}L_2(i) + \cdots + \alpha_{2,K}L_K(i), \\ &\vdots \\ I_K(i) &= \alpha_{K,1}L_1(i) + \alpha_{K,2}L_2(i) + \cdots + \alpha_{K,K}L_K(i), \\ &\qquad (i = 1, 2, \cdots, N), \end{aligned} \tag{1}$$

where $I_k(i)$ and $L_k(i)$ are, respectively, the values of the i-th pixels in the images I_k and L_k for $k = 1, 2, \cdots, K$ and N is the total number of pixels. Now, for given linear mixtures $\{I_k\}_{k=1}^K$ with unknown coefficient vectors $\{\boldsymbol{\alpha}_k\}_{k=1}^K$, we will estimate (constant times each of) the most likely source images $\{L_k\}_{k=1}^K$.

2.1. The MGNGC Measure

We introduce a multiple correlation, which we will refer to as the Multiple Generalized Normalized Gray-scale Correlation ($MGNGC$) measure, between an image f_K and the other $K-1$ images $\{f_k\}_{k=1}^{K-1}$ as,

$$MGNGC_K(f_1, f_2, \cdots, f_{K-1}; f_K) = \frac{\sum_{i=1}^N MNGC_{K,i}^2(f_1, f_2, \cdots, f_{K-1}; f_K) \prod_{k=1}^K V_i(f_k)}{\sum_{i=1}^N \prod_{k=1}^K V_i(f_k)}, \tag{2}$$

where $MNGC_{K,i}(f_1, f_2, \cdots, f_{K-1}; f_K)$ is the multiple correlation coefficient between the partial image (hereafter referred to as the "i-th partial image") composed of $W \times W$

pixels centered at a pixel i in f_K and the i-th partial images in $\{f_k\}_{k=1}^{K-1}$ and, by the definition of the multiple correlation coefficient, is expressed as

$$MNGC_{K,i}(f_1, f_2, \cdots, f_{K-1}; f_K) = \sqrt{1 - \frac{1}{\widetilde{r}^{i}_{K,K}}}, \quad (3)$$

where $\widetilde{r}^{i}_{K,K}$ is the KK-th element of the inverse matrix $(R^i_K)^{-1}$ of the correlation matrix

$$\boldsymbol{R}^i_K = \begin{pmatrix} 1 & r^i_{1,2} & \cdots & r^i_{1,K} \\ r^i_{2,1} & 1 & \cdots & r^i_{2,K} \\ \vdots & \vdots & \ddots & \vdots \\ r^i_{K,1} & r^i_{K,2} & \cdots & 1 \end{pmatrix}, \quad (4)$$

where $r^i_{k,l} \triangleq \frac{C_i(f_k, g_l)}{\sqrt{V_i(f_k)V_i(g_l)}}$ $(k, l = 1, 2, \cdots, K)$, where $V_i(f_k)$ is the variance of the i-th partial image in f_k and $C_i(f_k, g_l)$ is the covariance between the i-th partial images in f_k and g_l. The $MGNGC$ measure is a multiple-correlation-based extension of *generalized normalized gray-scale correlation* ($GNGC$) measure [8] and forms the basis of our algorithm to be discussed in the next subsection.

2.2. Algorithm

We present a method for separating linear mixtures of an arbitrary number of images based on the development of the previous subsection. Given as input a set, $\{I_k\}_{k=1}^{K}$, of K mixtures, our algorithm can be run in the following two steps.

Step.0 Initialize $\{I^{(t)}_k\}_{k=1}^{K}$ as follows and then set the iteration number t to 1.

$$\begin{pmatrix} I^{(0)}_1 \\ I^{(0)}_2 \\ \vdots \\ I^{(0)}_K \end{pmatrix} = \begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_K \end{pmatrix} \quad (5)$$

Step.1 Find minimizer $(\sigma^{(t)}_1, \sigma^{(t)}_2, \cdots, \sigma^{(t)}_{K-1})$ of the $MGNGC$ measure between $I^{(t-1)}_K - \sum_{k=1}^{K-1} \sigma_k I^{(t-1)}_k$ and $\{I^{(t-1)}_k\}_{k=1}^{K}$ as follows.

$$(\sigma^{(t)}_1, \sigma^{(t)}_2, \cdots, \sigma^{(t)}_{K-1}) = \operatorname*{argmin}_{\sigma_1, \sigma_2, \cdots, \sigma_{K-1}} MGNGC_K(I^{(t-1)}_1, \cdots, I^{(t-1)}_{K-1}; I^{(t-1)}_K - \sum_{k=1}^{K-1} \sigma_k I^{(t-1)}_k) \quad (6)$$

Step.2 Update $I^{(t-1)}_K$ to $I^{(t-1)}_K - \sum_{k=1}^{K-1} \sigma^{(t)}_k I^{(t-1)}_k$ and then permute cyclically $\{I^{(t-1)}_k\}_{k=1}^{K}$ as follows. Then let $t \leftarrow t + 1$ and return to step 1.

$$\begin{pmatrix} I^{(t)}_1 \\ I^{(t)}_2 \\ I^{(t)}_3 \\ \vdots \\ I^{(t)}_K \end{pmatrix} = \begin{pmatrix} I^{(t-1)}_K - \sum_{k=1}^{K-1} \sigma^{(t)}_k I^{(t-1)}_k \\ I^{(t-1)}_1 \\ I^{(t-1)}_2 \\ \vdots \\ I^{(t-1)}_{K-1} \end{pmatrix} \quad (7)$$

In the following, we will derive update equations for $\{\sigma_k\}_{k=1}^{K-1}$ in the Step 1. First we devide (4) into four blocks as,

$$\boldsymbol{R}^i_K = \left(\begin{array}{c|c} \boldsymbol{R}^i_{K-1} & \boldsymbol{r}^i_{K-1} \\ \hline (\boldsymbol{r}^i_{K-1})^T & 1 \end{array}\right), \quad (8)$$

where $\boldsymbol{R}^i_K$ is a $K-1$ dimensional square matrix and $\boldsymbol{r}^i_{K-1}$ is a $K-1$ dimensional vector. Hence, from the definition of the inverse matrix, $\widetilde{r}^i_{K,K}$ included in (3) is expressed as

$$\widetilde{r}^i_{K,K} = \frac{\det(\boldsymbol{R}^i_{K-1})}{\det(\boldsymbol{R}^i_K)}, \quad (9)$$

where $\det(\cdot)$ denotes the matrix determinant. By first assuming $\det(\boldsymbol{R}^i_{K-1}) \neq 0$ and then applying the formula [3] for the determinant of a block matrix to (8), we have

$$\det(\boldsymbol{R}^i_K) = \det(\boldsymbol{R}^i_{K-1}) \cdot \left(1 - (\boldsymbol{r}^i{}_{K-1})^T (\boldsymbol{R}^i_{K-1})^{-1} \boldsymbol{r}^i{}_{K-1}\right), \quad (10)$$

where $(\cdot)^{-1}$ denotes the inverse matrix. Using (10), we can rewrite (9) as,

$$\widetilde{r}^i_{K,K} = \frac{1}{1 - (\boldsymbol{r}^i{}_{K-1})^T (\boldsymbol{R}^i_{K-1})^{-1} \boldsymbol{r}^i{}_{K-1}}. \quad (11)$$

Hence, using (11), we can rewrite (3) as,

$$MNGC_{K,i}(f_1, f_2, \cdots, f_{K-1}; f_K) = \sqrt{(\boldsymbol{r}^i{}_{K-1})^T (\boldsymbol{R}^i_{K-1})^{-1} \boldsymbol{r}^i{}_{K-1}}. \quad (12)$$

Now, let us rewrite the objective function defined in (6) using (2) as (omitting subscript $t-1$),

$$MGNGC_K(I_1, I_2, \cdots, I_{K-1}; \widetilde{I}) = \frac{\sum_{i=1}^{N} \Psi_i}{\sum_{i=1}^{N} V_i(\widetilde{I}) \prod_{k=1}^{K-1} V_i(I_k)}, \quad (13)$$

where $\widetilde{I}$ and Ψ_i are, respectively, as follows.

$$\widetilde{I} = I_K - \sum_{k=1}^{K-1} \sigma_k I_k \,, \quad (14)$$

$$\Psi_i = MNGC^2_{k,i}(I_1, \cdots, I_{K-1}; \widetilde{I}) V_i(\widetilde{I}) \prod_{k=1}^{K-1} V_i(I_k). \quad (15)$$

Thus, using (12), we can rewrite (15) as,

$$\Psi_i = \left((\boldsymbol{u}^i_{K-1})^T (\boldsymbol{U}^i_{K-1})^{-1} \boldsymbol{u}^i_{K-1}\right) \prod_{k=1}^{K-1} V_i(I_k), \qquad (16)$$

where $\boldsymbol{U}^i_{K-1}$ and $\boldsymbol{u}^i_{K-1}$ are, respectively,

$$\boldsymbol{U}^i_{K-1} = \begin{pmatrix} 1 & u^i_{1,2} & \cdots & u^i_{1,K-1} \\ u^i_{2,1} & 1 & \cdots & u^i_{2,K-1} \\ \vdots & \vdots & \ddots & \vdots \\ u^i_{K-1,1} & u^i_{K-1,2} & \cdots & 1 \end{pmatrix}, \qquad (17)$$

$$\boldsymbol{u}^i_{K-1} = \left(u^i_{1,K}\,,\, u^i_{2,K}\,,\, \cdots\,,\, u^i_{K-1,K} \right)^T, \qquad (18)$$

where $u^i_{k,l} = NGC_i(I_k, I_l)\ \ (k, l = 1, 2, \cdots, K)$.

Taking partial derivatives of (16) with respect to σ_k, for $k = 1, 2, \cdots, K-1$, we get

$$\begin{aligned} \frac{\partial \Psi_i}{\partial \sigma_k} &= \Big(\frac{\partial \boldsymbol{u}^i_{K-1}}{\partial \sigma_k}\Big)^T \cdot \frac{\partial \Psi_i}{\partial \boldsymbol{u}^i_{K-1}} \\ &= 2\Big(C_i(I_k, I_K) - \sum_{m=1}^{K-1} \sigma_m C_i(I_k, I_m)\Big) \prod_{l=1}^{K-1} V_i(I_l) \\ &\qquad (k = 1, 2, \cdots, K-1). \end{aligned} \qquad (19)$$

So, the partial derivatives of the denominator, $\sum_{i=1}^{N} \Psi_i$, of (13) with respect to σ_k, for $k = 1, 2, \cdots, K-1$, are given by

$$\begin{aligned} &\frac{\partial}{\partial \sigma_k} \sum_{i=1}^{N} \Psi_i \\ &= 2\sum_{i=1}^{N} \Big(C_i(I_k, I_K) - \sum_{m=1}^{K-1} \sigma_m C_i(I_k, I_m)\Big) \prod_{l=1}^{K-1} V_i(I_l) \\ &\qquad (k = 1, 2, \cdots, K-1). \end{aligned} \qquad (20)$$

Further, one can easily see that for $k = 1, 2, \cdots, K-1$, the partial derivatives of the numerator, $\sum_{i=1}^{N} V_i(\widetilde{I}) \prod_{k=1}^{K-1} V_i(I_k)$, of (13) with respect to σ_k is also equal to (20). Hence, instead of setting the partial derivatives of (13) to zero, setting the partial derivatives of (20), with respect to σ_k, for $k = 1, 2, \cdots, K-1$, to zero, we get (the details of the derivation are described in the Appendix)

$$\begin{aligned} &\sum_{m=1}^{K-1} \Big(\sum_{i=1}^{N} C_i(I_k, I_m) \prod_{l=1}^{K-1} V_i(I_l)\Big)\sigma_m \\ &= \sum_{i=1}^{N} C_i(I_k, I_K) \prod_{l=1}^{K-1} V_i(I_l)\ \ (k = 1, \cdots, K-1). \end{aligned} \qquad (21)$$

(21) is equivalent to a set of $K-1$ linear simultaneous equations with unknowns $\{\sigma_k\}_{k=1}^{K-1}$. Let $\boldsymbol{\sigma}^{(t)} = (\sigma_1^{(t)}, \cdots, \sigma_{K-1}^{(t)})^T$ be the solution of (21) as in (6). Then, the update equations for $\{\sigma_k\}_{k=1}^{K-1}$ in Step.1 is obtained analytically as,

$$\boldsymbol{\sigma}^{(t)} = \left(\boldsymbol{A}^{(t-1)}_{K-1}\right)^{-1} \boldsymbol{b}^{(t-1)}_{K-1}, \qquad (22)$$

where the kl-th element, $a^{(t-1)}_{k,l}$, of the $K-1$ dimensional square matrix $\boldsymbol{A}^{(t-1)}_{K-1}$ is given by

$$\begin{aligned} a^{(t-1)}_{k,l} &= \sum_{i=1}^{N} C_i(I_l^{(t-1)}, I_k^{(t-1)}) \prod_{m=1}^{K-1} V_i(I_m^{(t-1)}) \\ &\qquad (k,\, l = 1, 2, \cdots, K-1), \end{aligned} \qquad (23)$$

and the l-th element, $b^{(t-1)}_l$, of the $K-1$ dimensional vector $\boldsymbol{b}^{(t-1)}_{K-1}$ is given by

$$\begin{aligned} b^{(t-1)}_l &= \sum_{i=1}^{N} C_i(I_l^{(t-1)}, I_K^{(t-1)}) \prod_{m=1}^{K-1} V_i(I_m^{(t-1)}) \\ &\qquad (\,l = 1, 2, \cdots, K-1). \end{aligned} \qquad (24)$$

3. EXPERIMENTAL RESULTS

3.1. Synthetic Images

We present experiments with synthetic mixtures of four known images. Fig. 1(a)∼(d) shows the source images. We mixed them using different mixing ratios (Fig. 1(e)∼(h)). For comparison, we show the results of the ICA-based separation method [2] and our separation method in Fig. 1(i)∼(l) and Fig. 1(m)∼(p), respectively. We can see that our approach gives very good results, while the separation results of the ICA-based approach are poor.

3.2. Real Images

We apply our method to layer extraction from images containing multiple reflections and transparency. We photographed a picture postcard in a glass-fronted bookcase (Fig. 2(a)). You can see transparency and double reflections due to light reflected from both the surfaces of the front side and rear side glasses (region surrounded by a rectangle of Fig. 2(a)). We took three photographs under three different illumination conditions by inserting a polarization sheet at the front or back of two glasses on the bookcase, or between them (Fig. 2(c)∼(e)). For acquisition of a ground truth transparency image we shot the same scene while shielding out some of the ambient light using a blackout curtain (Fig. 2(b)). The results for the mixed images (c)∼(e) in Fig. 2 are shown in Fig. 2(f)∼(h). The reconstructed image (f) in Fig. 2 is similar to the transparency image (b) in Fig. 2, and both of the ones (g)(h) in Fig. 2 are also relatively clear.

4. CONCLUSIONS

We have proposed a novel method of recovering a set of source images from a set of their linear mixtures of multiple images with unknown mixture coefficients using multiple correlation analysis. We have derived the separation algorithm, and shown its effectiveness through experiments with mixtures of synthetic and real images.

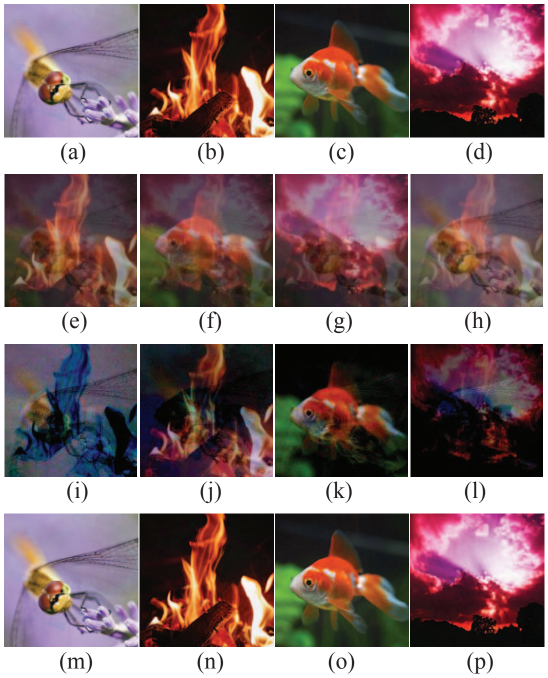

Fig. 1. Comparison of our method with the ICA-based method: (a)∼(d) source images, (e)∼(h) mixed images, (i)∼(l) reconstructed images using the ICA-based method, (m)∼(p) reconstructed images using our method.

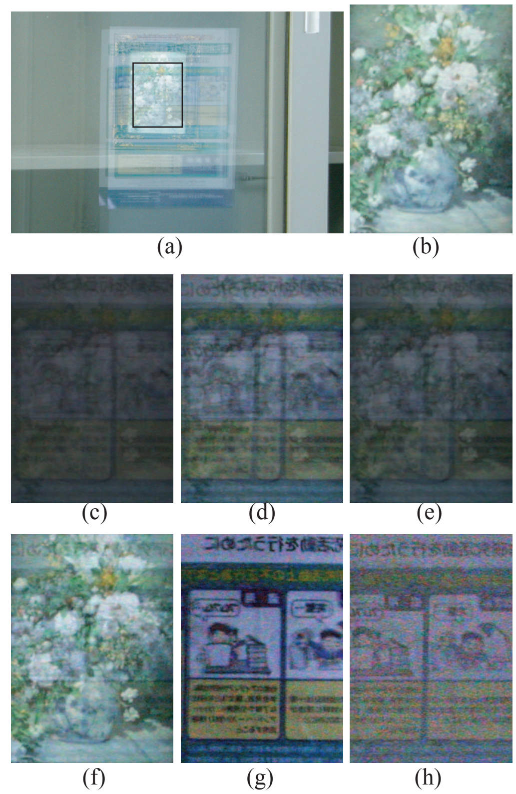

Fig. 2. Separation of layers from mixtures with nearly singular mixing matrices: (a)∼(d) source images, (e)∼(h) mixed images (mixing matrix $\mathbf{A}$), (i)∼(l) reconstructed images (mixing matrix $\mathbf{A}$), (m)∼(p) mixed images (mixing matrix $\mathbf{B}$), (q)∼(t) reconstructed images (mixing matrix $\mathbf{B}$).

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A. DERIVATION OF (21)

Let F and G be the denominator and the numerator of (13), respectively. Then, from (20), we get

$$\frac{\partial F}{\partial \sigma_k} = \frac{\partial G}{\partial \sigma_k} = 2\sum_{i=1}^{N}\Big(C_i(I_k, I_K) - \sum_{m=1}^{K-1}\sigma_m C_i(I_k, I_m)\Big)\prod_{l=1}^{K-1} V_i(I_l)$$

Taking partial derivatives of (13) with respect to σ_k and setting them to zero, we get

$$\begin{aligned} 0 &= \frac{\partial}{\partial \sigma_k}(MGNGC_K(I_1, I_2, \cdots, I_{K-1}; \widetilde{I})) = \frac{\partial}{\partial \sigma_k}\Big(\frac{F}{G}\Big) \\ &= \frac{1}{G^2}\Big(G\frac{\partial F}{\partial \sigma_k} - F\frac{\partial G}{\partial \sigma_k}\Big) = \frac{1}{G}\Big(1 - \frac{F}{G}\Big)\frac{\partial F}{\partial \sigma_k} \end{aligned} \tag{25}$$

Then, from the definition of $MGNGC$ and the independency of $\{I_k\}_{k=1}^{K}$, we have $F/G < 1$ Hence, we get $\partial F/\partial \sigma_k = 0$ in (25), which leads to the following equation.

$$\frac{\partial F}{\partial \sigma_k} = \sum_{i=1}^{N}\Big(C_i(I_k, I_K) - \sum_{m=1}^{K-1}\sigma_m C_i(I_k, I_m)\Big)\prod_{l=1}^{K-1} V_i(I_l) = 0$$

This equation can be rewritten as,

$$\sum_{m=1}^{K-1}\Big(\sum_{i=1}^{N} C_i(I_k, I_m)\prod_{l=1}^{K-1} V_i(I_l)\Big)\sigma_m = \sum_{i=1}^{N} C_i(I_k, I_K)\prod_{l=1}^{K-1} V_i(I_l)$$