

JOINT LINEAR-CIRCULAR STOCHASTIC MODELS FOR TEXTURE CLASSIFICATION

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ABSTRACT

In this paper, we investigate both linear and circular stochastic models in the context of texture discrimination. These models aim at representing the magnitudes and orientations obtained by a complex wavelet decomposition, such as the steerable pyramid. The novelty consists in considering specific parametric models for circular data such as von Mises and Ψ -distributions to describe the distributions of orientations. Particular attention is paid to the choice of a metric and to its adequation to the models. Indexing experiments are conducted to quantitatively evaluate the performances of the proposed models and of the chosen matrices, i.e. the L^1 and Kullback-Leibler distances.

Index Terms— texture, oriented pyramid decomposition, orientation, Gamma distribution, Ψ -distribution, Kullback-Leibler distance

1. INTRODUCTION

In conventional image processing tasks implying texture modeling such as filtering, classification, segmentation or synthesis, the main challenge is to find relevant features that capture the spatial information provided by the image. At the end of the 1990's, several works (e.g.[1]) related to texture modeling showed how relevant it is to model the filter bank responses obtained by multiscale oriented decomposition. The choice of a relevant model depends on the nature of the multiscale decomposition. In the case of complex wavelet decompositions, most approaches focus on the magnitudes or on the real and imaginary parts of the complex subbands using standard linear statistical models [2, 3, 4].

Yet, as a result of such complex filter bank decompositions, multiscale information about orientations in texture is directly available. Modeling such orientations imply the use of circular statistics i.e. statistics of directional data [5]. Directional data are a kind of cyclic data in which measures are embedded on the circle such as orientations in $[0, 2\pi[$ or directions in $[0, \pi]$. Recent works [6, 7, 8] propose to use the first and second circular moments or the circular histogram of angles to discriminate textures. None of these

works implements a complete stochastic modeling of the structural information on texture provided by orientation data.

In this paper, we aim at exploring the potential of both circular and linear stochastic models for texture discrimination. Circular and linear parametric models are chosen to describe orientation and magnitude information respectively. We consider that magnitudes and orientations are independent. The same principle was used in [8] where the empirical magnitude is modeled using a Gamma distribution and the orientation is described simply using the empirical histogram. In contrast, we propose here to make use of circular models to handle orientation data. We consider two types of parametric models respectively the von Mises and Ψ -distributions and investigate the appropriateness of some metrics or divergences. The efficiency of the couples “model-metric” is investigated within the image-retrieval framework described in [7]. Results are compared with the ones obtained in [8].

The article is organized as follows. In Section 2, we briefly review some related works proposed in literature dealing with orientation estimation. Then, in Section 3 we focus on magnitude and orientation modeling. In Section 4, we introduce the similarity measures. In section 5 we present the experimental setup and results. Finally, section 6 is dedicated to final remarks, conclusion, and prospects.

2. LOCAL MAGNITUDE AND ORIENTATION ESTIMATIONS

Directionality and coarseness are conventional features used to characterize texture regularity. They can be extracted from the image partitioned into frequency channels according to any complex wavelet decomposition. The image is decomposed using a set of $N_{or} \times N_{sc}$ oriented filters, where N_{or} and N_{sc} denote the numbers of orientations and scales. This decomposition results in $N_{or} \times N_{sc}$ complex oriented subbands $C_{k,l}(x,y)$ and two residual high-pass and low-pass bands. In the remaining of the paper, we consider only the $N_{or} \times N_{sc}$ oriented subbands. Low-pass or high-pass bands can exhibit multiple

orientations. Of course, several types of filters banks exist with various intrinsic properties; see [7] for instance.

Local orientations and associated magnitudes can be directly derived from complex coefficients. The moduli of the complex coefficients are related to the magnitude of texture patterns defined by $A_{k,l}(x, y) = |C_{k,l}(x, y)|$.

Local orientations can be deduced from the local increments of the coefficient phase $\varphi_{k,l}(x, y)$ defined by

$$\varphi_{k,l}(x, y) = \arctan 2 \left(\frac{\text{Im}(C_{k,l}(x, y))}{\text{Re}(C_{k,l}(x, y))} \right)$$

where $\varphi_{k,l} \in [-\pi, \pi]$.

Orientation at pixel (x, y) in band (k, l) is given by

$$\theta_{k,l}(x, y) = \arctan \left(\frac{\varphi_{k,l}(x, y+1) - \varphi_{k,l}(x, y)}{\varphi_{k,l}(x+1, y) - \varphi_{k,l}(x, y)} \right)$$

where $\theta_{k,l} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

In the following, parametric models for the probability density functions of $A_{k,l}(x, y)$ and $\theta_{k,l}(x, y)$ are proposed and used for texture characterization. We aim at evaluating the description capability of joint linear-modulus and circular-orientation stochastic models. Moreover, we argue that the parametric form is a well-founded alternative to the empirical discrete orientation histogram.

3. MODELS FOR MAGNITUDES AND ORIENTATION DISTRIBUTIONS

3.1 A model for magnitude distribution

In order to model the magnitude distribution, we use the Gamma distribution which has been proved in former works to be relevant for magnitude modeling [8]. The Gamma density function is defined by

$$f(A, \alpha, \beta) = A^{\alpha-1} \frac{e^{-(A/\beta)}}{\beta^\alpha \Gamma(\alpha)} \quad (1)$$

where $\alpha > 0$ is the shape parameter, and $\beta > 0$ is related to the scale of the distribution.

3.2 Models for orientation modelling

Appropriate models for orientation distributions are found in circular statistics. Such models are for instance the Von Mises [5] and the ψ -distributions [9].

3.2.1 Von Mises distribution

This distribution has interesting mathematical properties including optimal parameter estimators and mathematically tractable expressions for its log-likelihood and Kullback-Leibler divergence. It is defined by

$$\rho(\theta; \mu, \kappa) = \frac{1}{2\pi I_0(\kappa)} e^{\kappa \cos(\theta - \mu)} \quad (2)$$

where $-\pi \leq \mu \leq \pi$ and $\kappa \geq 0$ are called respectively the mean direction and the concentration, and I_0 denotes the modified

Bessel function of the 1st kind and order 0

$$I_0(\kappa) = \frac{1}{2\pi} \int_0^{2\pi} e^{\kappa \cos(\theta)} d\theta$$

3.2.2 ψ -Distribution

Recently, Jones and Pewsey [9] proposed a new family of symmetric distributions called the ψ -distributions with a density function of the form

$$\rho_\psi(\theta; \mu, \kappa) = \frac{(\cosh(\kappa\psi) + \sinh(\kappa\psi) \cos(\theta - \mu))^{1/\psi}}{2\pi P_{1/\psi}(\cosh(\kappa\psi))} \quad (3)$$

where $\kappa \geq 0$ and $-\pi \leq \mu \leq \pi$ are called the location and the concentration. Parameter $\psi \in \mathbb{R}$ describes the shape of the distribution. $P_{1/\psi}$ is the associated Legendre function of the first kind of degree $1/\psi$ and order 0. ψ -distributions are unimodal with mode at $\theta = \mu$. Special cases of the ψ -distribution are the uniform, von Mises, wrapped Cauchy and cardioid distributions [9].

4. SIMILARITY MEASURE

In order to evaluate the ability of the models presented in section 3 to describe both individual subbands and the whole texture, similarity measures are required. Different metrics can be used based for instance on the L^1 or the L^2 norm (Euclidian distance). However, the Kullback-Leiber distance (KLD) or relative entropy [10] arises in many contexts as appropriate measures of the distance between two distributions.

4.1 KLD between two Gamma distributions

For the Gamma distribution, the Kullback-Leiber divergence [4] is defined by

$$\begin{aligned} D_{KLG}(f_j, f_q) &= (\beta_j - 1) \Psi(\beta_j) - \log(\alpha_j) - \beta_j \\ &\quad - \log \Gamma(\beta_j) + \log \Gamma(\beta_q) + \beta_q \log \alpha_q \\ &\quad - (\alpha_q - 1) (\Psi(\beta_j) + \log(\alpha_j)) + \frac{\alpha_j \beta_j}{\alpha_q} \end{aligned} \quad (4)$$

where $\Psi(\cdot)$ is a ψ linear distribution and $\Gamma(\cdot)$ is a Gamma distribution f_j and f_q are respectively the Gamma pdf for images j and q .

4.2 KLD between two von Mises distributions

The KLD von Mises distribution is given by

$$D_{KLM}(\rho_j, \rho_q) = \log \left(\frac{I_0(\kappa_j)}{I_0(\kappa_q)} \right) + (\kappa_j - \kappa_q \cos(\mu_j - \mu_q)) \frac{I_1(\kappa_j) - I_1(-\kappa_j)}{2I_0(\kappa_q)} \quad (5)$$

where I_1 denotes the modified Bessel function of the first kind and order 1 and ρ_j and ρ_q are respectively the von Mises distribution of the image j and q .

4.3 KLD of empirical orientation histograms

To compare our results with [8], we defined, D_{hist} , the KLD of the empirical distribution of the quantized angles between two images I_q and I_j

$$D_{hist}(j, q, n) = \sum_r P_n^j(r) \log \left(\frac{P_n^j(r)}{P_n^q(r)} \right). \quad (6)$$

4.4 Similarity measure for Ψ -distributions

As no analytical form of the Kullback-leiber divergence is available for Ψ -distributions, we use a L^1 distance between the distribution parameters μ, κ, ψ to compare two Ψ -distributions. This implies that we use a L^1 distance with the Gamma distribution under the linear-circular study.

5. MODEL EVALUATION

5.1 The indexing framework

The discrimination capabilities of both orientation and magnitude statistics are now evaluated within the indexing framework used in [7]. Indexing is performed on a texture database extracted from the VisTex database [12]. The latter is composed of 40 images of size 512×512. Each original image is divided into 4 subimages, resulting in a final database of 160 texture samples of size 256×256. Each original texture being considered as a single class, the

database thus contains 40 classes and 4 samples per class.

The hyperparameters of the distribution (linear or circular) are estimated by least squares.

When submitted to the indexing framework, every sample is processed using the steerable pyramid described in [1], which decomposes the image using a set of $Nor \times Nsc$, where Nor and Nsc denote the numbers of orientations and scales of the pyramid. We use 2 scales and 6 orientations, yielding 12 complex subband images on which statistical features are computed.

Two experiments are carried out. In the first one, we analyse the performance of the model composed of a Gamma distribution for magnitudes and a ψ -distribution for orientations. In the second one, the ψ -distribution is replaced by the von Mises distribution. These results are compared to the retrieval effectiveness obtained in [8].

5.2 Results

Experiment 1: This experiment uses Gamma and ψ -distributions. As the KLD divergence is not available for the ψ -distribution, the L^1 distance is chosen instead. The retrieval rate is presented in table 1. All parameters are normalized using the appropriate standard deviation.

Table 1: Retrieval rates obtained on 256x256 images patches using the Gamma (1) and ψ -distributions (3) respectively for magnitudes and orientations. The chosen metrics is the L^1 norm, computed on distribution hyperparameters.

ψ -distribution μ, κ, ψ	Gamma distribution β, α	Gamma and ψ -distribution
56.2%	68.6%	70.7%

Experiment 2: In this experiment based on the Gamma and von Mises distributions, the KLD can be applied. The expressions of the KLD are given in (8) and (9).

We observed that the numerical values of the Kullback-Leibler divergence on Gamma distributions are higher than those obtained on von Mises distribution (results not reported here). In order to combine both distributions in a common metric, a specific kernel [11] has thus to be used which performs a non-linear transformation of the metric value. Herein, we consider the exponential kernel

$$D_K = 1 - \exp(-aD)$$

where D is the KLD defined in section 3 and a is a constant, i.e. the adaptation, which was given the value 4 in the following.

In the case of empirical orientation histograms, the KLD is of the same order of magnitude as the KLD of Gamma distributions. In this case, we thus give the constant a the value 1.

Tables 2 and 3 present the indexing results obtained using this KLD.

Table 2: Retrieval rates obtained on 256x256 images patches using the Gamma distribution for magnitudes and the von Mises distribution for orientations. Distances between patches are computed using the parametric expressions of the KLD given by (4) and (5).

D_{KvM}	D_{KGam}	$D_{KvM} + D_{KGam}$
75.1%	83.2%	86.5%

Table 3: Retrieval rates obtained on 256x256 images patches using the Gamma distribution for magnitudes and the von Mises distribution for orientations. Distances between patches are computed using the parametric expression of the KLD for the Gamma distribution (4) and the non parametric one for orientation distributions (6).

D_{Khist}	D_{KGam}	$D_{Khist} + D_{KGam}$
74.5%	83.2%	83.4%

5.3 Discussion

Table 1 summarizes the overall retrieval rates obtained when using either the hyperparameters of the Gamma distribution, those of the ψ -distribution, or all hyperparameters together. We observe that performances increase when both magnitude information and orientation information are combined.

In table 2, we observe that the use of the KLD allows an increase of the discrimination rate from 68,5%, with distance L^1 , to 83,2% with the KLD. Moreover, as in the case of the first experiment, when we combine the KLD of the von Mises distribution with the KLD of the Gamma distribution, the retrieval rate (86.5%) is increased.

Table 3 shows the retrieval rate obtained for KLD with the same kernel as the one used in the second experiment. We see that the retrieval rate also increases when combining the Gamma distribution and the angular empirical distribution.

Finally, when comparing table 2 to table 3, it appears that the parametric expression of the KLD on von Mises distributions allows better results (86.5%) than the non parametric KLD (83.4%).

6. CONCLUSION

An evaluation was carried out according to an indexing experiment to evaluate the capabilities offered by circular statistics for modeling orientation data obtained by multiscale oriented decompositions of textures.

It is shown that, combined with magnitude statistics, multiscale orientation information allows a significant increase of the discrimination capability. Besides, the

superiority of parametric circular models upon empirical distributions is also shown.

Finally, these results show that the performances of the classification system rely both on the selected models and on the metrics used for discriminate textures. This lead us to choose a circular model, the von Mises pdf, for which the KDL is mathematically tractable and can be easily computed.

Future work will concern joint parametric models of orientation and magnitude data. New models which take in consideration the dependence between linear and circular data will thus have to be addressed. Spatial dependences of either magnitudes, orientations or both will also have to be considered in order to take the spatial structure of textures into account.

7. REFERENCES

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