COPULAS BASED MULTIVARIATE GAMMA MODELING FOR TEXTURE CLASSIFICATION

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ABSTRACT

This paper deals with texture modeling for classification or retrieval systems using multivariate statistical features. The proposed features are defined by the hyperparameters of a copula-based multivariate distribution characterizing the coefficients provided by image decomposition in scale and orientation. As it belongs to the multivariate stochastic models, the copulas are useful to describe pairwise non-linear association in the case of multivariate non-Gaussian density. In this paper, we propose the d-variate Gaussian copula associated to univariate Gamma densities for modeling the texture. Experiments were conducted on the VisTex database aiming to compare the recognition rates of the proposed model with the univariate generalized Gaussian model, the univariate Gamma model, and the generalized Gaussian copula-based multivariate model.

***Index Terms*—** Image texture analysis, Information retrieval, wavelet decomposition, Gamma distribution, Gaussian copula.

1. INTRODUCTION

For numerous application domains, texture analysis is used in the workflow of image filtering, classification, segmentation, indexing, and/or synthesis. The main issue is to provide a unified and well-founded model of homogenous textures. When dealing with statistical texture modeling, many authors proposed to make use real or complex wavelet transform in a multiorientation and multiscale scheme [1], [2]. This transformation which consists in decomposing an image into a set of oriented and scaled subbands captures the directionality, the structuredness, and coarseness of a texture. Consequently, approaches based on orientation and scale features imply to work in high-dimensional space. The topic concerning the selection of a stochastic model to characterize this augmented data-space is the main issue discussed in this paper.

Several works use marginal densities to characterize separately each subband. Indeed, subband coefficients can be described by univariate Gaussian (Gaud), generalized Gaussian (gGaud) or Gamma (Gamd) densities [3], [4], [5]. Some recent works propose to use circular statistics, i.e. statistics of circular data [15] or joint linear-circular stochastic model [16] to characterize each subband coefficients. All these representation leads to a simple and attractive approach, defined by a limited set of parameters, but they do not provide a complete statistical description of the texture images. Indeed, to take into account the interscale and intrascale statistical dependencies between subband coefficients it is necessary to develop a statistical multivariate framework. The main problem is that the direct multivariate extension of the above model, e.g. gGaud or Gamd, are generally not analytically defined [6], [7], [8]. Thus, alternative models such as sub-Gaussian model, alpha-stable, Gaussian scale mixture or other non-Gaussian stochastic fields for joint and marginal statistics has been studied in previous works [2], [9]. However, none of these models is very reliable due to high number of associated hyperparameters and due to the very high computational complexity for their estimate.

In this paper, we focus on the multivariate statistical modeling using the copulas theory [11], [12], [13]. Especially, we will use a copula as a starting point for constructing multivariate models for texture characterization. Copula models have become increasingly popular for multivariate modeling in many fields where the multivariate dependence is of great interest. Copulas are useful especially with non-Gaussian random variables and play an important role in developing a unified likelihood framework to analyze discrete or continuous stochastic processes. To develop a tractable multidimensional statistical models based on Gamd or gGaud, we address copula-based multivariate model which is able to describe the previous dependencies and the local structure inside each subband.

The paper is organized as follows. The next section provides the proposed statistical multivariate models for homogenous texture. A brief review and main properties of copulas is discussed followed by a description of the model based texture features. In section 3, experimental results are given to evaluate the retrieval performances of the proposed model.

2. PARAMETRIC TEXTURE MODELING IN THE WAVELET TRANSFORM DOMAIN

2.1 A brief review of copulas

A copula is a multivariate cumulative distribution function (cdf) defined on the d-dimensional unit hypercube

$[0,1]^d$ such that every marginal distribution is uniform on the interval $[0,1]$. With a help of copula one can easily combine univariate marginals into a multivariate distribution. Thanks to the Sklar theorem [11], the copula theory allows us to analyze the dependence structure of multivariate distribution separately without studying marginal distributions. Precisely, let $X = [X_1, \cdots, X_d]$ be a d-dimensional random vector with the cdf F and margins $F_1, \cdots, F_d$. The multivariate cdf is given by

$$F(x_1,\cdots,x_d) = P(X_1 \le x_1, \cdots, X_d \le x_d), \ \forall (x_1,\cdots,x_d) \in R^d \quad (1)$$

Then there exists a copula C such that

$$F(x_1,\cdots,x_d) = C(F_1(x_1),\cdots,F_d(x_d)). \quad (2)$$

Conversely, if C is a copula function and $F_1,\cdots,F_d$ are univariate cumulative distribution functions, then the function defined in (2) is a joint cdf with margins $F_1,\cdots,F_d$. This theorem demonstrates that joint distribution of a random vector of variables and the associated marginal distribution are necessary linked by a copula. Moreover if the function C is continuous and differentiable, then the copula density is given by

$$c(u_1,\cdots,u_n) = \frac{\partial^d C(u_1,\cdots,u_d)}{\partial u_1 \cdots \partial u_d}. \quad (3)$$

Given that distribution F and copula C are absolutely continuous, the joint density function f of X is given as follows

$$f(x_1,\cdots,x_d) = c(F_1(x_1),\cdots,F_d(x_d))\prod_{i=1}^{d} f_i(x_i) \quad (4)$$

where f_i is the probability density function of X_i and c is the density of the copula C defined in (3). More details about copula theory can be consulted in [13].

2.2 Proposed copula-based multivariate models

In this subsection we attempt to introduce a copula for characterizing simultaneously the marginal distributions, and the local structure of each subband. For this purpose, from each ith subband noted $\{w_i(m,n)\}$ we consider a $(2p+1)\text{x}(2q+1)$ window. We concatenate neighbors' pixel in a column vector W_i of size $N = (2p+1)(2q+1)$ as follows

$$W_i = [w_i(m-p,n-q),\cdots,w_i(m+p,n+q)]^T \quad (5)$$

Under the spatial homogeneity assumption of each subband, observations of W_i can be obtained by moving the window across the subband in an overlapping manner.
To describe also the statistical interband dependencies, we consider the following d-dimensional random vector $W = [W_1^T, W_2^T, \cdots, W_B^T]^T$ where B is the total number of bands, and each component W_i is the N-dimensional vector described in (5). Thus, thanks to copula approach the multivariate density of the $d-$dimensional vector W with $d = NB$ is expressed as in (4).

$$f_W(\omega_1,\cdots,\omega_d) = c(u_1,\cdots,u_d)\prod_{i=1}^{d} f_i(\omega_i) \quad (6)$$

with $u_1 = F_1(\omega_1),\cdots,u_d = F_d(\omega_d)$, F_i and f_i are respectively the univariate marginal pdf and cdf of W. Among a wide variety of copulas, we propose a Gaussian copula density defined as follows:

$$\forall u \in [0,1]^d, \ c(u,\Sigma) = |\Sigma|^{1/2} \exp\left[-\frac{\tilde{u}^T(\Sigma^{-1} - I)\tilde{u}}{2}\right] \quad (7)$$

with $\tilde{u}$ being a vector of normal scores such that $\tilde{u}_i = \Phi^{-1}(u_i)$, and Φ is the cdf of the normalized Gaussian distribution. The matrix I implies the $d-$dimensional matrix identity and Σ is the correlation matrix.
Note that the multivariate Gaussian pdf is a special case of (6) when all margins are univariate Gaussian. In this case the matrix Σ is the Pearson correlation matrix. If the margins are non-Gaussian, the (j,k)th element of Σ represents the linear correlation of two normal scores

$$r(j,k) = corr[\Phi^{-1}(F_j(W_j)), \Phi^{-1}(F_k(W_k))] \quad (8)$$

where Φ is the cdf of normal distribution $N(0,1)$, W_j and W_k are respectively the jth and the kth univariate component of W. According to the structure of the vector W, the correlation matrix Σ is a block matrix structured as follows

$$\Sigma = \begin{bmatrix} \Sigma_1 & \Sigma_{11} & \cdots & \Sigma_{1B} \\ \Sigma_{21} & \Sigma_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \Sigma_{B-1),B} \\ \Sigma_{B1} & \cdots & \Sigma_{B(B-1)} & \Sigma_B \end{bmatrix} \quad (9)$$

where Σ_i is the correlation matrix associated to ith subband and Σ_{ij} reflects the spatial crosscorrelation matrix between the ith and jth subbands. Moreover, since each band is homogenous, the diagonal matrices Σ_i are structured as a $(p+1)\times(p+1)$ block Toeplitz matrices with Toeplitz blocks as follows

$$\Sigma_i = \begin{bmatrix} \Sigma_0^i & \Sigma_{-1}^i & \cdots & \Sigma_{-p}^i \\ \Sigma_1^i & \Sigma_0^i & \ddots & \vdots \\ \vdots & \ddots & \ddots & \Sigma_{-1}^i \\ \Sigma_p^i & \cdots & \Sigma_1^i & \Sigma_0^i \end{bmatrix} \quad (10)$$

Each submatrix Σ_k^i is a $(q+1)\times(q+1)$ Toeplitz one as

$$\boldsymbol{\Sigma}_k^i = \begin{bmatrix} r_i(k,0) & r_i(k,-1) & \cdots & r_i(k,-q) \\ r_i(k,1) & r_i(k,0) & \ddots & \vdots \\ \vdots & \ddots & \ddots & r_i(k,-1) \\ r_i(k,q) & \cdots & r_i(k,1) & r_i(k,0) \end{bmatrix} \quad (11)$$

where the term $r_i(k,l)$ is the spatial correlation of the ith subband at the lags (k,l). Each matrix $\boldsymbol{\Sigma}_{ij}$ in (9) has the same structure as $\boldsymbol{\Sigma}_i$ and contains the spatial crosscorelation between the i th and j th subbands.

As mentioned in the previous section, it is well-known that a univariate gGaud [3] or Gamd [4] would be a reasonable statistical representation in the multi-resolution approach. These univariate models are simple to fit and capable of characterizing the first order statistics properties of the band coefficients and its magnitudes respectively. Using copula theory, multivariate versions of these models can be obtained as follows

Multivariate Gamma model

To model all bands coefficients magnitudes by a multivariate Gamma model we use equation (4) with a Gaussian density copula in (7) and the univariate Gamd defined by

$$\forall x \in R^+,\ f(x,\gamma,\theta) = x^{a-1} \frac{e^{-(b)}}{b^a \Gamma(a)} \quad (12)$$

where $a > 0$, is a shape parameter, and $b > 0$ is related to the scale distribution, and $\Gamma(.)$ is the *Gamma function*, defined as follows

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt\ ,\ \forall z > 0 \quad (13)$$

Multivariate Generalized Gaussian Model

The multivariate dGaud can be obtained using the Gaussian copula and the univariate gGaud as marginal. The univariate gGaud is given by

$$\forall x \in R, f(x,\alpha,\beta) = \frac{\beta}{2\alpha\Gamma(1/\beta)} e^{-(|x|/\alpha)^\beta} \quad (14)$$

where α is a shape parameter, and β is related to the scale of the distribution. This marginal modeling allows us to cover super Gaussian $(\beta < 2)$, Gaussian $(\beta = 2)$ and a sub gaussian $(\beta > 2)$ densities.

2.3 Parameter estimation and Feature extraction

In a parametric approach, the feature extraction step consists of estimating the model parameters. For multivariate copula-based model, the maximum likelihood criterion for the model in (4) requires two stages of estimation methods. The first stage consists in estimating the parameters of marginal distribution by the maximum likelihood estimator. In the second stage, these parameters are used to transform the data in the unit hypercube and involve a maximum likelihood estimation of copula matrix. If we consider the coefficients of different subbands as independent, the correlation matrix of the Gaussian copula is a block diagonal matrix as follows

$$\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Sigma}_1 & 0 & \cdots & 0 \\ 0 & \boldsymbol{\Sigma}_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \boldsymbol{\Sigma}_B \end{bmatrix} \quad (15)$$

Furthermore, the joint pdf associated to the ith subband is given by

$$f_{W_i}(\omega, \boldsymbol{\Sigma}_i, \boldsymbol{\varepsilon}_i) = c_{W_i}\left(F^i(\omega_1), \cdots, F^i(\omega_n), \boldsymbol{\Sigma}_i\right) \prod_{k=1}^{n} f^i(\omega_k, \boldsymbol{\varepsilon}_i) \quad (16)$$

where c_{W_i} is a Gaussian copula density parameterized by the matrix $\boldsymbol{\Sigma}_i$, f^i is the univariate marginal pdf of the ith subband characterized by the parameters $\boldsymbol{\varepsilon}_i$, and F^i is the related cdf. As a result, the signatures at each image constitute a vector $\boldsymbol{\Theta}$ which contains the marginal parameters $\boldsymbol{\varepsilon}_i$ and a subset of elements from the matrix $\boldsymbol{\Sigma}_i$ of each band.

3. EXPERIMENTAL RESULTS

To compare the performances of the presented texture models, we now use the benchmark proposed by Do and vetterli in the framework of texture retrieval [3]. The retrieval scheme is applied on a set of texture images obtained from the MIT Vision Texture (VisTex [13]) database. From each of these texture images of size 512x512 pixels, 16 subimages of 128x128 pixels are created. A test database of 640 texture images is thus obtained. The objective of our experiments is to compare the retrieval performances of the following models:

- **MGamdGC**: Multivariate Gamd with Gaussian copula.
- **MgGaudGC**: Multivariate gGaud with Gaussian copula.
- **UGamd**: Univariate Gamd [4].
- **UgGaud**: Univariate gGaud [3].

For this purpose we use the steerable pyramid decomposition proposed in [2], [3] with $Nsc = 2$ levels and $Nor = 6$ orientations. The univariate and multivariate generalized Gaussian models are used to fit only the real part of the steerable pyramid components. For the univariate approaches **UGamd** and **UGaud**, the marginal parameters of each band are estimated using the ML technique described respectively in [3] and [4]. For each model the estimated parameters are concatenated to form a feature vector

$$\boldsymbol{\Theta}_{UGamd} = [a_1, b_1, \cdots, a_B, b_B] \quad (17)$$

$$\boldsymbol{\Theta}_{UgGaud} = [\alpha_1; \beta_1; \cdots, \alpha_B; \beta_B] \quad (18)$$

where $B = Nor \times Nsc$ is the total number of bands. For the two multivariate models the feature extraction is carried out using the autocorrelation samples within each matrix $\boldsymbol{\Sigma}_i$. In these experiments, the parameters p and q are fixed such as $p = q = 1$. Thus, the matrix $\boldsymbol{\Sigma}_i$ contains only the following 4×1 vector $\boldsymbol{R}_i = [r_i(0,1), r_i(1,0), r_i(1,1), r_i(-1,1)]$. Finally, these statistics and the marginal parameters of each band are concatenated to obtain feature vectors $\boldsymbol{\Theta}_{MGamdGC}$ and $\boldsymbol{\Theta}_{MgGaudC}$ as follows

$$\boldsymbol{\Theta}_{MgGamdGC} = [\boldsymbol{R}_1, \cdots, \boldsymbol{R}_d, \boldsymbol{\Theta}_{UGamd}] \tag{19}$$

$$\boldsymbol{\Theta}_{MgGaudGC} = [\boldsymbol{R}_1, \cdots, \boldsymbol{R}_d, \boldsymbol{\Theta}_{UGaud}] \tag{20}$$

We note that the number of the extracted features for multivariate models is given by

$$N_\Theta = Nsc.Nor[.p(2q+2)+2] \tag{21}$$

In order to determine similarity between two images in the database, we use the normalized L1 distance between the feature vectors $\boldsymbol{\Theta}^1$ and $\boldsymbol{\Theta}^2$, i.e.,

$$d(\boldsymbol{\Theta}^1, \boldsymbol{\Theta}^2) = \sum_{k=1}^{N_\Theta} \left| \frac{\Theta^1(k) - \Theta^2(k)}{\sigma(k)} \right| \tag{22}$$

where $\sigma(k)$ refers to the standard deviation of the component k of all the feature vectors in the database.

In the retrieval stage a query image is any one in the database. The relevant images for each query are the other 15 images obtained from the same original 512x512 image. The performance of each model is evaluated only in the average percentage of retrieving relevant images. In Table 1, we provide a comparison of performances when the signatures are computed by the four previous methods. The maximum and minimum percentage rates were 73.4% and 83.4%. From this table we first observe that the proposed multivariate models improve the performances of the retrieval system than univariate models. Second, from these results the copula based multivarite Gamma model has about 3% improved retrieval rate than generalized Gaussian multivariate model.

	UgGaud	**UgGamd**	**MgGauGC**	**MgGamGC**
Average retrieval rate (%)	73.4375	77.9785	80.3625	**83.4543**
N_Θ	24	24	72	72

Table 1: Average retrieval rates (%) comparison

CONCLUSION

In this paper we have addressed the problem of texture image modeling in a multiorientation and multiscale scheme. A multivariate Gamma model using Gaussian copula is proposed and its performances are evaluated for texture indexing. The experiments results show that the multivariate model driven by copula presents performances in the retrieval system better than that of marginal modeling. However, the Gaussian copula used in this work is just a solution among others. As a perspective we intend in future works to use other copulas with more flexible dependence.

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