

# APPROXIMATING 3D SHAPE USING BEZIER SURFACE

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## ABSTRACT

Estimating 3D shape of the object is an important research topic in the area of Computer Vision, with a wide range of applications. This paper introduces a new method for Focused-Based Passive Methods (like SFF) to approximate the 3D shape of the object using Bezier surface. The discrete nature of image sampling results in the loss of information between two consecutive images. Conventional approximation methods optimize or interpolate focus values. We have suggested interpolating depth values instead of focus value over a small patch on the object surface. The method approximates the surface more accurately and also reduces any noise caused by focus measure. The proposed method is tested and analyzed to demonstrate the effectiveness against traditional methods.

**Index Terms**— Shape from Focus, Bezier Surface, 3D Shape Approximation, Passive Methods.

## 1. INTRODUCTION

The 3D Shape Recovery methods are broadly divided Active Methods and Passive Methods. The Passive Methods include Shape from Shading, Shape from Motion (Motion Parallax), Stereo Vision, Depth from Focus, Depth from Defocus, Shape from Focus and Shape from Defocus (etc) are included.

A lot of research has been done on the image focus analysis to automatically focus the imaging system, or to obtain the sparse depth information from the observed scene. An important advantage of shape from focus or depth from focus is that, unlike stereo and motion, it is not confronted with correspondence problem; as a result it is computationally efficient, requiring only the use of simple focus measure operators.

In Shape from Focus (SFF) methods a sequence of images is used, taken by a single camera at different focus levels, to compute the depth of the object in the scene. Then, the entire image sequence is searched to find the best focused image frame for a particular point in the image space; and, the camera parameter settings for that image

frame are used to compute the distance of corresponding object point by using the Lens-Formula (1).

$$\frac{1}{f} = \frac{1}{\Delta_o} + \frac{1}{\Delta_i} \quad (1)$$

Where  $f$  is focal length,  $\Delta_o$  and  $\Delta_i$ , are the object and image distance from the lens, respectively.

SFF can be implemented through a variety of techniques including traditional methods, neural networks, dynamic programming etc, but all these techniques start with the estimation of depth map using focus measures.

A Focus Measure (FM) is defined as a quantity for locally evaluating the sharpness of a pixel, and FM operator calculates the best focused point in the image. Laplacian, Modified Laplacian, Sum of Modified Laplacian (SML), Tenenbaum, Gray Level Variance (GLV), Mean Method Focus Measure, Curvature Focus Measure, and M2 are the famous focus measures.

The discrete nature of image sampling results in the loss of information between two consecutive images. Yun and Choi [4] have summarized various approximation techniques. Shree K. Nayar [6] proposed the use of Gaussian Interpolation to get the focus values in between the image frames and the method is known as SFF.TR. Yun and Choi found that the SFF.TR does not consider the fact that an image of 3D object is also 3D in image space. Subbarao and Choi [5] proposed a new concept they refer to as SFF.FIS, applied on SML results and are based on planar surface approximations. Furthermore, Ahmad and Choi [3] proposed the use of DP on SML results for handling the computational complexity of FIS

In this paper we have proposed a new SFF method to approximate the 3D shape of the object using Bezier surface. An initial depth map is computed by using SML and then the surface is approximated by interpolating depth values using Bezier surface. The proposed method is then experimented and its performance is evaluated, by using synthetic and real images of objects. Comparative analysis demonstrates the effectiveness of the proposed method. Section 2 gives details of Image-Acquisition-System. Section 3 describes the proposed method. Section 4 gives the results of shape reconstruction of the objects and their comparisons with previous methods.

## 2. IMAGE ACQUISITION SYSTEM

This paper introduces a new Method for the 3D shape recovery in SFF methods. For experiments we have used three different objects (simulated cone, coin and LCD-filter). The simulated cone images are generated by computer simulations. The details of simulated cone images are given in [3]. The Coin images are the magnified images of Lincoln's head at the back of (US) One-Cent coin. The LCD images are microscopic images of LCD-TFT color filter. These images are obtained by microscopic control system called as MCS. The system consists of a personal computer with frame grabber board (Matrox Meteor-II) attached to a CCD camera (SAMSUNG CAMERA SCC-341) which is mounted on a microscope (NIKON OPTIPHOT-100S). Software in the computer acquire images by controlling the lens position through a stepper-motor driver (MAC 5000) having a 2.5 nm step length. All the images are taken by varying the object plane by ' $\Delta_o$ ', and are stored in a sequence on every step such that (a) the object moves towards (or away from) the lens assuring that the complete object is first defocused then gradually it focuses (on every point) and then it is again completely defocused, (b) there is no magnification when the images are taken [2]. Figure 1 shows sample images in the image stack for simulated cone, coin and LCD-TFT.

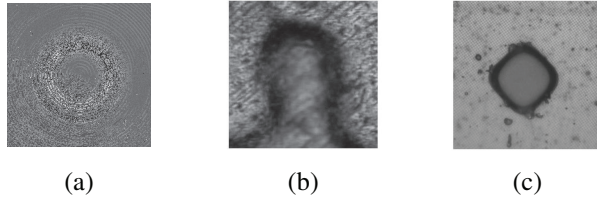


Figure 1. Sample images of (a) Simulated Cone, (b) Coin, and (c) LCD-TFT color filter

## 3. SHAPE APPROXIMATION USING BEZIER SURFACE

At first, we used Sum of Modified Laplacian (SML) to compute the focus value of the each pixel in the image stack. The Laplacian operator, being a point and symmetric operator, is suitable for accurate shape recovery. If we let  $I(i,j)$  be an image frame, then Laplacian of the image is obtained by adding second derivatives in the x- and y-directions, such that

$$\nabla^2 I(i,j) = \frac{\partial^2 I}{\partial i^2} + \frac{\partial^2 I}{\partial j^2} \quad (2)$$

If the image has rich textures with high variability at each pixel, SML can be calculated using (3), for a single pixel. In order to improve the robustness for weak-textured images, Nayar and Nakagawa [6] presented a Laplacian of

the image at  $(i,j)$  as the sum of values of (2) in a local window and called it as SML.

$$\nabla_{SML}^2 I(i,j) = \sum_{p(i,j) \in U(i_0,j_0)} \left( \left( \frac{\partial^2 I}{\partial i^2} \right)^2 + \left( \frac{\partial^2 I}{\partial j^2} \right)^2 \right) \quad (3)$$

The depth map is computed by finding the frame number  $[d_{i,j}]$  of the maximum value of the SML for each pixel, given by (4)

$$[d_{i,j}] = \arg[\max(\nabla_{SML}^2 I(i,j))] \quad (4)$$

After the depth map is estimated we optimize it using the Bezier Surface.

A Bezier surface is defined by a two-dimensional set of control points,  $d_{\alpha,\beta}$  where  $\alpha$  is in the range of 0 and  $g$ , and  $\beta$  is in the range of 0 and  $h$ , with  $g+1$  rows and  $h+1$  columns, defined by the equation (8)

$$\delta(u,v) = \sum_{\alpha=0}^g \sum_{\beta=0}^h B_{g,\alpha}(u) B_{h,\beta}(v) d_{\alpha\beta} \quad (5)$$

Where  $B_{g,\alpha}(u)$  and  $B_{h,\beta}(v)$  are the  $\alpha^{\text{th}}$  and  $\beta^{\text{th}}$  Bezier basis functions in the  $u$  and  $v$  directions, respectively and ' $\delta(u,v)$ ' is the Bezier surface. A Bezier surface is of degree  $(g,h)$  and has  $(g+1, h+1)$  control points. The surface passes through the control points only, at the four corners of the control net  $d_{0,0}$ ,  $d_{g,0}$ ,  $d_{g,h}$ , and  $d_{0,h}$ , and lies in the convex hull defined by its control points. Bezier surface is also affine invariant.

The Bezier basis functions  $B_{g,\alpha}(u)$  and  $B_{h,\beta}(v)$  are nonnegative for all  $g, h, i, j$  and  $u, v$  in the range of  $(0,1)$ , and are defined by (6) and (7) The sum of all  $B_{g,\alpha}(u)$  and  $B_{h,\beta}(v)$  is 1 for all the value of  $u$  and  $v$  in the range of 0 and 1. The basis functions are defined by the following equations.

$$B_{g,\alpha}(u) = \frac{g!}{\alpha!(g-\alpha)!} u^\alpha (1-u)^{g-\alpha} \quad (6)$$

$$B_{h,\beta}(v) = \frac{h!}{\beta!(h-\beta)!} v^\beta (1-v)^{h-\beta} \quad (7)$$

Since  $B_{g,\alpha}(u)$  and  $B_{h,\beta}(v)$  are degree ' $g$ ' and degree ' $h$ ' functions, therefore Bezier Surface of degree  $(g,h)$  will be obtained. As the parameters of  $g$  and  $h$  are in the range of 0 and 1, hence the Bezier surface maps the unit square to a rectangular surface patch.

A pixel is selected along with its eight-neighbors, and these nine-points are taken as the control points. The matrix representation of Bezier surface as a Cartesian product of two curves obtained by 2<sup>nd</sup> degree Bernstein Polynomial is given by.

$$\delta(u, v) = \begin{bmatrix} u^2 & u & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} d_{i-1,j-1} & d_{i-1,j} & d_{i-1,j+1} \\ d_{i,j-1} & d_{i,j} & d_{i,j+1} \\ d_{i+1,j-1} & d_{i+1,j} & d_{i+1,j+1} \end{bmatrix} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v^2 \\ v \\ 1 \end{bmatrix} \quad (8)$$

The corrected depth value of the pixel defined by the Bezier surface is taken as the final depth value. The process is repeated for all the pixels and finally the Bezier-Depth-Map is computed, which represents the 3D shape of the object.

$$[d_{i,j}^{NEW}] = \delta(\frac{1}{2}, \frac{1}{2}) \quad (9)$$

#### 4. RESULTS AND DISCUSSION

In this paper we used Bezier surface to approximate the 3D shape of the object, which is applied to the depth map obtained by SML.

Let us consider an object point (45, 45) on the Simulated Cone, as shown in figure 3(a) with its eight-neighbors. The initial depth obtained by using SML was [26 26 26; 27 28 28; 27 27 28]. These nine-points are taken as the control points for computing the Bezier surface, as shown in the Figure 3(b). The corrected depth values for the point (45,45) (after applying the Bezier surface are [26 26.75 27; 26 27.19 27.25; 26 27.5 28]. The center value in Figure 3(b) (i.e. 27.19) is taken as the corrected depth value for that point. The process is repeated for all the points in the depth map.

Figure 5(a) and 5(b) shows the simulated cone by SFF.TR and by SFF.DP, respectively. The Simulated cone obtained by the Bezier surface is shown in Figure 5(c).

We used results of SFF.TR and SFF.DP to compare with our method using the root mean square error (RMSE) and correlation. We found that RMSE is lowest for the proposed method whereas correlation coefficient is the highest as shown in Table I and Figure 3 and Figure 4.

Method	RMSE	Correlation
SFF.TR	6.6918	0.9655
SFF.DP	6.6132	0.9698
<b>SFF.BS</b>	<b>6.5977</b>	<b>0.9704</b>

Table I: Comparisons of Proposed Method with different methods for Simulated Cone.

Figure 6 shows the depth maps for the Coin and LCT-TFT, using proposed method (SFF.BS), SFF.TR and SFF.DP.

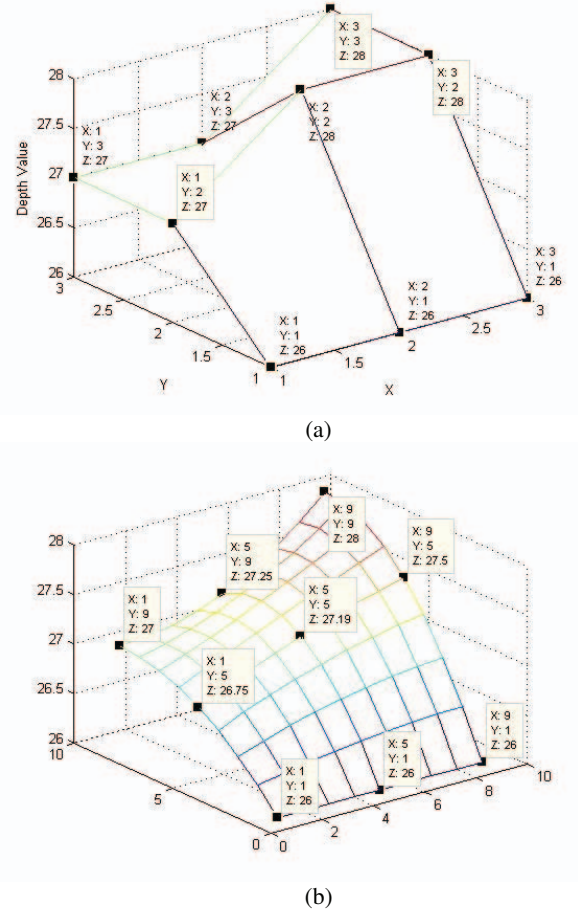


Figure 2. (a) Depth Map of point (45, 45) with its eight-neighbors using SML, (b) Bezier surface approximation of point (45, 45).

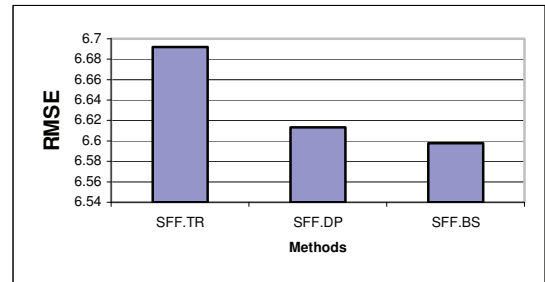


Figure 3. Comparison of RMSE of Different Methods.

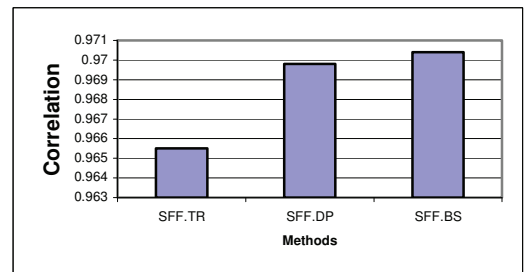


Figure 4. Comparison of Correlation of Different Methods.

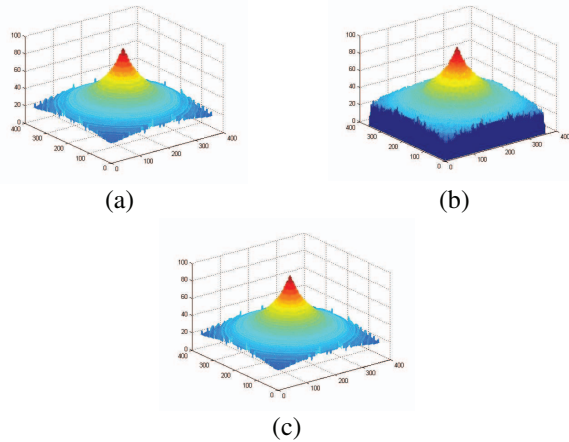


Figure 5. Depth Map of Simulated Cone using (a) SFF.TR, (b) SFF.DP, and (c) Proposed Method (Bezier Surface).

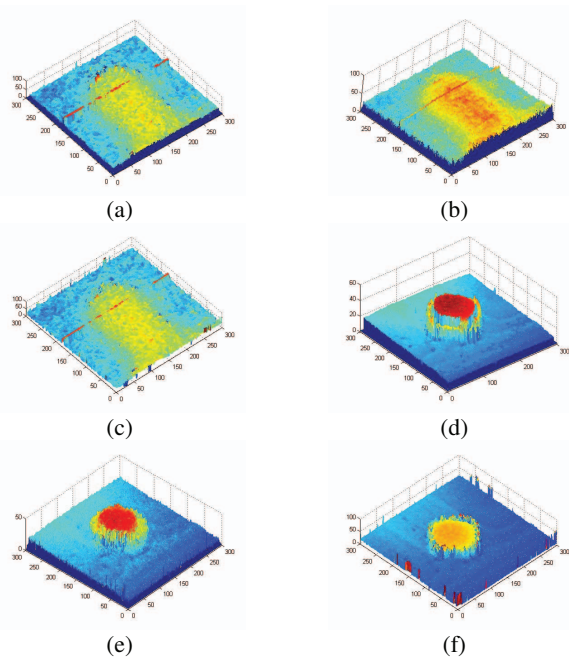


Figure 6. 3D Representation of Coin and LCT-TFT. (a & d) by SFF.TR, (b & e) by SFF.DP, (c & f) by Proposed Method SFF.BS.

## 5. CONCLUSION

In this paper we have proposed a new approximation method for Shape-from-Focus using Bezier surface. Initially, the depth map is computed using Sum of Modified Laplacian. Then each point on the depth map along with its eight-neighbors is taken and Bezier surface approximation is applied to it. The corrected depth of each point is taken as new depth map to represent the shape of the object. We have used RMSE and correlation to compare the proposed method with previous methods (SFF.TR and SFF.DP). The proposed method shows good results and it was found that the RMSE of the proposed method was less than previous methods, whereas, the correlation was better.

In Shape from Focus the discrete nature of image sampling results in the loss of information between two consecutive frames. This loss of information is carried out to next stage of shape approximation. The proposed method not only reduced this effect significantly, but also reduced noise (false detection of depth) caused by the focus measure. The results have shown noteworthy improvement by proposed method.

We computed the focus value of object point (45,45) with neighborhood of window size  $5 \times 5$  using Matlab 7.0.1 on Pentium-IV machine. The complexity of Bezier surface approximation varies with the selection of degree and length of parameters of polynomials. We applied 2nd order polynomials and the proposed algorithm iterates  $X \times Y$  times, where as the conventional methods search the whole focus volume i.e. they execute  $n(X \times Y)$  times, where  $n$  is the number of images in the sequence. Hence, the overall complexity of the proposed approach is comparable with existing methods.

## 6. ACKNOWLEDGEMENT

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