ROBUST DETECTION OF A SET OF OUTLIERS FOR IMAGE CHANGES BASED ON RERUNNING THE REGRESSION

Dong Sik Kim

School of Electronics and Information Engineering, Hankuk University of Foreign Studies, Gyonggi-do, 449-791, Korea

ABSTRACT

By comparing two images, which are captured with the same scene at different times, we can detect the image changes due to moving objects. To reduce the influence from the different intensity properties of the images, an intensity compensation scheme, which is based on the polynomial regression model, is employed. For an accurate detection of outliers alleviating the influence from a set of outliers, a simple technique that reruns the regression is employed. In this paper, the algorithm that iteratively reruns the regression is theoretically analyzed by observing the convergency of the estimates of the noise variance. Using an empirical compensation constant for the estimate is also proposed. The compensation enables the detection algorithm robust to the choice of thresholds for selecting outliers.

***Index Terms*—** Doubly truncated samples, outlier, regression.

1. INTRODUCTION

Suppose that two images contain the same scene, but are captured at different times and thus have non-corresponding objects. We call the objects the sets of outliers or the *outlier objects*. Such outlier objects are usually caused by different illumination conditions, the intensity saturation, specularity, shadows, or occlusions from moving objects of pedestrians, vehicles, or cloud. We can detect the outlier objects or the image changes by comparing the images. The surveillance system, which detects trespass, is an application of such techniques finding outlier objects. We can also detect a car on the road and find its license plate.

In this paper, the outlier objects are detected in the context of the *outliers*, which is widely used in the area of statistics [1], by formulating the *polynomial regression model* for the intensity compensation without prior knowledge of the camera parameters. Using the obtained regression function, we can compensate the intensity difference between images and reduce the influence from the intensity difference. Rerunning the regression [1, p. 162] after excluding the possible outliers that are selected in the previous stage can efficiently detect possible outliers at a low cost. However, it is difficult to describe the relationship between the detected outliers and the real outlier objects. Furthermore, the appropriate number of times for rerunning the regression is not known. In this paper, the rerunning approach is analyzed based on the notion of estimating the noise variance from the doubly truncated samples. We theoretically observe the convergence property of the estimate for the noise variance. The robust algorithm developed in this paper has the notion that the performance is insensitive to the selection of the thresholds. The proposed detection algorithm uses two thresholds, and is numerically analyzed by observing the robustness with respect to the thresholds. Note that the selection of the thresholds is not critical in the performance of the proposed algorithm.

This paper is organized in the following way. In Section 2, the outlier image model is formulated based on the polynomial regression for the intensity compensation. In Section 3, the approach that iteratively reruns the regression is theoretically analyzed by observing the convergency of the estimated noise variance. In Section 4, the robust performance of the detection algorithm is demonstrated for real images, and the paper is concluded in the last section.

2. OUTLIER IMAGE MODEL BASED ON THE POLYNOMIAL REGRESSION

In this section, an intensity compensation is performed based on the polynomial regression model to alleviate the intensity influence on detecting outlier objects.

Let u_i and v_i, for $i = 1, \ldots, m$, denote the pixel values in $\mathbf{R}$ for the reference and input images, respectively. For the intensity compensation consider a polynomial η given by

$$\eta(v; \boldsymbol{q}) := q_0 + q_1 v + \cdots + q_b v^b$$

for pixel values $v \in \mathbf{R}$, where $\boldsymbol{q}$ is a coefficient vector defined by $\boldsymbol{q} := (q_0, q_1, \ldots, q_b) \in \mathbf{R}^{b+1}$ with $b+1$ coefficients in $\mathbf{R}$. The *polynomial regression model* [1, p. 181] for the intensity compensation of input v_i with respect to the reference image u_i is now given by $u_i = \eta(v_i; \boldsymbol{q}^*) + \varepsilon_i$, for $i = 1, \ldots, m$, where $\boldsymbol{q}^* \in \mathbf{R}^{b+1}$ is a coefficient vector. Here, we suppose that ε_i are independent, and identically distributed random variables with mean zero and variance σ^2, where σ is a nonnegative constant. We call σ^2 the *noise variance*. By minimizing the sum of squares $\sum_{i=1}^{m} [u_i - \eta(v_i; \boldsymbol{q})]^2$ with respect to $\boldsymbol{q}$, we obtain a least square estimate $\boldsymbol{q}^o$ of $\boldsymbol{q}^*$ [1, (2.9)]. Hence, a compensated image is given by $\eta(v_i; \boldsymbol{q}^o)$. Note that $\eta(v; \boldsymbol{q}^o)$ should be a monotonically increasing function of v in order to prevent the inversion of the intensity in the compensated images [2].

We now consider an *additive outlier image model* for a mathematical observation on the influence of outliers [2]. Let $r_i \in \mathbf{R}$ denote the outlier image to be added. The outlier image model is then given by

$$\bar{u}_i = \eta(v_i; \boldsymbol{q}^*) + r_i + \varepsilon_i, \text{ for } i = 1, \ldots, m.$$

Let Λ denote the index set as $\Lambda := \{1, \ldots, m\}$, and let $\Omega^* \subset \Lambda$ denote the index set of the outlier objects, of which size is $n\ (< m)$. For a simple outlier image model, we suppose that $r_i = r$ if the ith pixel is an outlier, i.e., $i \in \Omega^*$, and $r_i = 0$ otherwise, where constant r is the pixel value of the outlier objects. Here, we call r/σ the object to noise ratio. We test the robustness of the detection algorithm by changing the ratio.

3. RERUNNING THE REGRESSION

To find outliers in the regression analysis, a measure is used to detect *possible outliers*, which will be examined for the final outliers [1, ch. 8]. The measure is defined as the residual $[\bar{u}_i - \eta(v_i; \boldsymbol{q}^*)]$, which implies the distance of the observation from its predicted value. To make the measure insensitive to the noise variance σ^2, we consider a scaled residual ρ_i, which is defined by $\rho_i := [\bar{u}_i - \eta(v_i; \boldsymbol{q}^*)]/\sigma$. For a positive constant T as a threshold, we presume that the pixels satisfying $|\rho_i| > T$ are possible outliers for detecting the outlier objects. In other words, if the magnitude of the image difference is greater than $T\sigma$, then the corresponding pixel is treated as a possible outlier. Rosin [3] proposed using $T = 3$ for the unimodal case.

3.1. Iterative Regression Analysis

In order to calculate the residual ρ_i for detecting possible outliers, we should obtain estimates of $\boldsymbol{q}^*$ and σ^2 using the samples $(\bar{u}_i, v_i)$ when u_i are not practically available. Hence, it is necessary to develop techniques that examine the influence of the outlier objects. A simple method is rerunning the regression after excluding the possible outliers that are selected in the previous stage [1],[4].

We now analyze the approach that is based on rerunning the regression by observing the estimates of the noise variance. Letting $\Omega \subset \Lambda$ denote an index set of possible outliers, an estimate of $\boldsymbol{q}^*$ is defined by

$$\boldsymbol{q}^o(\Omega) := \arg\min_{\boldsymbol{q}} \sum_{\ell \in \Lambda \setminus \Omega} [\bar{u}_\ell - \eta(v_\ell; \boldsymbol{q})]^2 .$$

We can also apply a constrained monotone regression in obtaining $\boldsymbol{q}^o(\Omega)$ [2]. For an estimation of the noise variance σ^2 similarly to [5], let us consider an estimate $s^2(\Omega)$, which is defined by

$$s^2(\Omega) := \min_{\boldsymbol{q}} \frac{1}{m(\Omega)} \sum_{\ell \in \Lambda \setminus \Omega} [\bar{u}_\ell - \eta(v_\ell; \boldsymbol{q})]^2$$

if $m(\Omega) \neq 0$, and $s^2(\Omega) := 0$ otherwise, where $m(\Omega)$ is the size of $\Lambda \setminus \Omega$. Let a positive constant μ denote the *compensation constant* for the estimate $s^2(\Omega)$ to compensate the possible bias from the estimate. The compensated estimate is then denoted by $[\mu s(\Omega)]^2$. We now introduce the algorithm that iteratively applies the regression analysis as follows:

Iterative Regression Algorithm
0) Set positive constants D, ϵ, and μ. Letting $k = 0$, choose the initial set as $\Omega_D^{(0)} = \emptyset$.
1) Calculate the residuals

$$\rho_i^{(k+1)} := \frac{\left[\bar{u}_i - \eta\left(v_i;\ q^o\left(\Omega_D^{(k)}\right)\right)\right]}{\mu s\left(\Omega_D^{(k)}\right)}$$

for $i = 1, \ldots, m$, and update the set from

$$\Omega_D^{(k+1)} := \left\{ i \in \Lambda \ :\ \left|\rho_i^{(k+1)}\right| > D \right\}. \tag{1}$$

2) If $\left| s^2\left(\Omega_D^{(k+1)}\right) - s^2\left(\Omega_D^{(k)}\right) \right| < \epsilon$, then stop. Otherwise, $k \leftarrow k + 1$ and goto Step 1).

3.2. Convergence of the Iterative Regression Algorithm

We now observe the convergence of the iterative regression algorithm by observing the estimate of the noise variance. For a positive constant D, let $B^{(k)} \in \mathbf{R}$ denote the kth threshold defined as $B^{(k)} := D\mu s\left(\Omega_D^{(k)}\right)$, for $k = 0, 1, \ldots$. The $(k+1)$th set $\Omega_D^{(k+1)}$ of (1) can then be rewritten by

$$\Omega_D^{(k+1)} = \left\{ i \in \Lambda \ :\ \left|\bar{\rho}_i^{(k+1)}\right| > B^{(k)} \right\}$$

for $k = 0, 1, \ldots$, where the residuals $\bar{\rho}_i^{(k+1)}$ are defined by

$$\bar{\rho}_i^{(k+1)} := \left[\bar{u}_i - \eta\left(v_i; \boldsymbol{q}^o\left(\Omega_D^{(k)}\right)\right)\right].$$

For the $k = 0$ case, we have the following relationship:

$$\begin{aligned} & s^2\left(\Omega_D^{(0)}\right) \\ & \quad \geq \frac{1}{m\left(\Omega_D^{(1)}\right)} \sum_{\ell \in \Lambda \setminus \Omega_D^{(1)}} \left[\bar{u}_\ell - \eta\left(v_\ell; \boldsymbol{q}^o\left(\Omega_D^{(0)}\right)\right)\right]^2 \\ & \quad \geq s^2\left(\Omega_D^{(1)}\right). \end{aligned}$$

Let $\Gamma_D^{(k+1)} := \left\{ i \in \Lambda \ :\ \left|\bar{\rho}_i^{(k+2)}\right| > B^{(k)} \right\}$, for $k = 0, 1, \ldots$. If $s^2\left(\Omega_D^{(k+1)}\right) \leq s^2\left(\Omega_D^{(k)}\right)$ holds for $k = 0, 1, \ldots$, then, from the definition of $B^{(k)}$, $B^{(k+1)} \leq B^{(k)}$ holds, and consequently $\Omega_D^{(k+2)} \supset \Gamma_D^{(k+1)}$ holds. Hence, we have the following relationship:

$$\begin{aligned}
&\frac{1}{m\left(\Gamma_D^{(k+1)}\right)} \sum_{\ell \in \Lambda \setminus \Gamma_D^{(k+1)}} \left[\bar{u}_\ell - \eta\left(v_\ell; \boldsymbol{q}^o\left(\Omega_D^{(k+1)}\right)\right)\right]^2 \\
&\quad \geq \frac{1}{m\left(\Omega_D^{(k+2)}\right)} \sum_{\ell \in \Lambda \setminus \Omega_D^{(k+2)}} \left[\bar{u}_\ell - \eta\left(v_\ell; \boldsymbol{q}^o\left(\Omega_D^{(k+1)}\right)\right)\right]^2 \\
&\quad \geq s^2\left(\Omega_D^{(k+2)}\right). \qquad (2)
\end{aligned}$$

We also have the following relationship:

$$\begin{aligned}
&s^2\left(\Omega_D^{(k+1)}\right) \\
&\quad \geq \frac{1}{m\left(\Gamma_D^{(k+1)}\right)} \sum_{\ell \in \Lambda \setminus \Gamma_D^{(k+1)}} \left[\bar{u}_\ell - \eta\left(v_\ell; \boldsymbol{q}^o\left(\Omega_D^{(k+1)}\right)\right)\right]^2. \qquad (3)
\end{aligned}$$

Therefore, from (2) and (3), we have

$$\left[\mu s\left(\Omega_D^{(k)}\right)\right]^2 \geq \left[\mu s\left(\Omega_D^{(k+1)}\right)\right]^2, \text{ for } k = 0, 1, \cdots$$

which implies that the compensated noise variance monotonically decreases and converges to a limit as $k \to \infty$ since $[\mu s(\Omega)]^2 \geq 0$. The limit of the estimate, however, is not known. In fact, for the case of small thresholds, e.g., $D < 2.0$, when $\mu = 1$, we numerically have $s^2\left(\Omega_D^{(k)}\right) \to 0$ in general. In other words, almost all pixels are selected as the possible outliers, which is meaningless. Even though we have other limits, we cannot guess whether the estimate is close enough or not to the real noise variance σ^2.

3.3. Compensated Estimate Using Doubly Truncated Samples

By using a part of the observation, we can estimate the real noise variance under special cases, such as $r = 0$. Kim and Lee [2] proposed using a constant, which is calculated with a normal distribution function. Here, we generalize the constant for a given distribution function F. We consider a doubly truncated variance, which is defined by $P_D^{-1} \int_{-D}^{D} z^2 dF(z)$, where F is a distribution function with mean zero, and $P_D := \int_{-D}^{D} dF$. Here, we assume that $P_D \neq 0$. We now define a further general constant λ_D for a given D as follows:

$$\lambda_D := \frac{P_D^{-1} \int_{-D}^{D} z^2 dF(z)}{\int z^2 dF(z)}$$

which satisfies $\lambda_D < 1$, and depends on the shape of F. Supposing that the distribution function of the noise ε_i is equal to F, let a set Ω_D be defined as $\Omega_D := \{i \in \Lambda \ : \ |\varepsilon_i/\sigma| > D\}$. The estimate of the noise variance based on the doubly truncated samples in Ω_D satisfies [2, Appendix]

$$E\{s^2(\Omega_D)\} \leq [1 - (1 - P_D)^m]\, \sigma^2 \lambda_D. \qquad (4)$$

Here, $\sigma^2 = \int z^2 dF(z)$. Since $\lambda_D < 1$ and $[1 - (1 - P_D)^m] < 1$, we have $E\{s^2(\Omega_D)\} < \sigma^2$. Hence, the estimate $s^2(\Omega_D)$ is a biased estimate of the real noise variance σ^2.

For a given D, let us use λ_D to choose the compensation constant as $\mu = \lambda_D^{-1/2}$. Let $\Omega_D^{(\infty)}$ denote a limit of (4) as $k \to \infty$. If the resultant vector is equal to the real one, i.e., $\boldsymbol{q}^o\left(\Omega_D^{(\infty)}\right) = \boldsymbol{q}^*$, and $\Omega_D^{(\infty)} = \Omega_D$, then the compensated estimate satisfies

$$\frac{s^2\left(\Omega_D^{(\infty)}\right)}{\lambda_D} = \frac{1}{\lambda_D \cdot m\left(\Omega_D^{(\infty)}\right)} \sum_{\ell \in \Lambda \setminus \Omega_D^{(\infty)}} \varepsilon_\ell^2 \qquad (5)$$

for $r = 0$. From (4), the expectation of (5) is approximately σ^2 for large m. Hence, the compensated estimate by using λ_D could be a necessary condition for finding an unbiased estimate when $r = 0$. Note that the estimate is independent of the threshold D, which is of importance in designing a robust detection algorithm since the performance is not sensitive to the selection of the threshold D. Furthermore, estimating the real noise variance is also important for the unimodal case [3].

3.4. Empirical Compensation Constant

In practical images for the detection of the outlier objects, acquiring the distribution function F of the noise is not easy. Hence, we propose using an *empirical compensation constant*, which is calculated from the observation $(\bar{u}_i, v_i)$. Let a possible outlier set $\hat{\Omega}_D$ be defined by

$$\hat{\Omega}_D := \left\{ i \in \Lambda \ : \ \left| \frac{\bar{u}_i - \eta(v_i; \boldsymbol{q}^o)}{s(\emptyset)} \right| > D \right\}$$

where $\boldsymbol{q}^o$ is an unbiased estimate of $\boldsymbol{q}^*$ defined by $\boldsymbol{q}^o := \boldsymbol{q}^o(\emptyset)$. The *empirical compensation constant* is then given by $\mu = \hat{\lambda}_D^{-1/2}$, where $\hat{\lambda}_D$ is an empirical constant defined by

$$\hat{\lambda}_D := \frac{\left[m(\hat{\Omega}_D)\right]^{-1} \sum_{\ell \in \Lambda \setminus \hat{\Omega}_D} [\bar{u}_\ell - \eta(v_\ell; \boldsymbol{q}^o)]^2}{s(\emptyset)}.$$

4. OUTLIER OBJECT DETECTION

Using the estimates $\boldsymbol{q}^o(\Omega_D^{(\infty)})$ and $s^2(\Omega_D^{(\infty)})/\hat{\lambda}_D$, which are obtained by the iterative regression algorithm with the compensated estimate for a fixed D, we calculate the modified residuals:

$$\rho_i^{(\infty)} := \frac{\left[\bar{u}_i - \eta\left(v_i; \boldsymbol{q}^o\left(\Omega_D^{(\infty)}\right)\right)\right]}{\hat{\lambda}_D^{-1/2} s\left(\Omega_D^{(\infty)}\right)}$$

for $i = 1, \ldots, m$. We then detect the final possible outliers by comparing the residuals with a fixed threshold T (> 0), which can be different from D. In other words, the samples that satisfy $|\rho_i^{(\infty)}| > T$ are regarded as the possible outliers. Note that D is used for obtaining an estimate of the noise variance and T is used for detecting possible outliers. In order to remove possible outliers that form small objects or holes in an object, we apply the morphological filtering technique [6, ch.9] to the final possible outliers using a square structuring element.

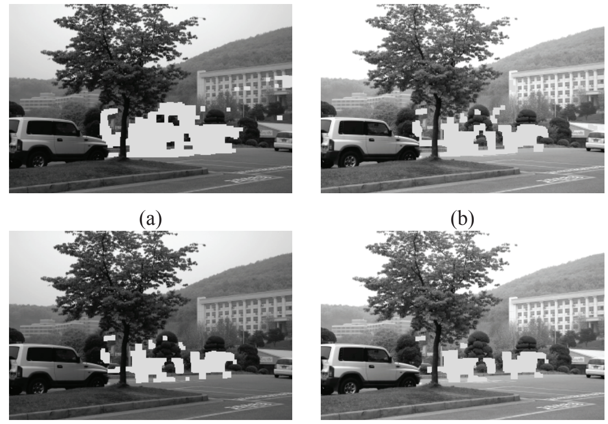
(a) (b)

Fig. 1. Images for numerical experiments. (a) Reference image with the outlier object (bus). (b) Input image (Image A).

In order to evaluate the performance of the iterative regression algorithm, which is based on the empirical compensation constant $\hat{\lambda}_D$, we use real images as in Fig. 1. Fig. 1(a) is a reference image having an outlier object (bus) and Fig. 1(b) is an input image. By using the empirical compensation constant we can obtain a stable estimate for the noise variance, which is insensitive to the choice of the threshold D. The detected results are shown in Figs. 2 and 3 for $D = 2.5$ and 4.0. Here, we use two images, Images A ((a) and (c)) and B ((b) and (d)), of which intensity conditions are different, and the opening and closing operation having the structuring element size of 5×5. In Fig. 2, the compensation constants λ_D are calculated from the normal distribution function. We can notice from Fig. 2 that the shapes of the detected outlier objects are slightly different depending on the values of thresholds D as well as the input images. However, for the case of the empirical compensation constant $\hat{\lambda}_D$ as in Fig. 3, the detected shapes are uniform. Hence, we can conduct a robust detection of outlier objects independently of the noise variances of input images.

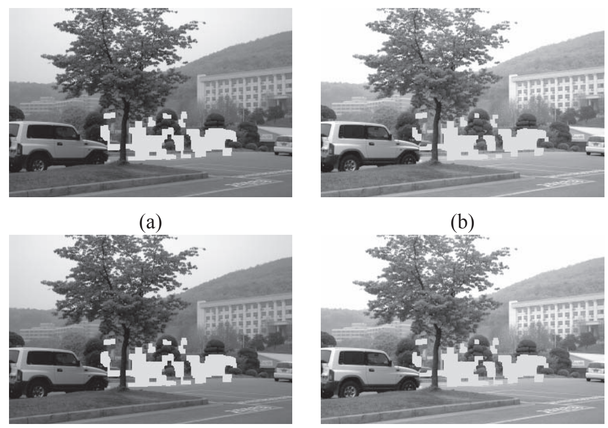
5. CONCLUSION

The polynomial regression model and residuals are used to detect outlier objects. Rerunning the regression can successfully detect the outlier objects at low cost. In this paper, an algorithm that iteratively reruns the regression is theoretically analyzed by observing the convergence property of the estimates of the noise variance. Using an empirical compensation constant for the threshold is proposed to devise a robust detection algorithm to the choice of thresholds for selecting outliers.



(a) (b) (c) (d)

Fig. 2. Input images with detected outlier object based on λ_D that is calculated with the normal distribution ($T = 3.3$). (a),(b) $D = 2.5$. (c),(d) $D = 4.0$.



(a) (b) (c) (d)

Fig. 3. Input images with detected outlier object based on the empirical compensation constant $\hat{\lambda}_D$ ($T = 1.5$). (a),(b) $D = 2.5$. (c),(d) $D = 4.0$.

6. REFERENCES

[1] A. Sen and M. Srivastava, *Regression Analysis, Theory, Methods, and Applications*, Springer-Verlag, NY, 1990.

[2] D. S. Kim and K. Lee, "Block-coordinate gauss-newton optimization and constrained monotone regression for image registration in the presence of outlier objects," *IEEE Trans. Image Processing*, vol. 17, no. 5, pp. 769–810, May 2008.

[3] P. L. Rosin, "Unimodal thresholding," *Pattern Recognition*, vol. 34, no. 11, pp. 2083–2096, 2001.

[4] A. Atkinson and M. Riani, *Robust Diagnostic Regression Analysis*, Springer, NY, 2000.

[5] D. A. Belsley, E. Kuh, and R. E. Welsch, *Regression Diagnostics: Identifying Influential Data and Sources of Collinearity*, Wiely, NY, 1980.

[6] R. C. Gonzalez and R. E. Woods, *Digital Image Processing, 2nd ed.*, Prentice Hall, NY, 2002.