PHASE UNWRAPPING FOR INTERFEROMETRIC SAR USING MULTIBASELINE JOINT DATA GROUP [1]

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ABSTRACT

Phase unwrapping is the key problem in building the digital elevation model (DEM) of a scene from interferometric synthetic aperture radar (SAR) system data. In this paper, we propose a method of phase unwrapping based on the model of the multibaseline joint data group. The method can not only adaptively coregister the SAR images, but also accurately provide the accurate estimation of the terrain unwrapped phase in the presence of the large coregistration errors. Moreover, the improvement in computational complexity is achieved by using the multibaseline joint data group. The method is investigated by simulations, and results show successful phase unwrapping even if the image coregistration error is close to one pixel.

Index Terms—phase unwrapping, interferometry, multibaseline joint data group, Cramer-Rao lower bounds (CRLB), digital elevation model (DEM)

1. INTRODUCTION

Phase unwrapping is the key problem in building the digital elevation model (DEM) of a scene from interferometric synthetic aperture radar (SAR) system data [1,2]. Almost all existing conventional InSAR phase unwrapping methods [3-5] have no capability to resolve the conflict between height sensitivity and interferometric phase aliasing as well. Whereas, the multibaseline InSAR systems (with two or more cross-track baselines) have the ability to overcome these drawbacks associated with single-baseline InSAR systems and significantly increase the ambiguity intervals of interferometric phases without degrading the height accuracy. Therefore, the exploitation of multibaseline InSAR for facilitating phase unwrapping and high-quality DEM reconstruction is widely investigated in the literature [6-9].

In InSAR data processing for the generation of the DEM of a terrain, image coregistration is likewise a fundamental task in image processing used to match two or more SAR images. When the required coregistration accuracy is not reached, the obtained interferometric phase will be too noisy to be unwrapped. Accordingly, it is necessary to find a registration strategy robust to the large coregistration error to unwrap the terrain interferometric phase.

In this paper, we propose an idea of the robust phase unwrapping method based on the multibaseline joint data group. The essence of the method is based on the combination joint pixel approach, array processing technique and optimization algorithm, which is quite different from that of the interferogram filtering. Moreover, the dimensions of the covariance matrix of the proposed method only relate to the number of the array phase centers and are uncorrelated with how to choose the pixel window sizes to construct the multibaseline joint data group. Based on this idea, the method can be performed in two steps: the first step is construction of the multibaseline joint data group from all of the neighboring pixels within a rectangular window after the coarse coregistration. The second is to estimate the optimized covariance matrix, and then the unwrapped phase can be obtained by using the adaptive eigenspace-like beamformer.

2. SIGNAL MODEL AND PROBLEM STATEMENT

Consider a multibaseline InSAR system, composed of a uniform linear array (ULA) of M two-dimensional phase centers. The obtained M SAR images collected by the sensors of the array, denoted as $\mathbf{x}(i)$, of a pixel pair i (corresponding to the same ground area) can be modeled as

$$\mathbf{x}(i) = \mathbf{a}(\varphi_i) \odot \mathbf{s}(i) + \mathbf{n}(i) \tag{1}$$

where $\mathbf{x}(i)$ denotes the complex data vector, which can be modeled as a joint zero-mean complex circular Gaussian random vector [1,2], $\mathbf{s}(i)$ is the complex magnitude vector (i.e., the complex reflectivity vector received by the satellites), and $\mathbf{n}(i)$ is the additive noise term, and $\odot$ denotes the Hadamard product. Furthermore, $\mathbf{a}(\varphi_i) = \left\{ e^{j(m-1)\varphi_i/(M-1)} \right\}_{m=1}^{M}$ represents the array steering vector (or the spatial steering vector) of the pixel pair i, and φ_i is an unknown deterministic parameter representing the unwrapped phase for the i^{th} examined resolution cell, i.e., the phase difference between the two furthest phase centers in the array.

For the convenience of presenting the proposed method, as

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shown in Fig.1, where rectangles represent the SAR image pixels, and i is the centric pixel pair (i.e., the desired pixel pair whose absolute phase to be estimated).When we construct the joint complex pixel vector, the selection of the pixel window sizes is tradeoff between the computational complexity and the lack of enough samples to estimate the covariance matrix. In this paper, the proposed method can provide the robust unwrapped phases (or the terrain heights) even in the presence of the large image coregistration errors, and has the ability to overcome the conflict associated with the computational complexity and the lack of the independent and identically distributed (i.i.d.) samples. As shown in Fig.1, we define the multibaseline joint data group as

$$\mathbf{X}(i)=\left[\mathbf{x}_1(i),\mathbf{x}_2(i),\cdots,\mathbf{x}_M(i)\right]^T \quad (2)$$

$$\mathbf{x}_m(i)=\left[x_m(i-4),\cdots,x_m(i),\cdots,x_m(i+4)\right],m=1,2,\cdots,M \quad (3)$$

where the superscript T stands for vector transpose. By using equations (2) and (3) one can write the corresponding joint covariance matrix as follows:

$$\mathbf{C}_{\mathbf{X}}(i)=E\left[\mathbf{X}(i)\mathbf{X}(i)^H\right]=\sigma_x^2(i)\mathbf{A}(\varphi_i)\mathbf{A}^H(\varphi_i)\odot\mathbf{R}_S(i)+\sigma_n^2\mathbf{I} \quad (4)$$

where $\mathbf{R}_S(i)$ is called the joint correlation coefficient matrix of the pixel pair i. $\mathbf{I}$ is an $M\times M$ identity matrix, and $E[\cdot]$ denotes the statistical expectation operator, the superscript H denotes vector conjugate-transpose, $\sigma_x^2(i)$ is the echo power of the pixel pair i and σ_n^2 is the noise power. And

$$\mathbf{A}(\varphi_i)=\left[\mathbf{a}(\varphi_{i-4}),\mathbf{a}(\varphi_{i-3}),\cdots,\mathbf{a}(\varphi_i),\cdots,\mathbf{a}(\varphi_{i+4})\right]^T \quad (5)$$

For simplicity and without loss of generality, we assume that the neighboring pixels have an identical terrain height. Then the array steering vectors of the pixel pair in the multibaseline joint data group become identical, i.e.,

$$\mathbf{a}(\varphi_{i-4})=\mathbf{a}(\varphi_{i-3})=\cdots=\mathbf{a}(\varphi_i)=\cdots=\mathbf{a}(\varphi_{i+4}). \quad (6)$$

In practice, considering the presence of the image coregistration error, we use not only the current pixel i but also its neighboring pixels, as shown in Fig.1, selected a 3×3 window to jointly estimate the covariance matrix. And in fact many windows can be used to construct the joint complex pixel vector, such as a 2×2 window, or even a one-dimensional window (if we have *a priori* knowledge of the direction of image misregistration). Contrary to the existing interferogram filtering methods, the dimensions of the covariance matrix in (4) are $M\times M$, regardless of the selection of the pixel window sizes. Accordingly, to provide a satisfactory unwrapped phase estimate, we can select a more reasonable window to construct the multibaseline joint data group.

From equations (5) and (6), we can see that $\mathbf{A}$ is a Vandermonde matrix, and $\mathbf{A}\mathbf{A}^H$ is a Hermitian matrix. Consequently, the beamforming problem is formulated as follows:

$$\mathbf{A}\mathbf{A}^H=\begin{bmatrix} 1 & e^{-j\varphi_i/(M-1)} & \cdots & e^{-j\varphi_i} \\ e^{j\varphi_i/(M-1)} & 1 & \cdots & e^{-j(M-2)\varphi_i/(M-1)} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j\varphi_i} & e^{j(M-2)\varphi_i/(M-1)} & \cdots & 1 \end{bmatrix} \quad (7)$$

$$\mathbf{P}=\mathbf{a}^H(\varphi_i)\mathbf{C}_{\mathbf{X}}(i)\mathbf{a}(\varphi_i) \quad (8)$$

The maximum output provides an estimate of the signal power and the unwrapped phase estimate is given by the scan value of φ_i that achieves this maximum, namely

$$\begin{aligned}\mathbf{P}&=\mathbf{a}^H(\varphi_i)\left(\sigma_x^2\mathbf{A}(\varphi_i)\mathbf{A}^H(\varphi_i)\odot\mathbf{R}_S(i)+\sigma_n^2\mathbf{I}\right)\mathbf{a}(\varphi_i)\\&=M\cdot\left(\sigma_x^2\mathbf{a}^H(\varphi_i)\mathbf{R}_S(i)\mathbf{a}(\varphi_i)+\sigma_n^2\mathbf{I}\right)\end{aligned} \quad (9)$$

Accordingly, the problem of interest herein is the estimation of the unwrapped phases from the covariance matrix $\mathbf{C}_{\mathbf{X}}(i)$ with unknown $\sigma_x^2(i)$, $\mathbf{R}_S(i)$ and σ_n^2. To describe the details of our approach, we first introduce the correlation matrix which contains the noise subspace, and then carry out the projection of the array steering vector onto the correlation matrix to estimate the unwrapped interferometric phases (or the terrain heights). Accordingly, we define our processing procedure as the eigenspace-like beamformer.

3. PROCESSING PROCEDURES

The proposed method can be described as follows:

Step1) The SAR images are coarsely coregistered first by using the cross correlation information of the SAR image intensity or other methods after SAR imaging of the echoes received by each satellite.

Step2) If the SAR images are accurately coregistered, the construction of the multibaseline joint data group $\mathbf{X}(i)$ is shown above in (2). The corresponding sample covariance matrix is given in (4). In practice, the coregistration error always exists in the SAR images, thus the construction of the weighted multibaseline joint data group can be rewritten as

$$\mathbf{X}(i,w_{opt})=\left[\mathbf{x}_1(i),\overline{\mathbf{x}}_2(i,w_{opt}^{(2)}),\cdots,\overline{\mathbf{x}}_M(i,w_{opt}^{(M)})\right]^T \quad (10)$$

where

$$\overline{\mathbf{x}}_m(i,w_{opt}^{(m)})=\left[\hat{x}_m(i-4),\cdots,\hat{x}_m(i),\cdots,\hat{x}_m(i+4)\right],m=2,\cdots,M \quad (11)$$

$$\hat{x}_m(k)=\mathbf{w}_{opt}^{(m)H}\mathbf{x}_m^T(k),k=i-4,i-3,\cdots,i,\cdots,i+4 \quad (12)$$

$\mathbf{w}_{opt}$ is the optimal weight vector, which is developed in [10](also see [12]).The corresponding covariance matrix, in fact, can be estimated by the sample covariance matrix in (13) of i.i.d. samples.

$$\hat{\mathbf{C}}_{\mathbf{X}}(i,w_{opt})=\frac{1}{2K+1}\sum_{k=-K}^{K}\mathbf{X}(i+k,w_{opt})\mathbf{X}^H(i+k,w_{opt}) \quad (13)$$

where $2K+1$ is the number of i.i.d. samples from the neighboring pixels. The noise eigenvector $\boldsymbol{\beta}_n^{(l)}, l=(2,\cdots,6)$ can be obtained from the eigen-decomposition of the sample optimized covariance matrix.

Step3) Since the joint correlation coefficient matrix can not be obtained from the received echo data, we make the amplitude (i.e., the absolute value) of the estimated optimized covariance matrix as the correlation coefficient matrix, i.e.,

$$\hat{\mathbf{R}}_S(i,w_{opt}) = \left|\hat{\mathbf{C}}_{\mathbf{X}}(i,w_{opt}) - \hat{\sigma}_n^2\mathbf{I}\right| \quad (14)$$

where $\hat{\sigma}_n^2$ denotes the noise power (the noise power can be estimated by the mean of the noise eigenvalues). By eigen-decomposing $\hat{\mathbf{R}}_S(i,w_{opt})$ one can obtain the principal eigenvector $\boldsymbol{\beta}_s$.

We introduce the $M \times M$ correlation matrix as

$$\mathbb{R} = \left(\boldsymbol{\beta}_s\boldsymbol{\beta}_s^H\right) \odot \left(\sum_{l=2}^{6}\boldsymbol{\beta}_n^{(l)}\boldsymbol{\beta}_n^{(l)H}\right)^H \quad (15)$$

Step4) Using equation (15), the unwrapped phase estimation can be obtained by using the eigenspace-like beamforming technique as

$$\hat{\varphi}_i = \arg\min_{\phi_i}\left\{\mathbf{a}^H(\phi_i)\cdot\mathbb{R}\cdot(\mathbf{a}(\phi_i)\right\} \quad (16)$$

The minimum in (16) corresponds to the estimate of the absolute interferometric phase.

By using the above four steps, the terrain unwrapped phases can be recovered after the SAR image pixel pairs are processed separately.

4. PERFORMANCE INVESTIGATION

In this section we evaluate the performance of the proposed method. Assuming a multibaseline cross-track interferometer system with six two-dimensional phase centers aligned to form a uniform linear array. We use a real SAR image to generate the reflectivity of each SAR pixel and simulate the mountainous terrain. The SNR of the SAR images is $17dB$ and the correlation coefficient of each pixel pair is computed according to the cross-track baseline length, the local terrain slope and the SNR [1,2].

Let us compare the performance of the proposed method with the method in [11]. It is well known that the dominant computational complexity of an algorithm is determined by that of the eigen-decomposition or inversion of the covariance matrix, and both these computational cost are equal to $O(M^3)$,where $O(\cdot)$ denotes "order of". And a larger pixel window has more degrees of freedom, and thus can perform a finer coregistration of SAR images, but it suffers from the computational complexity and the lack of enough samples to estimate the covariance matrix. Therefore, the selection of the pixel window sizes is a tradeoff between these considerations. When we select a 3×3 window to construct the multibaseline joint data group, the dimensions of the covariance matrix using the method in this paper are $M \times M$ .And the dimensions of that in [11] are $9M \times 9M$. Furthermore, the dimensions of the covariance matrix of the proposed method only relate to the number of the array phase centers and are uncorrelated with how to choose the pixel window sizes to construct the multibaseline joint data group. Accordingly, we can conclude that the overall computational cost of the proposed method is much lower than that of the joint subspace method.

We discuss likewise the eigenspectra of the covariance matrix for different coregistration errors. In Fig.2, we plot the eigenspectra of the covariance matrix for the different coregistration errors (the coregistration errors of the m^{th}($m=2,3,\cdots,6$) SAR image with respect to the first SAR image). Fig.2(a) is the eigenspectra of the covariance matrix for coregistration errors of [0.5,0.8,1.0] pixels, respectively. Fig.2(b) shows the eigenspectra for accurate coregistration and coregistration errors of [0.5,0.8,1.0] pixels after the optimization on the radar echo using the proposed method. From Fig.2(a) and Fig.2(b), we can observe that the phase noise is suppressed greatly by the proposed method.

To further verify the robustness of the proposed method to the different image coregistration errors, we reconstruct the DEM of the terrain via the obtained unwrapped phase. In the case of accurate coregistration of six SAR images, Fig.3(a) is the unwrapped phases by using the standard Copan beamforming (SCB). Fig.3(b) and Fig.3(c) are the reconstructed DEM and the height error map between the reconstructed DEM and the originally simulated terrain, respectively. Fig.4 plots the unwrapped phases, the reconstructed DEM and the height error map in the presence of coregistration error of 0.5 pixels using the SCB, respectively. When the image coregistration errors reach one pixel, the interferogram obtained by the SCB is very noisy.

In the presence of accurate coregistration and coregistration error of 0.5 pixels between the SAR images, Fig.5 and Fig.6 show the results by using the proposed method, respectively. When the image coregistration errors reach one pixel, the corresponding pixel pairs are completely decorrelated, and the proposed method can still accurately estimate the unwrapped phases, as shown in Fig.7. On the contrary, there are no interferometric fringes in the interferogram obtained by the SCB. Comparing Figs.5-7 with Fig.3 and Fig.4, we can observe that the large coregistration error has almost no effect on the interferogram obtained by the proposed method. The results from Figs.5-7 manifest that the method can provide the accurate unwrapped phases in the presence of the large image coregistration errors.

5. CONCLUSIONS

In this paper, we present a method for robust phase unwrapping based on the multibaseline joint data group.

The method can adaptively coregister the SAR images by using all the neighboring pixel information available, and provide the accurate estimation of the terrain unwrapped phase in the presence of the large coregistration errors. Moreover, the method has the ability to overcome the conflict associated with the computational complexity and the lack of the independent and identically distributed samples. Theoretical analysis and experimental results show that the proposed method can provide the accurate estimation of the terrain interferometric phase in the presence of the large coregistration errors.

6. REFERENCES

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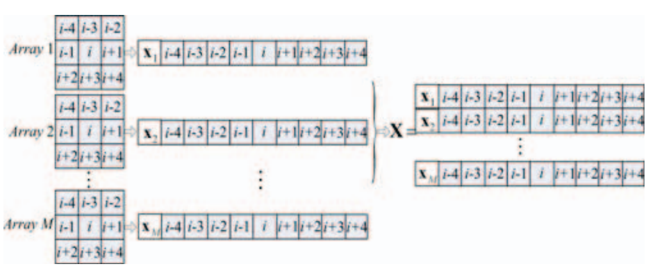

Fig.1. Construction of the multibaseline joint data group.

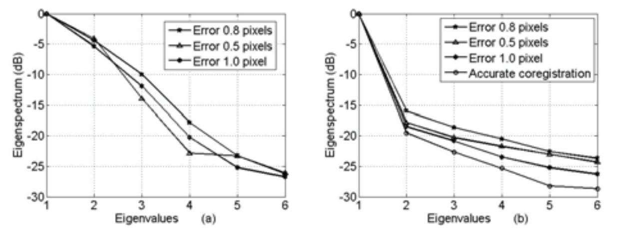

Fig.2. Eigenspectra of joint covariance matrix (a) Computed with the conventional method,(b) Computed with the proposed method.

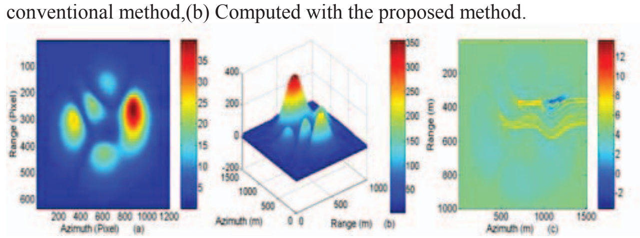
Fig.3. Results obtained by the SCB for accurate coregistration: (a) Unwrapped interferogram, (b) Reconstructed terrain, (c) Height error.

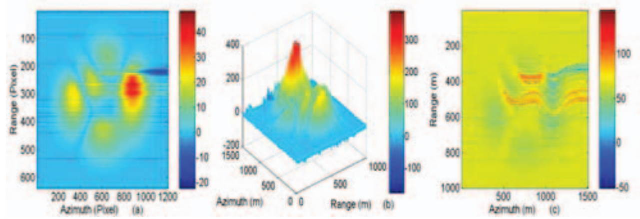
Fig.4. Results obtained by the SCB for coregistration error of 0.5 pixels: (a) Unwrapped interferogram, (b) Reconstructed terrain, (c) Height error.

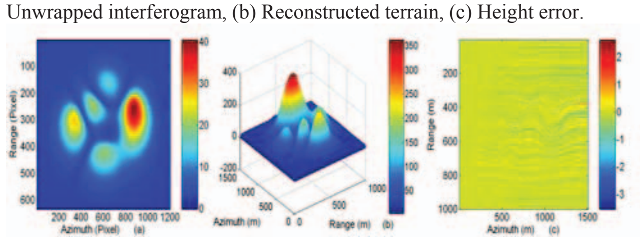
Fig.5. Results obtained by the proposed method for accurate coregistration: (a) Unwrapped interferogram, (b) Reconstructed terrain, (c) Height error.

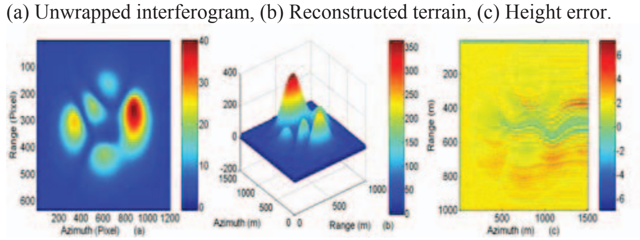
Fig.6. Results by the proposed method for coregistration error of 0.5 pixels: (a) Unwrapped interferogram, (b) Reconstructed terrain, (c) Height error.

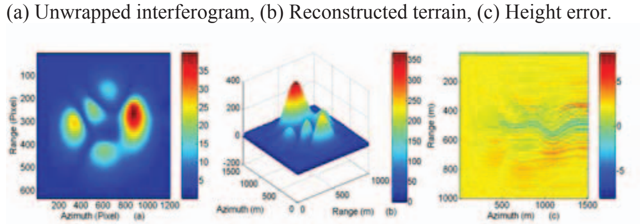
Fig.7. Results by the proposed method for coregistration error of 1.0 pixels: (a) Unwrapped interferogram, (b) Reconstructed terrain, (c) Height error.